River monitoring from satellite radar altimetry in the Zambezi River Basin

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Abstract

Satellite radar altimetry can be used to monitor surface water levels from space. While current and past altimetry missions were designed to study oceans, retracking the waveforms returned over land allows data to be retrieved for smaller water bodies or narrow rivers. In this study, retracked Envisat altimetry data was extracted over the Zambezi River Basin using a detailed river mask based on Landsat imagery. This allowed for stage measurements to be obtained for rivers down to 80 m wide with an RMSE relative to in situ levels of 0.32 to 0.72 m at different locations. The altimetric levels were then converted to discharge using three different methods adapted to different data-availability scenarios: first with an in situ rating curve available, secondly with one simultaneous field measurement of cross-section and discharge, and finally with only historical discharge data available. For the two locations at which all three methods could be applied the accuracies of the different methods were found to be comparable, with RMSE values ranging from 5.5 to 7.4 % terms of high flow estimation relative to in situ gauge measurements. The precision obtained with the different methods was analyzed by running Monte Carlo simulations and also showed comparable values for the three approaches with standard deviations found between 8.2 and 25.8 % of the high flow estimates.

1 Introduction

Hydrological models are widely used by water resources managers to obtain river flow predictions and significant effort has gone into the improvement of models’ predictive capabilities. One of the key steps of modeling is the calibration/validation phase in which modeled and measured quantities are compared. For hydrological models, this is typically carried out using in situ discharge measurements. However, in spite of the usefulness of such datasets, river discharge monitoring has globally declined over the past few decades leaving many of the world’s river basins un-or-sparsely monitored.
Remote sensing data has stepped in to complement or replace in situ model input data for many hydrologically relevant datasets such as precipitation (e.g., Stisen and Sandholt, 2010), temperature, reference evapotranspiration (Schmugge et al., 2002) and topography (with the Shuttle Radar Topography Mission, Farr et al., 2007) among others (see Tang et al., 2009 for a review). Remote sensing data can also be used to obtain data useful in the calibration/validation step of modeling such as actual evapotranspiration (e.g., Stisen et al., 2008), soil moisture (Wagner et al., 1999; Aubert et al., 2003) or total water storage (e.g., Tapley et al., 2004; Milzow et al., 2011).

While no current remote-sensing technique is capable of directly measuring discharge, radar altimeters measure water surface elevation over rivers which can then be converted to discharge. Past and current satellite altimetry missions including Geosat, TOPEX/Poseidon, ERS-1, ERS-2 and Envisat were primarily designed to study oceans and ice caps but they have none the less been collecting large amounts of data over inland waters (Calmant et al., 2008). The first applications of continental altimetry focused on lakes (e.g., Morris and Gill, 1994; Birkett, 1995). Koblinsky (1993) showed the potential for using radar-altimetry in river level monitoring in the Amazon Basin where Geosat-derived river levels were found to have a 70 cm root mean square error (RMSE) relative to in situ measurements. Birkett (1998) studied the performance of TOPEX/Poseidon over floodplains and large rivers in many basins and showed that for rivers of widths above 1.5 km, the TOPEX/Poseidon altimeter was able to track river levels, finding a mean RMSE relative to gauged level of 60 cm over the Amazon River. Most studies only consider large lakes and rivers as the retracking algorithms used rely on the echoes returned from the water surface to be ocean-like in shape (Berry et al., 2005). Berry et al. (2005) showed that by retracking individual echo shapes, data could be retrieved from a much larger proportion of the earth’s river systems, and from smaller water bodies, than was previously thought. In a study of the Mekong River, considering rivers with widths of down to 400 m, Birkinshaw et al. (2010) found RMSE values of 0.44 to 0.65 cm for retracted Envisat data.
Because river discharge rather than water level is usually the variable of interest to hydrologists, the next step after acquiring river stage measurements is conversion to discharge. In traditional in situ river discharge monitoring, a rating curve relating water level to discharge is established by simultaneous measurement of flow and discharge at different flow levels and a curve is fitted through the measured points (Chow et al., 1988). Water levels are typically recorded on a daily basis and discharge obtained through the rating curve. The same approach can be used for the conversion of altimetry data to discharge when such a rating curve is available at the location of the crossing of the satellite track over the river system (crossings will be referred to as virtual stations or VS in the remainder of the article). This can be done either by applying the in situ rating curve to the altimetry-derived water levels or by developing a specific rating curve directly relating altimetry water levels to in situ discharge (e.g., Kouraev et al., 2004; Zakharova et al., 2006). For situations where no in situ data is available at the VS, Bjerklie et al. (2003) proposed a method based on remote sensing data only which relies on the measurement of hydraulic data from space and multiple regression analysis of discharge measurements to derive the discharge equations. Using hydrological models calibrated with in situ data at other locations in the basin, Leon et al. (2006) and Getirana et al. (2009) developed methods to derive rating curves at VS locations based on altimetric levels and modeled discharges.

Radar altimeter data has also been used to improve modeled discharge estimates in calibration and assimilation frameworks. Getirana (2010) showed that altimetric river levels could be used in the automatic calibration of a hydrological model of the Branco River Basin provided knowledge of the stage-discharge relationship at the measurement location. Studies preparing for the upcoming Surface Water Ocean Topography Mission (SWOT) have shown that combining virtual swath altimetric measurements with hydrodynamic models in a data assimilation framework improved modeled depth and discharge on river stretches where the bathymetry is assumed to be known (Andreadis et al., 2007; Biancamaria et al., 2011).
This study focuses on altimetry data for monitoring of water levels and discharge in the Zambezi River Basin (ZRB). The ZRB covers 1,390,000 km² and is the largest river basin in Southern Africa. It is one of Africa’s main water resources providing water for human consumption, irrigation of crops as well as hydropower and is shared between eight countries. This study aims to present a realistic assessment of the potential for altimetry in the basin.

Results were obtained over the basin for rivers with a minimum width of 80 m using river masks derived from Landsat imagery and stage-to-discharge capabilities were systematically assessed for three different approaches corresponding to three different data availability scenarios: existence of an in situ rating curve, availability of one simultaneous measurement of stage and discharge from a field visit and availability of historical discharge data.

2 Materials and methods

2.1 Altimetry data and extraction

The altimetric stage data used for this study is the RAT (Radar AlTimetry) product derived at the Earth and Planetary Remote Sensing Lab (EAPRS) from the 18 Hz Envisat waveforms (Berry et al., 2005). Each waveform is the average of 100 consecutive individual echoes along the orbit. The averaging is done on board the satellite to produce the 18 Hz waveforms with 369 m along-track spacing. To obtain the RAT product, the waveforms are retracked using one of 12 possible retrackers. The different retrackers allow for altimetric heights to be derived from complex waveforms. The footprint of each waveform is of approximately 2–10 km over oceans but is significantly smaller over land surfaces. If a water body is located within the footprint it will usually dominate as the return signal is much stronger over water than over land.

The data obtained from EAPRS covers the entire area of the Zambezi Basin. The first step was to select data corresponding to virtual stations. The return period for En-
visat is of 35 days, meaning that a data point is available every 35 days at each VS, provided there is no loss of lock on the underlying terrain by the altimeter. The coordinate of each RAT data point is that of the first of the 100 averaged waveforms. RAT points therefore contain data from waveforms located up to 369 m from their location in the along-track direction. For each virtual station, RAT data points were therefore selected if the river was located within 365 m from the point in the along-track direction. For larger rivers where more than one data point was available per satellite overpass, the selection distance from the river was reduced in order to limit the contamination from other surfaces. Detailed river masks at VS locations were extracted from Landsat imagery by computing the Normalized Difference Vegetation Index (NDVI). Typical NDVI values for open water range between $-1$ and $-0.1$. A threshold for open water delineation was determined for each VS after visual inspection of the imagery. Threshold values were found to be between $-0.1$ and $-0.3$ depending on the surroundings and the size of the river.

An additional selection was conducted based on the backscatter coefficient (sigma0 expressed in dB). This is especially useful for locations where the river width varies greatly between wet and dry seasons as seasonal variations in river width were not directly taken into account in the extraction procedure. A threshold value of 20 dB was chosen.

For each VS time-series, outliers were removed by applying the following procedure:

- The mean value, $m$ and the standard deviation, $\sigma$, of the altimetry elevation $h$ were computed.

- For each altimetry value, $h_i$, the point was rejected if $|h_i - m| > 3 \cdot \sigma$

Except for the highly controlled flows in reaches downstream of reservoirs, rivers in the Zambezi show a strong seasonal signal. This was exploited for outlier removal by splitting the time-series in low- and high-flow series based on the month of the measurement. High- and low-flow months for each VS were determined as follows: for measurements taken on month $N$, the number of altimeter values below and above...
the average observed at the station were compared. Months with a majority of values above average were classified as wet and months with a majority of values below average as dry. The previously described outlier removal was then applied to the wet and dry time-series separately.

Where more than one data point per satellite pass was available after the selection steps described above, a weighted average of the heights recorded for the same pass was computed. The weights were chosen based on the retracker used to process the data point with higher weights being attributed to those corresponding to water wave-form shapes, i.e. retrackers 5, 8 and 10 (Berry et al., 1997). Retrackers 5, 8 and 10 were given weights of 2, 2 and 4, respectively and all other retrackers were given weights of 1.

### 2.2 Virtual station quality control

In order to assess the quality of individual virtual stations and estimate the errors, in situ data was used when available. Data was obtained from the Zambian Department of Water Affairs (DWA) and the Global Runoff Data Centre (GRDC). The stations from the GRDC dataset contain either monthly or daily flow data. Stations in the DWA dataset have either daily water levels or both daily water levels and flows. There are 98 GRDC stations in the Zambezi River Basin, of which only 34 have data up to the years 2000 (the latest reported year for one station being 2006). More station data was available in Zambia thanks to the DWA dataset in which more stations and longer time-series are reported, though large gaps are present in most station time-series.

For virtual stations where an in situ station was available along the same stretch of river with no or little water input from tributaries between the virtual station and in situ station, quality was assessed by calculating the RMSE between the altimetry and in situ datasets.

Some of the altimetry stations were located up to 200 km from the nearest gauge and the wave celerity had to be taken into account. This was done by finding the delay which maximized the correlation ($CC$) between the in situ and altimetry datasets. The
correlation between the altimetry data and the in situ data was calculated as follows:

\[
CC(d) = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{h(t_i) - \overline{h}}{\sqrt{\frac{1}{n} \sum_{i=1}^{n} (h(t_i) - \overline{h})^2}} \cdot \frac{y(t_i + d) - \overline{y}}{\sqrt{\frac{1}{n} \sum_{i=1}^{n} (y(t_i + d) - \overline{y})^2}} \right)
\]  

(1)

Where \( n \) is the number of coincident measurements, \( h \) is the altimetry derived water level, \( y \) is the in situ measured water level, \( d \) is the delay in days and the overline denotes the average of a series.

Flow cross-sectional area typically varies on short spatial scales and it is therefore not expected to reach a perfect fit between measured and altimetry derived values.

In the case where inflow from tributaries is negligible between the two measurement points and assuming negligible lateral inflow, conservation of flow was assumed along the reach:

\[
Q_{1(t=t1)} = Q_{2(t=t1+d)}
\]  

(2)

With the kinematic wave approximation and considering a wide river (a river is wide if its width is more than 10 times the depth) yields (Dingman, 2002):

\[
\frac{B_1 \cdot Y_1^{5/3}}{n_1} \cdot \sqrt{S_1} = \frac{B_2 \cdot Y_2^{5/3}}{n_2} \cdot \sqrt{S_2}
\]  

(3)

Where \( B \) is the water-surface width [m], \( Y \) is the average depth [m], \( S \) is the channel bed slope and \( n \) is Manning’s roughness \([s \cdot m^{-1/3}]\). Assuming that bed slope and Manning’s roughness are constant in time at each location, the equation can be rewritten as:

\[
Y_1 = C \cdot \left( \frac{B_1}{B_2} \right)^{3/5} \cdot Y_2
\]  

(4)
where $C$ is constant. At each location, the depth $Y$ can be written as $Y = h + a$ where $h$ is the water level and $a$ is a reference depth, and assuming a time invariant width ratio, we get:

$$h_{1(t=t1)} = \alpha \cdot h_{2(t=t1+d)} + \beta$$

(5)

Where $\alpha$ and $\beta$ are fitting parameters which were determined by fitting the amplitude of the time series at the two locations such that the 10th and 90th percentiles for the two series were equal.

The RMSE for the amplitude adjusted and non-adjusted time series were then computed to assess the virtual station’s quality:

$$\text{RMSE}_{\text{amplitude adjusted}} = \sqrt{\frac{1}{n} \sum_{1}^{n} \left( h_{1(t=t1)} - (\alpha \cdot h_{2(t=t1+d)} + \beta) \right)^2}$$

(6)

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{1}^{n} \left( h_{1(t=t1)} - h_{2(t=t1+d)} - (\overline{h_1} - \overline{h_2}) \right)^2}$$

(7)

In cases without amplitude adjustment, the averages are subtracted from the values because there is no common height reference between the in situ and altimetry datasets.

### 2.3 Discharge computation

In order to assess the potential over the whole Zambezi Basin, discharge was computed in different ways depending on data availability at different virtual stations. The data-availability categories chosen were the following: VSs with available in situ rating-curves, VSs which were visited during a field campaign where river cross sections and...
one level-discharge pair were measured and VSs with only past or average monthly flows available on the reach or at the outlet of the subbasin. The kinematic wave approximation was assumed valid for all discharge computations in this study.

2.3.1 Method 1 – in situ rating curves

For virtual stations which coincide with an in situ gauging station, the rating curve from the existing station was used directly. As the in situ and altimetry levels do not have a common height reference, the altimetry dataset was shifted by the difference between the in situ and altimetric means.

The in situ rating curves are power laws of the form:

\[ Q = a \cdot (h - c)^b \]  

Where \( Q \) is the discharge \([m^3/s]\), \( h \) the water level \([m]\) and \( a, b \) and \( c \) are fitting parameters which were obtained from the DWA database.

2.3.2 Method 2 – field data

Over the months of May and June 2010, a field campaign was carried out in the Zambian part of the ZRB and 13 virtual stations visited. The selection criteria of virtual stations to visit were based on accessibility, river width and proximity to a monitoring station.

For each virtual station, the cross section and flow were measured at one or two locations (as the crossing area between the ground track and the river can be up to a few kilometers wide). This was done using a tagline, weight and propeller (USGS Type AA-MH current meter) for narrow rivers (up to 120 m wide) and an Acoustic Doppler Current Profiler (ADCP, RiverRay, Teledyne RD Instruments) for the wide rivers. With the tagline method, depth was sampled every 5 to 10 m and velocity measurements were taken at 0.8 and 0.2 times the total depth at each point. The velocities were then averaged and
integrated over the cross sectional area to obtain discharge (see full description of the velocity area method in Dingman, 2002).

In order to develop a rating curve using only measured quantities, Manning’s equation was applied (Dingman, 2002):

\[ Q = \frac{A^{5/3}}{P^{2/3} \cdot n} \cdot \sqrt{S_f} \]  \hspace{1cm} (9)

Where \( Q \) is the river discharge \([m^3 \text{ s}^{-1}]\), \( A \) is the cross sectional area \([m^2]\), \( P \) is the wetted perimeter \([m]\), \( n \) is Manning’s roughness \([s \text{ m}^{-1/3}]\) which is assumed constant and \( S_f \) is the friction slope which is equal to the bed slope \( S_0 \) in the kinematic wave approximation (Chow et al., 1988). At the time of measurement we have:

\[ Q_m = \frac{A_m^{5/3}}{P_m^{2/3} \cdot n} \cdot \sqrt{S_0} \]  \hspace{1cm} (10)

Where the \( m \) index indicates values measured in the field. Manning’s equation can then be rewritten as:

\[ Q = Q_m \cdot \left( \frac{A}{A_m} \right)^{5/3} \cdot \left( \frac{P_m}{P} \right)^{2/3} \]  \hspace{1cm} (11)

The cross section being known, \( A \) and \( P \) can be exclusively expressed as functions of water depth. The altimetry to depth conversion is done by acquiring the altimetry water level on the day closest to the day of measurement and using it as a reference.

2.3.3 Method 3 – historical flow data

Dingman and Sharma (1997) showed that for a wide range of rivers, a good estimate of discharge can be obtained by applying the following rating curve:

\[ Q = 1.564 \cdot A^{1.173} \cdot R^{0.4} \cdot S^{-0.0543} \cdot \log_{10}(S) \]  \hspace{1cm} (12)
Where $A$ is the cross-sectional area $[m^2]$, $R$ is the hydraulic radius $[m]$ and $S$ is the water-surface slope. This equation has the advantage of relying only on morphological characteristics of the river. However, there is no remote sensing technique capable of measuring river depth which is needed to obtain cross-sections. Therefore the use of this equation implies making assumptions on channel geometry. The assumption made here was that of a rectangular cross section. Channel width was read from Landsat imagery by dividing the channel area by the channel length. In the kinematic wave approximation, the water surface slope is equal to the bed slope (Chow et al., 1988). In order to determine the bed slope, elevation was extracted from the Shuttle Radar Topography Mission (SRTM) data along a 20 km stretch of river around the VS and the slope determined by linear regression.

In order to convert altimetry measurements to depth, a reference depth was derived. For many reaches in the Zambezi River Basin, the only type of data available is monthly discharge data covering all or part of the time period between 1950 and 1980 from GRDC stations or average monthly discharges for the major subbasins. This data was exploited based on the observation that inter-annual flow variations in uncontrolled reaches are low in the dry months. The average in situ flow value for the driest month was calculated from the dataset and the flow equation was solved for $d$. Rewriting the flow equation in terms of channel depth and width yields:

\[ Q_{\text{low}} = 1.564 \cdot (W \cdot d_{\text{low}})^{1.173} \cdot \left( \frac{W \cdot d_{\text{low}}}{W + 2 \cdot d_{\text{low}}} \right)^{0.4} \cdot S_0^{-0.0543 \cdot \log_{10}(S_0)} \]  

(13)

Where $Q_{\text{low}}$ is the average flow for the driest month of the year $[m^3 \cdot s^{-1}]$ and $d_{\text{low}}$ is the depth associated to this flow value $[m]$. The average altimetry height for the driest month, $h_{\text{low}}$, was then extracted and the altimetry to depth conversion carried out as follows:

\[ d = h - (h_{\text{low}} - d_{\text{low}}) \]  

(14)
2.4 Uncertainty analysis

For each of the discharge computation methods presented, the magnitude of the uncertainties in the measured quantities was estimated and their impact on the final discharge estimate analyzed using Monte Carlo simulations. For each method, 1000 runs were carried out by randomly sampling the uncertain quantities from normal distributions where the measured value was taken as the mean and the standard deviation of the distribution were determined as described in the next paragraphs. The in situ rating curves were used as the benchmark for quality assessment and therefore not considered uncertain.

For all three methods, the standard deviation of the altimetric stage was taken as the RMSE calculated in the non-adjusted case (see Sect. 2.2).

In method 2, for the field tagline measurements, the standard deviations on the measured depths, distances and velocities for sampling were determined to be 5 cm, 50 cm and 5 mm s$^{-1}$, respectively based on the field procedure. For the ADCP measurements, the standard deviations on the measured flow and cross section are outputs of the measurement and were directly used as such.

For method 3, uncertainty on the measured slope was determined from the fit of the linear regression from the SRTM data (see Sect. 2.3.3), and uncertainty on maximum width was determined to be 10 m. The standard deviation on the low flow value was computed from the sample of average monthly flows available for the driest month at each in situ station.

3 Results

3.1 In situ and altimetric river level

The satellite ground track was found to cross the river network in the Zambezi at 423 points. These were located on rivers, lakes and floodplains. The RAT data was
extracted using the river mask derived from Landsat imagery at all locations where the river width was more than 70 m. Smaller rivers where many RAT points with high sigma0 values were observed were also considered. After elimination of time-series with too few data points, too many outliers or unrealistically high annual level variations, 31 virtual stations were identified as useable. 20 of these were located on the same stretch as a gauging station with recent level and/or flow data. The locations of the VSs and useable gauging stations are presented in Fig. 1.

Comparing VS and in situ data yielded RMSE values between 0.34 and 0.72 m for VSs coinciding with gauge locations. For VSs where the gauge was located further on the same reach, RMSE values between 0.27 m and 1.07 m without amplitude adjustment and from 0.24 m to 1.56 m RMSE with amplitude adjustment were found (Table 1).

In most cases where the gauge was located far from the virtual station, the amplitude adjustment improved the RMSE. However this was not always the case in part because of the time resolution of the altimetry dataset and the uncertain estimation of the delay between gauge and virtual station. With one data point every 35 days, the altimetry dataset may miss important peaks of short duration leading to an erroneous rescaling of the altimetry data. Figure 2a shows an example of this problem occurring at VS 299.

Figure 2b however shows an example where using the amplitude adjusted altimetry dataset allowed for a better quality classification. Considering only the unfitted RMSE of 1.01 m for VS 187 (see Table 1) would classify the virtual station as of poor quality while Fig. 2b justified the classification of the virtual station as “good”.

The amplitude adjusted and non-adjusted RMSE values therefore need to be combined in the classification of virtual stations and visual inspection was necessary in some cases. For VSs located far from the gauging station used for comparison, the RMSE values were not directly used for the classification, rather as indicators, as the assumption of negligible inflow between VS and gauge (see Sect. 2.2) is no longer valid for some of the distances considered.

In order to classify virtual stations not located near gauging stations, quality was assessed by visually comparing data from altimetry virtual stations located along the
same reach. Figure 3 shows an example where the virtual stations were classified as “good”. The mean of each time series was subtracted from the measured levels to account for the different elevations at the two VSs.

This method however was not used in highly controlled reaches as the comparison of non-simultaneous data points could no longer be carried out due to the high daily (or even sub-daily) flow variations from reservoir releases.

It should be noted that on reaches with highly controlled flows, no good quality virtual stations were identified partly due to the fact that the outlier removal approach used is more efficient for time-series with a strong seasonal signal (see Sect. 2.1). However, even in the event of a good quality virtual station being located on such a reach, the utility of one measurement point every 35 days in an environment with very high daily variations in flow would be limited.

Figure 4 shows the classification based on the criteria described above. The label “good” corresponds to a VS with an expected error of less than 40 cm, “moderate” to a VS with an expected error of less than 70 cm and “bad” to a VS with an expected error of more than 70 cm. Due to lack of both in situ and neighboring VS, some of the VSs could not be classified.

3.2 Discharge

Discharge estimates were carried out for all of the analyzed virtual stations with the three methods described previously where the necessary data was available. For all methods, the in situ measured discharge was used as the benchmark for the discharge prediction based on altimetry. In this section, results are presented both in terms of accuracy and precision. The RMSE values present the expected deviation of the mean modeled discharge relative to the in situ measured flows and the standard deviations (std) present the spread between the model runs for each method. Due to incomplete in situ discharge time series, the “high flow” values used to illustrate the RMSE and the std in terms of percentages do not necessarily correspond to each other as the highest
level simultaneously observed in situ and by the altimeter is not always the highest level observed by the altimeter overall.

Method 1, using in situ rating curves, was only applied to the four virtual stations located at an in situ rating curve. For the virtual stations classified as good, the std on the discharge calculations were found to be between 25.4 and 69.4 m$^3$ s$^{-1}$ with an RMSE ranging from 19.9 to 69.4 m$^3$ s$^{-1}$ relative to the in situ data which corresponds to 5.7 to 6.7 % of the wet season flow (Table 2).

Method 2 was applied to 4 of the 5 selected virtual stations where field data was collected. The 5th, VS 237, could not be used because no data was collected by the satellite during the pass corresponding to the time of the field measurement.

The field-derived and in situ discharge time-series were first inspected. Good agreement was found for VS 150, VS 222 and VS 309. At VS 109 the field rating curve was found to significantly underestimate discharge (Fig. 5).

At the gauge near VS 109, a second rating curve based on pre-1984 in situ measurements was also available (dashed grey line in Fig. 6) and was found to be much closer to our field-derived rating curve.

The shape of the field rating curve depends on two major variables: the measured flow on the day of the field visit and the value of the altimetry height on that day. With the available data, we were not able to determine whether the measurements were faulty or which rating-curve produced the more accurate results and VS 109 was therefore left out for further analysis of method 2. It should also be noted that the in situ rating curves at VS 109 are available only to 5.61 m while the maximum in situ stage measured is 8.35 m and no comparisons could therefore be carried out at high flow levels.

For the remaining 3 VSs, RMSE values ranging from 42.9 to 59.9 m$^3$ s$^{-1}$ relative to the in situ data were found which corresponds to 4.6 to 7.4 % of the high flow values and with standard deviations on the high flow estimates between 16.6 and 25.8 % of the value (Table 3).

In order to apply method 3, using historical data as low-flow calibration, the values of average minimum monthly flow and the standard deviation were calculated from the
available in situ data. For operational gauges, the time-series was used up to the year 2000. While the std on the low flows were typically found to represent a high percentage of the low flow values (from 16% to 120% of the low flow values), the impact of this uncertainty on the std of flow estimations was found to be relatively low with values ranging from 3.3% to 8.1% of the high flow estimates (Table 4).

Applying method 3 yielded RMSE values on computed flows between 54.5 and 123.6 m$^3$/s$^{-1}$ corresponding to 5.9 to 7.3% of the wet season flow with std values on the high flow estimates between 16.2 and 17.6% of the flow for VSs located at the same place as a gauge (lines on white background in Table 5). The method was also applied to VSs up to 90 km from the nearest gauge yielding RMSE values between 34.7 and 608.9 m$^3$/s$^{-1}$ which correspond to 4.4 to 23.4% of the high flow with std values for the high flow estimate between 14.9 and 57.2% of the computed value. It should be noted that these flow estimates are not meant as estimates at the gauge locations but at the VS locations.

4 Discussion and conclusions

While previous river altimetry studies focused on wide rivers, from 200 m to a few km wide, using retracked altimetry and a detailed river mask enabled us to extract accurate stage measurements for rivers down to 80 m widths and useable levels were also extracted for one 40 m wide river. Using an outlier removal strategy based only on the seasonality and spread of the altimetry time-series itself, RMSE values of between 0.32 and 0.72 m were obtained for VS locations coinciding with a gauging station. These values are comparable to the 0.44 to 0.76 m reported by Birkinshaw et al. (2010) in their study of the Mekong River using the same retracking methodology, though the Mekong study concerned wider rivers (over 450 m wide).

For the rivers studied, the RMSE of 0.34 to 0.72 m on stage translated to 8.2 to 20.7% error on flow estimation during the high flow season using the in situ rating
curve which is an acceptable level of error considering the errors expected for rating-curve derived discharge measurements.

For cases where no pre-existing rating curves were available, two methods were developed and tested. Both methods relied on obtaining a reference flow/cross section pair either by carrying out field work or by assuming a rectangular channel and using historical low-flow values as a reference.

The second method, using field data, yielded RMSE values between 4.6 and 7.4 % of the high flow values for three of the VS considered with standard deviations on the high flow estimates between 16.6 and 25.8 % of the high flow values. These results are comparable to those found utilizing an existing rating curve, which is promising as it implies that any accessible river-ground track crossing of interest can become a virtual flow monitoring station based on one field visit to the site. However, at the last VS, the field-derived rating curve systematically underestimated high flows and further study of the area would be needed in order to establish which rating curve provides the best discharge estimates.

Using the historical flow method to estimate discharge yielded RMSE values between 48.7 and 128.5 m$^3$s$^{-1}$ corresponding to 5.5 to 7.5 % of high flow values at the VSs in question. Standard deviations on the high flow estimates were found to be between 16.2 and 17.6 % of the flow value for VS locations coinciding with a gauging station. Some of the VSs however showed much higher standard deviations on high flow estimates, up to 57.2 % of the calculated value for relatively narrow rivers with high uncertainties on slope measurements from SRTM. However, the RMSE between flows estimated at these VSs and flow at the gauging stations further along the stream remained quite low with values ranging from 4.4 to 23.4 % on high flow estimates showing the possibility of applying this method when the historical flow data record available was from a gauge located in our case up to 90 km along the same reach as the VS thereby increasing the number of potential virtual discharge gauging stations.

The advantage of this method is that with the decline in in situ gauging over the past 20 yr, in many areas historical flow data exists while more recent data may be non-
existent. Averaged monthly flows are data typically found in openly available reports (e.g., The World Bank, 2010) and could be used in such an approach, while more detailed datasets are not always made available by the organizations in charge of data collection. One major drawback is the rectangular cross section hypothesis which does not allow for the inclusion of complex river profiles, the flooding of banks etc. This could be improved by adapting the cross-sectional profile by obtaining time-series of river widths, which can also be obtained from remote sensing imagery. This will in particular be aided by the launch of the Surface Water Ocean Topography (SWOT) mission which will be dedicated to hydrology and provide measurements of river width and surface slope as well as stage (Durand et al., 2010).

This study has shown the high potential for stage and discharge monitoring in the Zambezi River Basin from radar altimetry. While the 35-day time resolution of Envisat remains insufficient for many applications, it is expected that combining this data with basin-scale hydrological modeling in a data assimilation framework will improve discharge estimates on finer time and spatial scales than the satellite resolution. The advantage of such an approach would be the real-time updating of flows at VS locations as well as at other locations along the same river stretch where the flow data is needed thereby increasing the value of radar altimetry level data for water managers.

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References

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Table 1. RMSE, distance to gauge and high flow channel width. (d) indicates the gauge is located downstream, and (u) upstream from the virtual station.

<table>
<thead>
<tr>
<th>VS</th>
<th>RMSE [m]</th>
<th>RMSE as % of total amplitude</th>
<th>Nb. of points of comparison</th>
<th>Approx. distance to gauge [km]</th>
<th>Approx. channel width [m]</th>
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<tr>
<td></td>
<td>Amplitude adjusted?</td>
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<td>No</td>
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The index ^1 after the channel width value indicates the virtual station is located in a channel with a floodplain, the index ^2 indicates the presence of pools adjacent to the channel and the index ^3 indicates a braided river channel.
Table 2. Results for discharge calculations using method 1.

<table>
<thead>
<tr>
<th>VS Number</th>
<th>RMSE [m$^3$ s$^{-1}$]</th>
<th>RMSE % of high flow</th>
<th>STD [m$^3$ s$^{-1}$]</th>
<th>STD % of high flow estimation</th>
<th>Nb. of points of comparison</th>
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Table 3. Results for discharge calculations using method 2.

<table>
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<th>RMSE [m³ s⁻¹]</th>
<th>% of high flow</th>
<th>RMSE [m³ s⁻¹]</th>
<th>% of high flow estimation</th>
<th>STD</th>
<th>% of high flow of comparison</th>
<th>Nb. of points</th>
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Table 4. In situ average low monthly flow values and standard deviations.

<table>
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<th>VS Number</th>
<th>( Q_{\text{low}} ) [m(^3) s(^{-1})]</th>
<th>STD ( Q_{\text{low}} ) [m(^3) s(^{-1})]</th>
<th>% of high flow</th>
<th>STD</th>
<th>Years of data</th>
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Table 5. Results for discharge calculations using method 3. Bold entries signal VS location not coinciding with that of the gauge used for comparison.

<table>
<thead>
<tr>
<th>VS Number</th>
<th>RMSE [m$^3$ s$^{-1}$]</th>
<th>RMSE % of high flow</th>
<th>STD [m$^3$ s$^{-1}$]</th>
<th>STD as % for high flow estimation</th>
<th>Distance to gauge [km]</th>
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<tr>
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Fig. 1. Location of the virtual stations and monitoring stations with data past the year 2002 in the basin.
Fig. 2. Comparison of altimetry derived water levels and in situ water levels for VS 299 (a) and VS 187 (b).
Fig. 3. Data from altimetry VS 003 (70 m wide) and VS 190 (75 m wide) located on the Lung-webungu River in North Western Zambia.
Fig. 4. VS location and quality in the Zambezi River Basin.
**Fig. 5.** Comparison of measured and simulated discharge from altimetry and field measured cross section and flow with 95% confidence intervals.
Fig. 6. In situ and field rating curves at VS 109.