**Hydrology and
Earth System
Sciences
Discussions**

Interactive comment on "Copula-based downscaling of spatial rainfall: a proof of concept" *by* M. J. van den Berg et al.

M. J. van den Berg et al.

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We thank the referee for her insightful comments. Our response can be found below.

Referee 2: 1. The overall performance of the method will depend heavily on the goodnes-of-fit of the marginal distributions. The marginal distributions used here assume independence of the observed values at each scale. Have the authors tested the validity of this assumption? I would expect this not to be the case, especially at the fine scale. In this light, I find the statement in lines 11-12 on page 210 slightly misleading. The copula will incorporate dependence structure between the coarse scale and the fine scale and thus the local distribution of the depths at the fine scale can vary as a function of the depth at the coarse scale. However, the local dependence structure within data at the fine scale will not change, as the marginals stay the same.

We are not aware of any specific independence requirements for the Gamma distribution. However, we do acknowledge that, generally, a probability density function is estimated from independent observations. However, this is generally not possible when investigating spatially continuous rainfall, as independent observations are impossible. To combat this, the probability distributions can be established based on a random sample from the image. This gives the same distribution as the one estimated from the full image. Hence, it has been assumed that the effects of dependence are negligible and they have been ignored in this paper.

As for the second part, we have clarified the sentence to reflect that we only adapt the local distribution function, not the local dependence function (which is not under consideration in this paper)

> Therefore, a novel method is explored that describes the dependence between the coarse, and the fine-scale rainfall depth using a copula. Furthermore, it allows for modeling the sub-pixel probability distribution within one coarse-scale pixel.

Referee 2: 2. The authors could be more precise in their description of the data they analyse in the paper. There is no mention of the size of the entire data set and when in-sample estimation is performed, the authors don't state how large a part of the data set was used for the estimation. Furthermore, as all dry events are removed from the data set, it should be stated how much data was removed.

We have expanded the data section with a table that describes the data and a section that describes the data selection:

> The dates, at which radar imagery is available, are selected based on rain-gage readings from a network across Belgium, where the rainfall was required to have a minimum Peak-over-Threshold return period of 10 years

to be included in the dataset. This resulted in a dataset consisting of the days listed in Table 2. For these days, the number of rainy hours has been listed, along with the average intensity for each rainstorm, and the ratio of dry pixels to wet pixels. Images that did not display rainfall were removed from the dataset as they hold no information.

The images for which no rainfall was detected have no impact on the analysis, and as such are not reported.

Whenever we perform a validation, we have clarified how many images are used where this was not the case. To do this, we have changed P. 220 l. 12-13 to

> We therefore derived an empirical copula for the rainfall field of the same storm as observed 5 hours prior to the image to be downscaled; this approach is followed unless mentioned otherwise.

and p. 218 l. 12 to

> This relation has been fitted to a series of ten randomly selected images, and the results are shown in Figure 6.

Referee 2: 3. *The extension of the method to include the dry observations might be out of the scope of this paper. However, I feel that the discussion of this could be somewhat improved. The authors propose to use the function f given in equation (6) to model the fraction of dry fine scale pixels for a coarse scale pixel as a function of the observed value in the coarse scale pixel. However, high resolution fine scale data is needed in order to estimate the parameters of the function f. This is in contradiction to the statement in lines 17-20 on page 208 where it is stated that such data is usually not available. In relation to this, see also my technical correction nr. 5 below.*

Technical correction 5. *Page 219, lines 8-16: the marginal distributions and the copula need not be continuous, see e.g. Hoff (2007). There are certainly complications from having discrete data in a copula approach, but there is nothing wrong with it.*

The inclusion of intermittency in the model is necessary for any downscaling method, as intermittency is paramount in rainfall modeling. As such, it is within the scope of the paper. Furthermore, although fine-scale information is generally not available for training, this direct method of fitting is sufficient for the proof of concept. In future work, a more advanced method of fitting will be explored that would remove the necessity of fine-scale data for validation.

We are aware of the existence of discrete copulas, however, the discrete nature of the data is an artifact of the method of measurement. Also, discrete copulas are not as well developed as continuous copulas, which would result in difficulties in future work. Finally, including the intermittency in a single model of either marginal distributions or the copula is probably possible. However, such an approach would unnecessarily complicate the approach.

Referee 2: 4. *I believe it would be informative to also include a performance measure which demonstrates how well the estimated distributions fit to the individual data points. As noted by the authors, classical measures such as the absolute error and the root mean square error only take into account the distance between the median or the mean of the distribution, respectively, and the corresponding data point. However, there are performance measures which account for both the location and the spread of the distribution. One such example is the continuous rank probability score (CRPS). Furthermore, the multivariate extension of the CRPS, the energy score, can be applied to assess the performance of the full bivariate model, see Gneiting et al. (2008). That is, the energy score can be applied to compare the fit of the bivariate copula model and the bivariate independence model which has the same marginals as the copula but assumes independence between the coarse scale and the fine scale data. Here, the energy score must the calculated using Monte Carlo samples from the respective*

bivariate distribution. Such a comparison should nicely demonstrate the gain in performance offered by the copula and it would especially be of interest for the discussion of the temporal robustness of the copula in Section 5.4.4 and of the storm dependence of the copula in Section 5.4.5.

To demonstrate the gain of the method over an assumption of independence, it is easiest to simply replace the assumed copula with the independence copula and repeat the tests. However, this is the same as simply taking the estimated marginal distribution and comparing it with the empirical distribution. This has already been done in Section 5.4.2, specifically Figures 9, 10 and 11.

Referee 2 : TECHNICAL CORRECTIONS:

1. Page 214, equation (7): there is a variable v on the right-hand side that doesn't appear on the left-hand side in the equation.

This should be v_m, this has been changed.

2. Page 217, line 9: the authors state that the gamma distribution "exhibits a good fit" to the spatial rainfall fields. How is this quantified? Would e.g. be possible to add a 95% confidence interval to the MLE estimate shown in Figure 3?

We added a 95% confidence interval to the figure.

3. Page 217, equation (13) and line 18: in line 18, it is stated that $\theta_\lambda = t\theta_{\lambda'}$. However, I believe that equation (13) should be

$$t\Gamma(k, \theta_\lambda) = \Gamma(k; t\theta_\lambda) = \Gamma(k; \theta_{\lambda'}); \tag{1}$$

which means that $\theta_{\lambda'} = t\theta_\lambda$.

Corrected

4. Page 218, lines 5-8: there is a word missing from this sentence.

Corrected

6. Page 221-222: It would be helpful for the reader if the description of the EMD included information on whether it is a positively or a negatively oriented measure.

The EMD is a distance function and as such takes positive values only.

7. Page 224, lines 19-24: As far as I understand, the fine scale data is not used at any stage of the modeling procedure (and rightfully so, as the method aims to provide a downscaling procedure for situations where fine scale data is only sparsely available). There is thus no reason to expect that the method will recover the empirical distribution of the fine scale data exactly at the zero time-lag. It can only be expected to provide a result that is very similar to the empirical distribution.

At the first time step, the model is fitted to the image that is downscaled. Because of this, if the method were perfect, it would recover the empirical distributions. However, fitting imperfections and interpolations result in a less than perfect result. This is not a problem, but an illustration of the error incurred because of the method.

8. Page 226, line 1: please correct this line.

Corrected

9. Page 231, Figure 2: please expand the description of the figure in the caption.

We have expanded the caption with

> Starting at the coarse-scale distribution, one obtains the probability (rank) of the value used for conditioning. This value is used in the copula to find the conditional probability of the fine-scale ranks, which are then 'projected' using the fine-scale distribution to find the conditional probability of the sub-pixel rainfall depths.

10. Page 237, Figure 8: please add an explanation of the title of each plot to the caption.

The new caption reads:

> Sixteen plots of actual coarse-scale pixels with their marginal (field-wide) distribution (thick, black), the empirical distribution of their sub-pixels (thin, dashed) and the modeled distribution of their sub-pixels (thin, solid). The rank of the coarse-scale observation used for conditioning is displayed above each plot.