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Hydrology and
Earth System
Sciences
Discussions

Interactive comment on "Transient drawdown solution for a constant pumping test in finite two-zone confined aquifers" *by* C.-T. Wang et al.

C.-T. Wang et al.

hdyeh@mail.nctu.edu.tw

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Response to the comments of reviewer 2

1. This paper develops an analytic solution for the estimation of the transient drawdown distribution due to constant-flux pumping from a finite-radius well with skin effect in a bounded confined aquifer. The current solution assumes two pairs of transmissivity/storage coefficient values, one pair for the skin zone (up to some radius r1) and another for the rest of the aquifer. The current solution generalizes a previous solution developed by some of the authors (Yeh, Yang and Peng, Adv. Water Resour, 26) which was developed for infinite aquifers only. Another major advantage of the present solution is that it includes an infinite series that can be accurately estimated with little effort.

The infinite series is much easier to compute than the solution of Yeh et al. (2003) which is defined in terms of an integral which is difficult to evaluate accurately because of the singularity at the origin.

2. The mathematical derivation, which uses Laplace transforms and Bromwich contour integral, appears to be valid. The mathematical solution used in particular constitutes an interesting contribution beyond what is already published in the literature. The combination of the Laplace transforms and Bromwich contour integral should be of interest to the readers of HESS. Although the presentation of these methodologies for the derivation of the drawdown solution should be of interest from an academic point of view, I believe the main limitation of this work is that the problem of confined circular aquifer with a pumping well located at the center of the aquifer and zero drawdown at the outer edge of the aquifer is not commonly encountered. The authors should therefore elaborate on the applicability of the solution. Reply: Thank for the comment. This comment is similar to the one given by the first reviewer. Therefore, our reply presented herein is a modified version of the one we respond to the first reviewer. There is no infinite boundary existed in the real-world problems. In practice, a radius of influence of the well R is commonly adopted as the outer edge of the aquifer in the groundwater community for deriving the steady-state confined flow equation, e.g., Thiem equation (see, e.g., Batu, 1998, p. 116, Eq. (3-10); Bear, 1979, p. 306, Eq. (8-7); Schwartz and Zhang, 2003, p. 222, Eq. (9.5)). The variable R was originally defined as the distance from the central line of the well to the outer boundary in the previous manuscript. Now, it has been changed to "the radius of influence of the pumped well defined as a distance measured from the center of the well to a location where the pumping drawdown is very close to zero" on page 8 (three line after Eq. (2)) in this revised manuscript. Note that Bear (1979) mentioned three semi-empirical and two empirical formulas for the estimation of R for homogeneous aquifers. In addition, Schwartz and Zhang (2003, p. 222) also provided a formula to estimate R from the field hydraulic-head measurements in homogeneous aquifers.

To address the applicability of the new solution, a new paragraph is added at the end of Introduction section and also listed below: “This new time-domain solution can be applied to: (1) predict the spatial and/or temporal drawdown distributions in both the skin and formation zones with known aquifer (five) parameters such as the outer radius of the skin zone and the transmissivity and storage coefficient for each of the skin and aquifer zones, (2) determine the aquifer parameters if coupled with an optimization algorithm in the pumping test data analyses, (3) verify numerical codes in the prediction of the drawdown distribution in two-zone aquifer systems, and (4) perform the sensitivity analysis and assess the impacts of parameter uncertainty on the predicted drawdown.” In addition, both the "Introduction" and "Potential application" sections have been slightly modified to improve the clarity of the presentation. Note that the modifications made in these two sections are all marked in blue color in the revised manuscript.

3. Under certain simplifying assumptions (such as late time, infinite aquifer, absence of the skin zone), the solution reduces to previously derived solutions (e.g., Thiem solution). However, to further demonstrate the validity of the solution, I would propose that the authors consider comparing, under general conditions, the analytic solution they have developed to the corresponding numerical solution. Reply: Thanks for the suggestion. The present solution has been verified with the Theis solution addressed in Comment 12. The numerical approaches often introduce truncation error due to grid discretization and approximation errors in the simulations of the well radius and outer boundaries. Since this manuscript is a technical note rather a full paper, we think the use of the Theis solution might be good enough to demonstrate the verification of the present solution.

4. The presentation of the paper is generally good. The use of the English Language is generally good, but the paper in some locations needs an additional round of editing. The title and abstract reflect well the scope of the paper. The Introduction section has a good summary of the published works related to analytic solutions of the drawdown

distribution due to pumping for different configurations. Another interesting feature of the developed solution is that it is expressed in terms of non-dimensional parameters which can help in the analysis of the solution. Reply: Thank for the suggestion. We have edited our English writing.

Specific comments: 5. Page 9309, line 16 and page 9310, lines 11-12: The authors should elaborate on the statement that the accuracy is to the fifth decimal point. Is this a general statement or specific to the problem solved in the paper? Perhaps some figure showing the accuracy as a function of n can be included in the paper. Reply: Thank for the comment. It is considered as a general statement. We had compared all the drawdowns predicted from the present solution with those from the inversion results obtained from the Laplace-domain solution and the modified Crump algorithm with accuracy to the fifth decimal. Figure A shows the curve of dimensionless drawdown versus dimensionless time at rho= 1 when kappa = 5, rho1=3 and rhoR =100. This figure indicates that the predicted drawdown approaches a constant value when the summation term n is larger than 50.

6. Discussion of Figures 2 and 3: There is no mention of the storage coefficients used for the skin zone and the outer zone of the aquifer. Reply: Thank you. We have added gamma=S2/S1=1 in these two figures. Note that S1 and S2 can be taken as 0.0001.

7. Page 9310 line 24 to page 9311 line 2: The results and discussion section of the papers are too brief. In the potential applications section, the authors mention the possibility of using the developed solution to estimate the parameters of the problem (transmissivity and specific storage of the skin and outer zone, and distance to the edge of the skin zone). I believe the addition of such an application would significantly enhance the manuscript. Reply: Thank for the comment. We add following text at the end of the first paragraph in the “Potential application” section to address such an application. “It is of interest to note that Yeh et al. (2009) developed a numerical approach composed of the drawdown solution developed by Yeh et al. (2003) and the algorithm of simulated annealing. The approach was used to analyze 84 hypothetical drawdown

data sets which included 14 different scenarios and each scenario contained 6 cases. The analyzed results demonstrated that their approach could give reasonably good estimations to the thickness of the skin zone and four aquifer parameters at the same time." Since Yeh et al. (2009) had introduced the detailed development of parameter estimation approach based on the solution developed by Yeh et al. (2003). In addition, their drawdown data analyses in the estimation of these five parameters were rather comprehensive. We are afraid that the application of present solution in parameter estimation added in the present paper may just iterate what had been done in Yeh et al. (2009). We hope that the addition of the text described above may be helpful to the interested readers for the application of the present solution.

8. Page 9311 lines 23-24: The authors state that the non-dimensional time matches well up to tao<100. I think that this is not a general comment, but specific to the problem at hand, in particular dependent on the distance to outer aquifer boundary. The authors should rephrase this statement and consider showing other cases to justify their answer. Reply: Thank you for the comment. We have added "and rhoR= 20, 30, and 50" in line 20 of page 9311.

Additional Comments 9. Gamma and omega (page 9305 line 6) are not defined. Reply: Thanks for the comment. They should be changed to phi and psi .

10. In the mathematical derivation (page 9306, equations 20-24 and other places, it would be helpful if the dimensions of the different variables are specified. Reply: This comment is not clear to us. We add "following lumped variables" before Eq. (20) and other places.

11. Equation 31, page 9308: s1 is non-dimensional therefore, use another symbol. omega_n is not defined. Reply: Thanks for the comments. s1 should read s1D and omega n should read beta n.

12. I suggest that the authors add the Theis solution to Figures 2 and 3. Reply: Figures 2 and 3 illustrate the curves of dimensionless drawdown versus dimensionless time. The dimensionless time in these two figures is defined as in which the well radius is in the denominator. Accordingly, the Theis solution (of neglecting the well radius) can not be plotted on Figures 2 and 3. We therefore plot Figure B (shown below) for the predicted drawdown at r = 10 m to show the comparison between the Theis solution and present solution for different value of R. This figure indicates that the present solution match the Theis solution very well when R is large and t is small. To avoid overlapping, we do not consider including this figure in the manuscript. Note that the values of pumping rate and aquifer parameter used to draw Figure B are taken from Wang et al. (2004, p. 986).

13. Page 9312, line26: The word "surprisingly" is not needed here. The large-time solution is expected to approach the steady-state solution. Reply: Thank you. We have removed it.

14. Page 9312 lines 10-11: change to: "... indicating that the drawdown is sensitive to contrast in transmissivity for positive skin cases". Reply: It is well taken.

References Batu V (1998), Aquifer Hydraulics, John Wiley & Sons, New York. Bear J (1979), Hydraulics of Groundwater, McGraw-Hill Inc., New York. Schwartz FW, Zhang H (2003), Fundamentals of Ground Water, John Wiley & Sons, NY. Wang, XS, Chen CX, Jiao JJ (2004) Modified Theis Equation by considering the bending effect of the confining unit. Advances in Water Resources, 27, 981-990. Yeh HD, Yang SY, Peng HY (2003), A new closed form solution for a radial two-layer drawdown equation for groundwater under constant flux pumping in a finite radius well, Advances in Water Resource, 26, 747-757.

Please also note the supplement to this comment:
http://www.hydrol-earth-syst-sci-discuss.net/8/C5760/2012/hessd-8-C5760-2012-supplement.pdf

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Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 8, 9299, 2011.

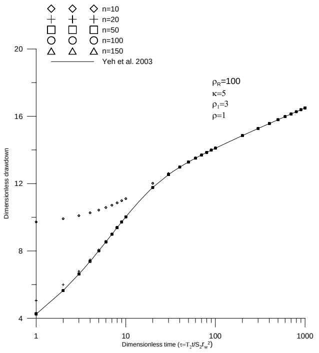

Figure A The dimensionless drawdown calculated using different value of *n*.

Fig. 1. Figure A The dimensionless drawdown calculated using different value of n.

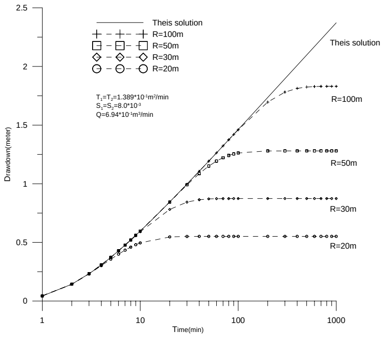

Figure B Temporal drawdown distributions predicted by the Theis solution and the present solution for different value of *R*.

Fig. 2. Figure B Temporal drawdown distributions predicted by the Theis solution and the present solution for different value of R.