An algorithm for delineating and extracting hillslopes and hillslope width functions from gridded elevation data

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Abstract

The subdivision of catchments into appropriate hydrological response units is an essential step in rainfall-runoff modelling, with the hillslope serving as a common fundamental unit for this purpose. Hillslope-based modeling approaches can utilize, for instance, the hillslope width function as a one-dimensional representation of three-dimensional landscapes by introducing profile curvatures and plan shapes. An algorithm was developed to delineate and extract hillslopes and hillslope width functions based on a new approach to calculating average profile curvatures and plan shapes from digital terrain data. The proposed algorithm does not depend on the resolution of the gridded elevation data, and it classifies hillslopes according to the nine elementary landscapes introduced by Dikau (1989). This algortihm was tested on two contrasting (flat and steep) catchments in Quebec, Canada. Good results were obtained based on criteria of monotonicity of the width functions and conservation of the hillslope and catchment surface areas.

1 Introduction

The representation of rainfall-runoff processes in hydrological modelling is highly dependent on spatial scale, landscape properties, and other factors (Moore et al., 1993; Grayson and Blöschl, 2000; Beven, 2001). The subdivision of a catchment into appropriately defined and extracted runoff response units represents an important first step in hydrological modelling, and the hillslope is viewed as one of the most common units used for such purposes (e.g., Fan and Bras, 1998; Troch et al., 2003). Hillslopes can be defined as either headwater or lateral flow units that encompass the area drained above or to the left or right side of a river segment, respectively.

The hillslope width function (HWF) is defined as the width of the hillslope from the river segment to the divide. The direction along the transect begins at the river segment and increases to the divide (see section 2.3 for further details). HWFs play a central role in recently developed hillslope-based models that collapse the three-dimensional (3D) landscape of a given hydrological unit into a one-dimensional (1D) representation (Fan and Bras, 1998; Troch et al., 2003). By introducing profile curvature and plan shape, the HWF can, by its 1D width variation, capture convergent, divergent, and uniform hillslope shapes as well as concave, convex, and straight profiles. Plan shape is defined as the tangential curvature that is perpendicular to the slope gradient, while profile curvature refers to the rate of change of slope (Schmidt et al., 2003).

To calculate profile curvature and plan shape, most terrain analysis software uses a quadratic equation on a 3x3 matrix as proposed by Zevenbergen and Thorne (1987). However, this method is highly sensitive to local variations in input data and digital elevation model (DEM) resolution, leading to greater scatter in spatial patterns of curvature, especially for flatter areas (Schmidt et al., 2003). This leads to overestimation of some features within a hillslope and to DEM resolution effects on the profile curvature and plan shape. Thus, there is a need to develop a method that is able to calculate the average plan shape and profile curvature independently of DEM resolution.

Whereas several terrain analysis algorithms have been presented in the literature for extracting the principal geomorphologic characteristics of catchments, such as slope, topographic index, and overland flow paths (e.g., LandSerf, Woods et al., 1995; TAPES, Gallant and Wilson, 1996; TARDEM/TauDEM, Tarboton, 1997; LANDLORD, Florinsky et al., 2002; TAS, Lindsay, 2005; LANDFORM, Klingseisen et al., 2008; PHYSITEL, Rousseau et al., 2011), the delineation of hillslopes, including extracting the width function and taking into account plan shape and profile characteristics, has not attracted as much attention and existing methodologies (e.g., Cochrane and Flanagan, 2003; Bogaart and Troch, 2006; Zhong et al., 2009) are not as standardized.

In this paper, a method to delineate hillslopes and extract the width function is presented, together with an application to two watersheds in Quebec, Canada. The procedure comprises three steps: (i) delineation of hillslopes; (ii) calculation of the profile curvature and plan shape and their association with the nine elementary landscapes introduced by Dikau (1989); and (iii) calculation of the HWF and optimisation of the final hillslope shape according to various criteria. These steps were coded up as an algorithm implemented in PHYSITEL (Rousseau et al., 2011), a geographic information system (GIS)-based pre-processor for the HYDROTEL (Fortin et al., 2001; Turcotte et al., 2003) distributed hydrological model.

2 Methodology

2.1 Delineation of hillslopes

The starting point for the hillslope delineation approach presented in this work is DEM-based drainage network extraction procedures (e.g., Orlandini et al., 2003) that calculate flow paths over an entire watershed. These procedures provide a flow direction matrix that gives the direction of flow for each cell (or pixel) according to the steepest descent direction, a flow accumulation matrix that identifies the upstream grid cell number that flows into each cell, and a subdivision of the watershed into subbasins.

With this extracted information, the hillslope delineation algorithm proceeds as follows. For each headwater subbasin, the area drained by the first pixel of the river segment is designated as a headwater hillslope, while the remaining area defines two lateral hillslopes, one on either side of the river segment. To simplify the analysis, the algorithm considers only those pixels with a flow direction directly towards a current river cell; in so doing the area drained on either side of the river segment can be easily computed. To get these pixels, the algorithm takes as arguments the previous, current, and next river cells. Then, from the flow direction matrix, the eight neighbor cells of the current cell are considered (Fig. 1), in clockwise and counterclockwise directions, until the algorithm finds the next or previous river segment cell. Every cell with flow direction directly towards the current cell is retained and put in one of two tables, right or left for the clockwise and counterclockwise directions, respectively (in Fig. 1, for example, cell number 6 gets placed in the left table). If the algorithm arrives at an intersection of two or more river segments, it can occur that a cell falls into both tables. To resolve this problem, it was decided, without loss of consistency or generality, to assign such cells to the right table for the river segment in question (see for example cell 4 in Fig. 2 and also the resulting delineation shown in Fig. 3). Once every river segment of the network matrix has been scanned, the table for each specific river segment identifies all the cells that will drain the hillslope (Fig. 3). Finally, the algorithm redraws the hilllsope matrix.

Fig. 4 summarizes the algorithm for the hillslope delineation process, while Fig. 5 illustrates different steps of the procedure as applied to the BEREV watershed example that will be presented in more detail in Section 3.

2.2 Determination of plan shape and profile curvature

Once all hillslopes in a watershed have been delineated, characterization of the plan shape and profile curvature represents the next step. The plan shape corresponding to elevation lines taken parallel to the average flow direction for a given river segment is calculated using the DEM. To characterize an elevation line as convergent, divergent, or uniform, a straight reference line is drawn between the first and last cells of that elevation line. An elevation line is then designated as convergent if the majority of its cells falls far enough below the reference line, divergent if its cells fall far enough above the reference line, and uniform otherwise. An arbitrary value of 1 m was set in the algortihm as the threshold above and below the reference line. However, the user may enter another value according to the precision of the DEM. A value of 5 m, for example, was used for the catchment examples in this study. When all the elevation lines have been processed, a convexity ratio is calculated for each hillslope as the number of convergent elevation lines relative to the total number of lines.

An analogous procedure is used to characterize the profile curvature, with the elevation and reference lines in this case taken perpendicular to the average river flow direction. An elevation line is then designated as concave if the majority of its cells fall far enough below the reference line, convex if its cells fall far enough above the reference line, and straight otherwise. The same precision value as in the plan shape analysis is used. When all the elevation lines have been processed, a convexity ratio is calculated for each hillslope as the number of convex elevation lines relative to the total number of lines.

The resulting convexity percentage ratios for the plan shape and the profile curvature, respectively, are then used as inputs to a fuzzy logic algorithm to determine membership in one of the nine elementary landform classes described by Dikau (1989) (Fig. 6). The input and output membership functions are shown in Fig. 7 and the rule matrix is given in Table 1. The algorithm was tested on theoretical three-dimensional forms.

2.3 Extraction of hillslope width functions

Two criteria were applied in the final step of the algorithm for delineating hillslopes and extracting width functions: monotonicity of the HWF and conservation of surface area. The first criterion is imposed in view of the potential application of the algorithm as a pre-processing step for the hillslope-storage Boussinesq model (Paniconi et al., 2003). This hydrological model requires that the width function be monotonically increasing (convergent plan shape), monotonically decreasing (divergent plan shape), or constant (uniform shape) in order to avoid flow singularities along the lateral boundaries of the hillslope. The second criterion, applied to individual hillslopes and to the overall watershed, provides a measure of mass conservation when the resulting hillslopes are used in watershed-scale, rainfall-runoff modelling applications.

The simplest geometric forms that ensure monotonic width functions are triangles for headwater hillslopes, which drain to a single river cell, and quadrilaterals for lateral hillslopes, which drain to a river segment.

2.3.1 Lateral hillslopes

Fig. 8 illustrates the procedure for HWF extraction in the case of lateral hillslopes. The first segment (AD) of the quadrilateral is defined as a line connecting the first and last cells of the river segment and parallel to the average flow direction in the river. Points B and C are then defined by following the boundary cells on the left and right sides, respectively. The algorithm preserves monotonicity and counts the number of hillslope cells inside the quadrilateral to obtain the quadrilateral surface. Then, the algorithm tries to match the original surface area of the hillslope as closely as possible by adjusting the position of points B' and C' according to the slope defined by the segments AB and DC. This will increase or decrease the hillslope surface area (see Fig. 8). The relative accuracy in terms of the second criterion is calculated as follows:

$$\text{relative precision (\%)} = \frac{\text{modelled surface - actual surface}}{\text{actual surface}} \times 100 \qquad (1)$$

A new vector is created with the coordinates of the four points that correspond to the quadrilateral vertices. This vector is then used to calculate the width of the hillslope from the divide to the river segment at an increment equal to the DEM cell size. The HWF is exported as a text file that contains, for each hillslope, the distance from the river segment to the divide and the width at the divide. The HWF extraction algorithm for lateral hillslopes is summarized in Fig. 9.

2.3.2 Headwater hillslopes

Fig 10 illustrates the procedure for HWF extraction in the case of headwater hillslopes, which are always convergent. With point A coincident with the river cell, the triangle is oriented in a way that best respects the general flow direction within the actual hillslope. The algorithm then starts at point A and examines the next cells on the left and right sides, proceeding upslope until the number of cells inside the triangle exceeds the number of cells in the original hillslope. The last two cells examined then get designated as points B and C. Analogously to the lateral hillslope case, the algorithm adjusts points B’ and C’, a relative accuracy is calculated, and the extracted information is exported as a text file.

3 Application

3.1 Description of the study catchments

The Chateauguay River, a tributary ofthe St. Lawrence River, drains a 2500 km^2 transboundary territory that lies 57% within the province of Quebec (Canada) and 43% within the state of New York (USA) (Côté et al., 2006). The des Anglais watershed is the largest subcatchment of the Chateauguay River watershed and has a land cover that is predominantly forest in the south and agricultural to the north. The watershed has a drainage area of 690 km^2, an average annual discharge of 300 x 10^6 m^3, and an elevation range from 30 m to 400 m (Sulis et al., 2011). The aquifer system in this region is part of the St. Lawrence Lowlands and consists of Cambrian to Middle Ordovician sedimentary rocks that are slightly deformed and fractured. Unconsolidated sediments of glacial and post-glacial origin overlay the bedrock aquifer and are of varying thickness, reaching 40 m in the northernmost portion (Tremblay, 2006). These sediments are in turn overlain by Quaternary deposits of silty till and soils that are characterized as mainly weathered Quaternary sediments (Lamontagne, 2005), with the exception of bogs and swamps that overly Champlain sea sediments in the northeastern part of the catchment. The climate is characterized as semi-humid with a mean annual temperature of 6.3 °C and an average annual precipitation of 958 mm (Canadian Daily Cilmate Data, 2004). The DEM used for our analysis has a 90 m horizontal resolution and a 5 m vertical resolution and consists of 497 x 592 cells. The projection is Universal Transversal Mercator (NAD 83) zone 18.

The second catchment selected for analysis is the “Bassin expérimental du ruisseau des Eaux-Volées” (BEREV). It has a drainage area of 9.2 km^2 and is situated 80 km north of Quebec City. It is part of the Montmorency forest in the high hills of the Laurentian mountain chain and has a land cover composed principally of balsam fir with some black and white spruce and white birch. High hills dominate the landscape and the elevation ranges between 990 m and 560 m. The surface geology is composed of glacial and fluvio-glacial tills of depth between 0 and 18 m. The underlying formation is a crystalline mother rock of Precambrian origin composed of charnockitic gneiss. The organic litter has an average thickness of 8 cm and the root depth is 30 cm on average in a podzol ferrohumic soil. This soil layer is very permeable compared to the underlying till and very rapid shallow subsurface flow is often observed. The BEREV catchment discharges into the Montmorency River (Lavigne, 2007). The DEM used for our analysis has a 5 m horizontal resolution and 5 m vertical resolution and consists of 825 x 799 cells. The projection is Quebec modified Transversal Mercator (NAD 83) zone 7.

3.2 Results

The proposed algorithm was used to subdivide the des Anglais and BEREV watersheds into three and eight hillslopes, respectively (Fig. 11 and 12). The extracted plan shapes and profile curvatures are presented in Tables 2 and 3. Figs. 11 and 12 illustrate, visually, the match between the original and extracted hillslope width functions. A more quantitative assessment is provided in Tables 2 and 3, where the resulting convexity ratios for plan shape and profile curvature indicate that 2 out of 3 of des Anglais hillslopes represent a flat area and most of the BEREV hillslopes represent an overall steep watershed. Tables 2 and 3 also give the elementary landform class (Fig. 6) attributed to each extracted hillslope according to the rule matrix of Table 1. These results indicate that the algorithm provides the correct Dikau form according to the linguistic variable and the rule matrix.

Tables 4 and 5 give the results of the extraction procedure in terms of the surface area conservation criterion. The overall surface area for the des Anglais and BEREV watersheds was well preserved, with underestimation of, respectively, 0.656% and 0.180%. The individual hillslopes also conserved surface area very well, with the highest error reaching only 1.019% for one of the hillslopes of the BEREV watershed. These results also show that divergent plan shapes lead to HWF that are generally less representative of the surface area because of the limit imposed by the trapezoidal form.

Tables 6 and 7 present the plan shape and profile curvature convexity percentages and elementary landform classification with an arbitrary resolution on elevation data of 1 m instead of 5 m for the des Anglais and BEREV watersheds. Comparing these results with those of Tables 2 and 3, we note that the convexity ratio is closer to 0.5, affecting even the elementary landform classification of des Anglais hillslope No. 2. Inaccurate elevation data therefore tends to flatten the plan shape and profile curvature, yielding a lower elementary landform class.

4 Conclusions

This paper has described the development of an algorithm to delineate and extract hillslopes and hillslope width functions from gridded elevation data. The algorithm was applied to the des Anglais and BEREV watersheds in Quebec, Canada. Some problems were encountered with the use of this algorithm over a flat watershed such as the des Anglais. When rivers are too sinuous or two or more pixels wide, the algorithm has some difficulties finding the contour cells. Other problems concern the intersection of two or more rivers that converge into one single river. However, sometimes it is possible to solve some of these issues by changing directions of some cells in the flow accumulation matrix. This leads to small changes in the limits of hillslopes and often helps the algorithm work properly.

Further work still needs to be done to increase the robustness of the algortihm developed in this study. Also, a comparison of the methodology presented here to other approaches for hillslope extraction (e.g., Cochrane and Flanagan, 2003; Bogaart and Troch, 2006; Zhong et al., 2009) should be conducted. Finally, the methodology needs to be tested and applied in a hydrological modelling context. To do so, it is important to compare the hydrological dynamics of the original hillslopes to that of the hillslopes derived from the algorithm. This could be done using a detailed model (e.g., CATHY, Camporese et al., 2010) applied in a single or multiple-hillslope context. This will lead to an assessment of the adequacy of some of the choices and approximations made in the algorithm (flow direction, plan shape, profile curvature, elementary form classification, use of simple geometric forms for the HWF, etc.), of some of the fundamental hypotheses (e.g., monotonicity), and of the guiding criteria (conservation of area). Also, it could be interesting to compare the results of a hydrological model (e.g., HYDROTEL, Fortin et al., 2001; Turcotte et al., 2003; Turcotte et al., 2007) run in its “standard” discretization mode (a watershed subdivided into subwatersheds) and in a discretization mode based on the hillslopes derived from the algorithm presented here. This type of testing can begin to address some of the pros and cons of passing from a “subwatershed” to a “hillslope”-based discretization and conceptualization of a watershed.

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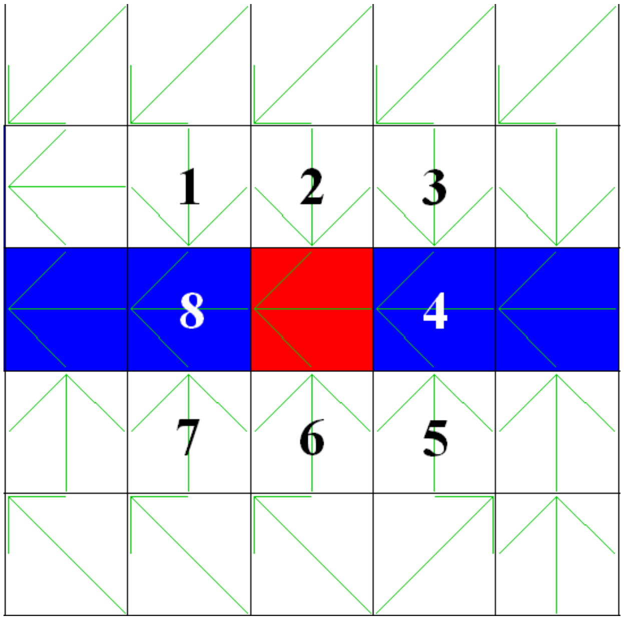

Figure 1. The eight neighbouring cells of a current (red) river network cell (blue).

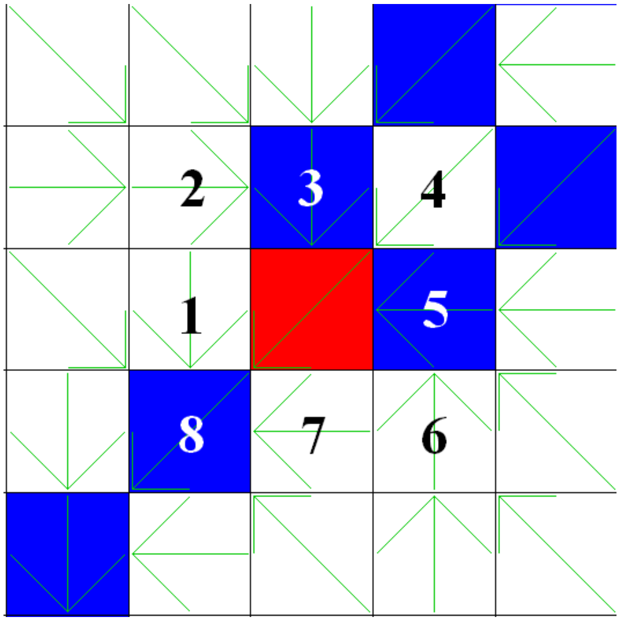

Figure 2. Representation of a cell (no. 4) that could be in two different tables according to the hillslope delineation algorithm.

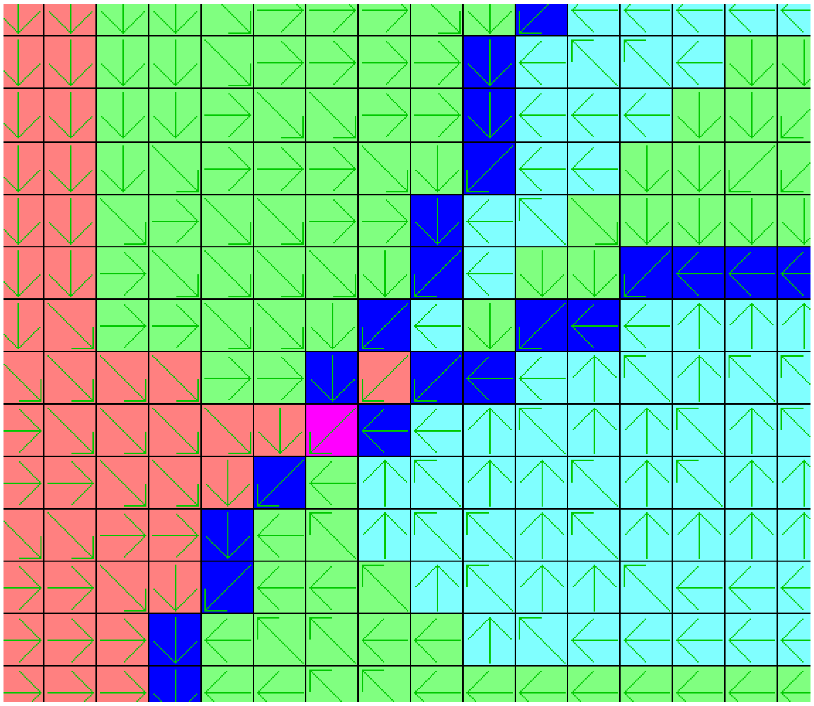

Figure 3. Hillslope delineation result.

```
With accumulation matrix and river network matrix.
while (current cell != river network last cell – 1)
    if current cell == first river network segment cell
        associate headwater hillslope with cells drained by river network current cell
    end
    else
        With previous, current and next river cell
        Clockwise:
        if neighbor cell != river segment cell
            if neighbor cell flow direction == toward current cell
                add neighbor cell to right table
            end
            neighbor cell = neighbor cell + 1
        end
        Counterclockwise
        If neighbor cell != river segment cell
            if neighbor cell flow direction == toward current cell
                add neighbor cell to left table
            end
            neighbor cell = neighbor cell + 1
        end
    end
    previous cell = river network previous cell + 1
    current cell = river network current cell + 1
    next cell = river network next cell + 1
end
```

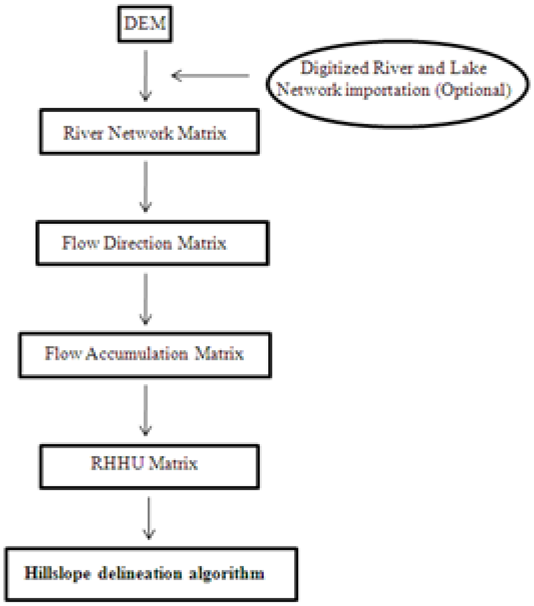

Figure 4. Hillslope delineation algortihm.

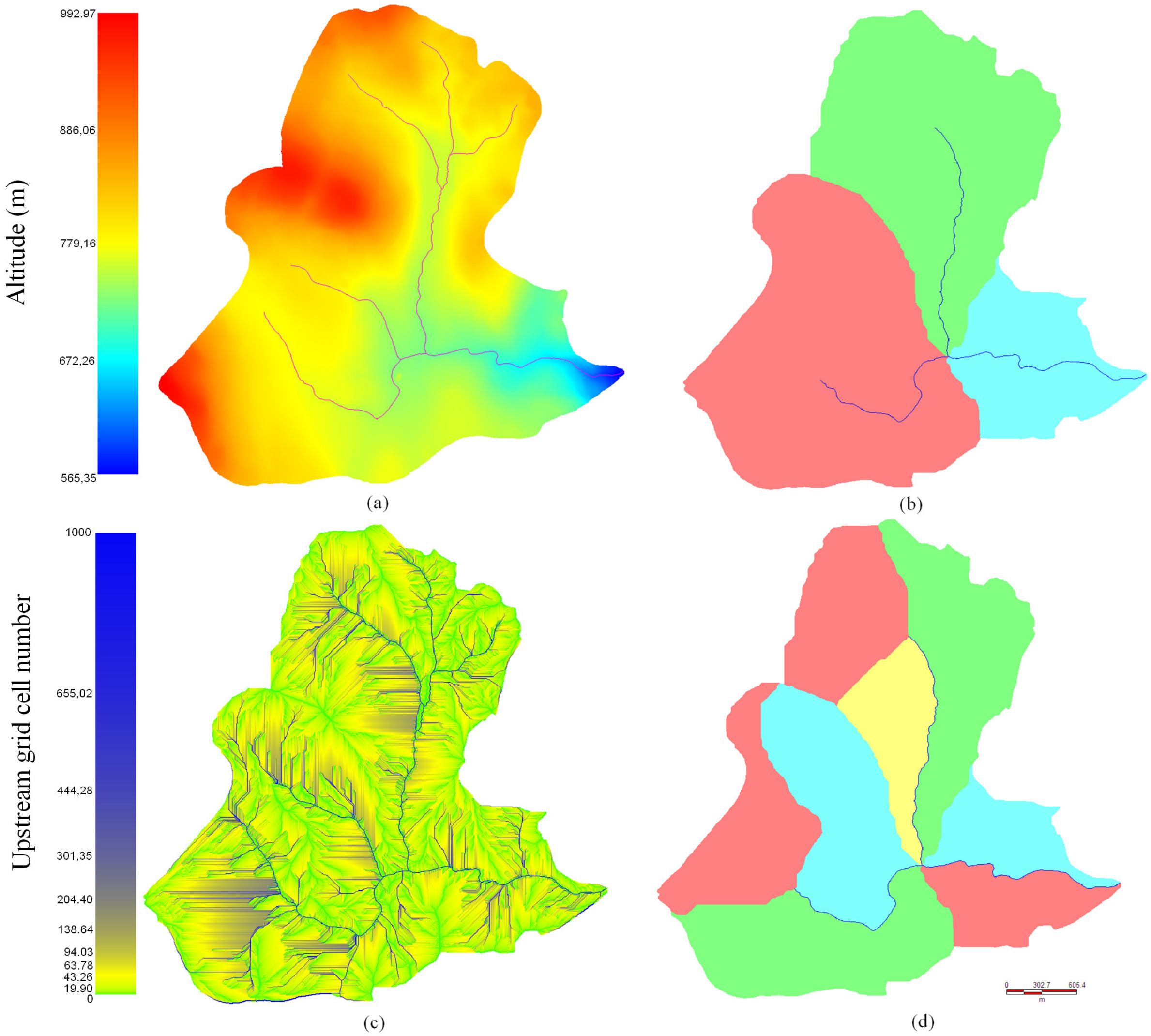

Figure 5. DEM and river network (a), subwatersheds (b), flow accumulation matrix (c), and hillslopes (d) for the BEREV watershed in PHYSITEL.

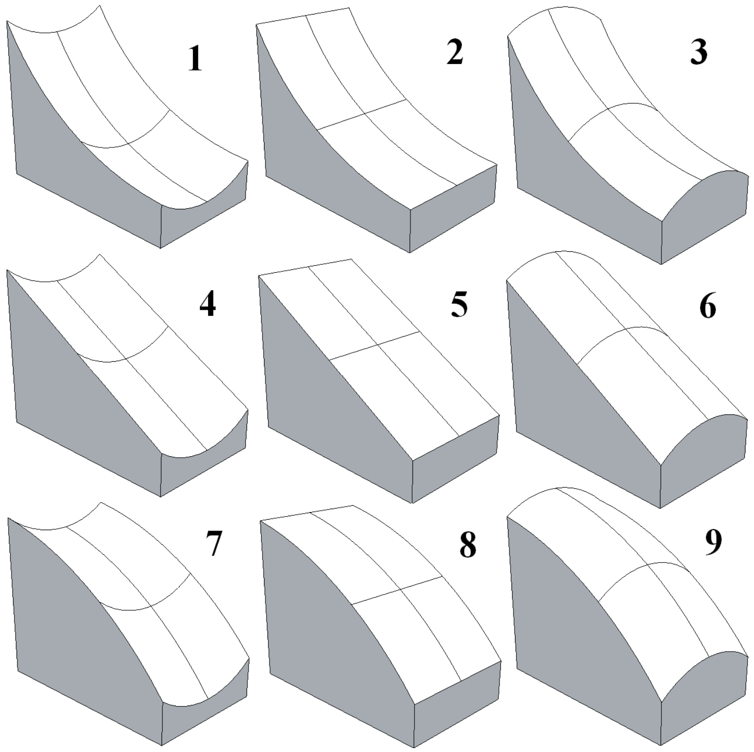

Figure 6. Three-dimensional view of the nine different hillslopes used in this study (Dikau, 1989).

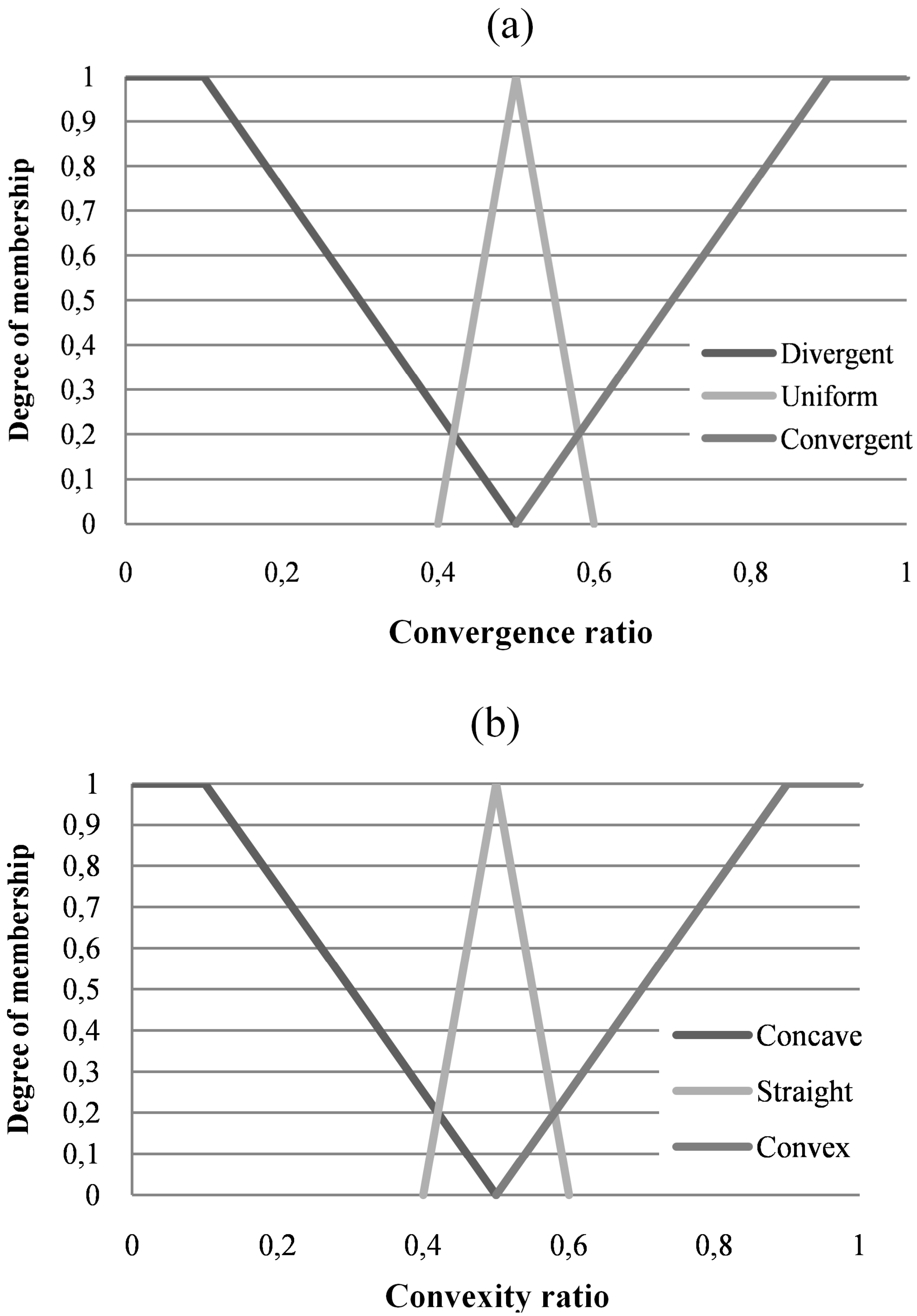
(a)
Degree of membership
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0
0,2
0,4
0,6
0,8
1
Convergence ratio
Divergent
Uniform
Convergent
(b)
Degree of membership
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0
0,2
0,4
0,6
0,8
1
Convexity ratio
Concave
Straight
Convex

Figure 7. Fuzzy logic input (a) plan shape, (b) profile curvature linguistic variables.

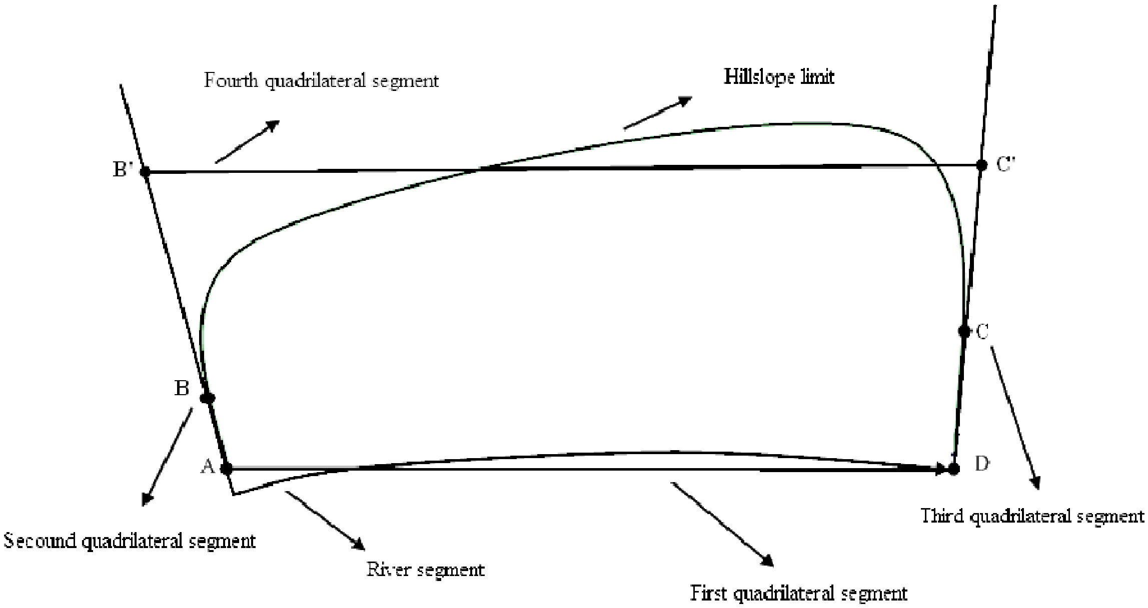

Figure 8. Segments of a quadrilateral modelling of a lateral hillslope with convergent plan shape.

Find original point A, B, C and D

```
Point A = First river segment cell
Point D = Last river segment cell

Adjust point A and D parallel to averarge river
flow direction

Old # hillslope cell inside quadrilateral = 0

while cell left != cell right

    for index left contour cell

    cell left =

      left contour cell at index left contour cell

        if cell left respect plan shape

          cell B =  cell left

        end

    index left contour cell =

      index left contour cell + 1

    end

    for index right contour cell

    cell right =

       right contour cell at index right contour cell

        if cell right respect plan shape

          cell C =  cell right

        end

        index right contour cell =

          index right contour cell + 1

    end

    if # hillslope cell inside quadrilateral >
          old # hillslope cell insidequadrilateral

          Point B = cell B
          Point C = cell C
          old # hillslope cell inside quadrilateral =
               # hillslope cell inside quadrilateral
    end

end
```

Optimize quadrilateral according criterion 2

```
Slope AB = Ax –Bx / Ay – By
Slope CD = Cx – Dx/ Cy – Dy

if slope AB < slope CD
    step = 1 / absolute value of slope AB
end
else
    step = 1 / absolute value of sope CD
end

difference = modeled surface – real surface

if difference < 0
     Expand quadrilateral with step
    Bigger = true
end
else
     Shorten quadrilateral with step
    Bigger = false
end

olddiffrence = difference
difference = new modeled surface – real surface

while | oldStep – step < = 0,5 |

     olddiffrence = difference
    difference = new modeled surface – real surface
    oldStep = step

    if oldDifference > 0 and difference < 0
          step = step + step/2
    end
    else if oldDifference > 0 and difference > 0
          step = step - step/2
    end
    else
          step = 2 X step
    end

    if Bigger
        Shorten quadrilateral with step
    end
    else
        Shorten quadrilateral with step
    end

end
```

Figure 9. HWF extraction algorithm.

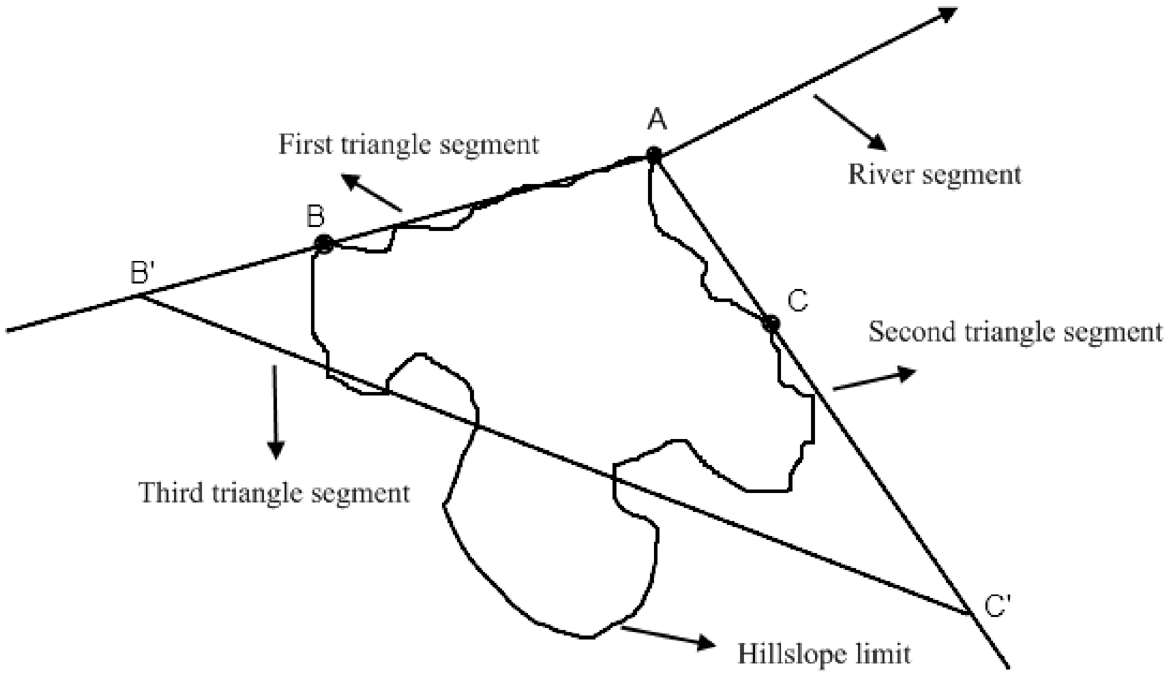

Figure 10. Segments of a triangular modelling of a headwater hillslope.

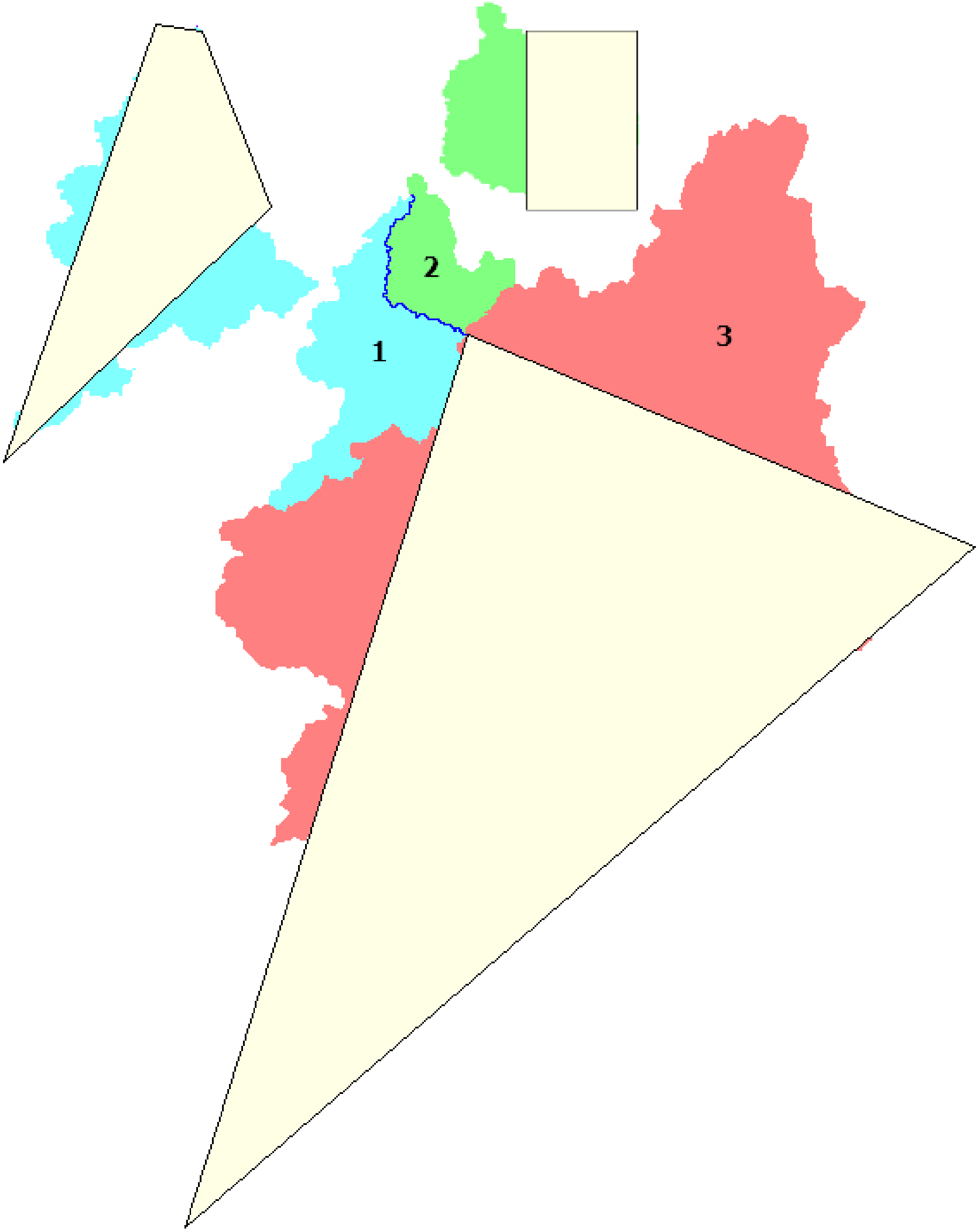

Figure 11. HWF results for the Des Anglais watershed.

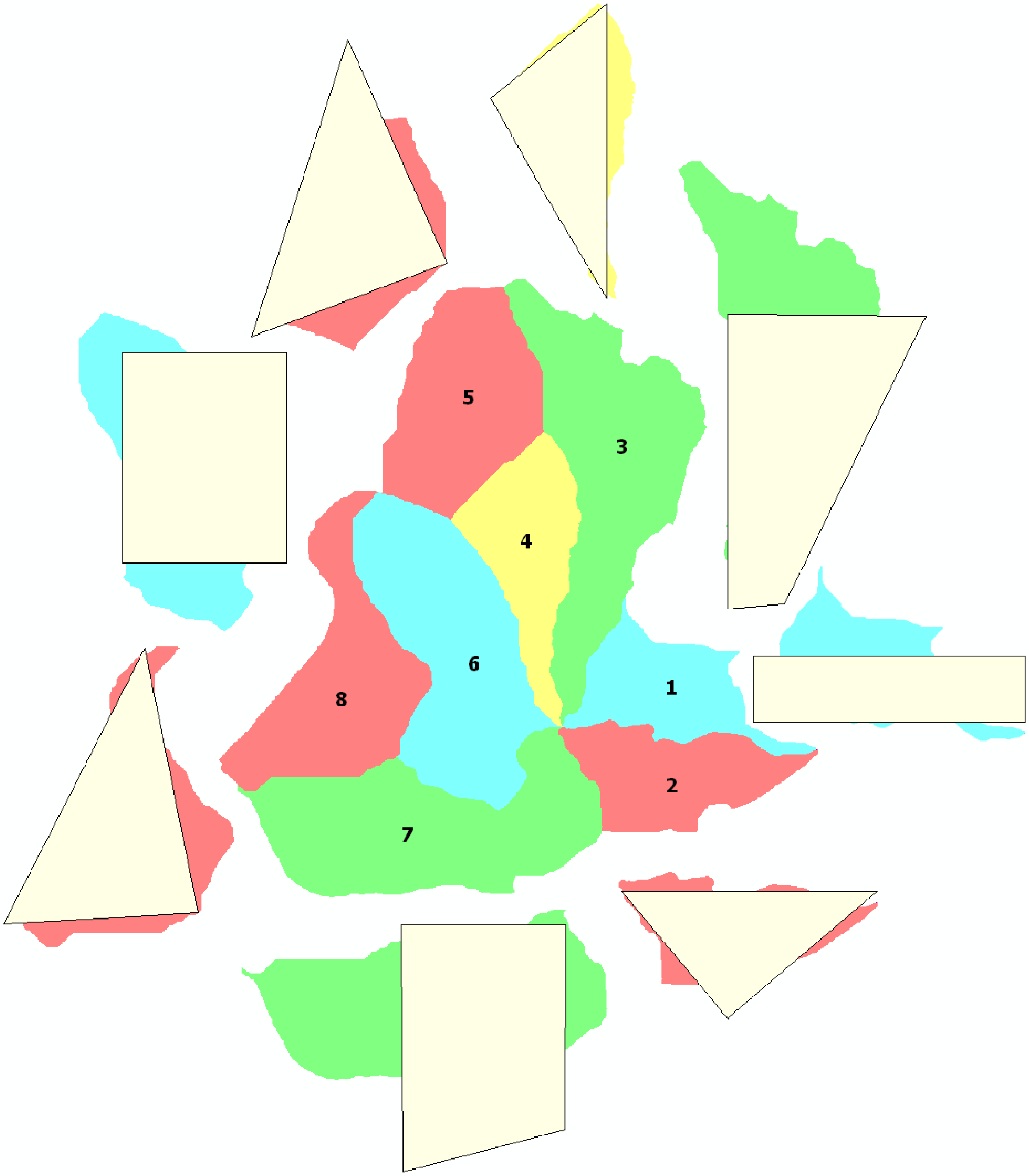

Figure 12. HWF results for the BEREV.

Table 1. Fuzzy rule matrix

IF Curvature/shape		THEN
Profile	Plan	Form
Concave	Divergent	1
Concave	Uniform	2
Concave	Convergent	3
Straight	Divergent	4
Straight	Uniform	5
Straight	Convergent	6
Convex	Divergent	7
Convex	Uniform	8
Convex	Convergent	9

Table 2. Plan shapes and profile curvatures for each hillslope of the des Anglais watershed and associated Dikau form

Hillslope	Plan shape (% convexity)	Profile curvature (% convexity)	Dikau form
1	Convergent (24.2)	Concave (39.1)	1
2	Uniform (52.0)	Straight (53.2)	5
3	Uniform (52.0)	Straight (56.5)	5

Table 3. Plan shapes and profile curvatures for each hillslope of the BEREV watershed and associated Dikau form

Hillslope	Plan shape (% convexity)	Profile curvature (% convexity)	Dikau form
1	Uniform (50.3)	Straight (49.8)	5
2	Divergent (64.2)	Convex (63.0)	9
3	Divergent (77.6)	Convex (69.0)	9
4	Divergent (62.9)	Convex (59.6)	9
5	Convergent (29.3)	Convex (74.7)	7
6	Uniform (45.8)	Concave (29.0)	2
7	Divergent (62.6)	Concave (39.2)	3
8	Convergent (37.5)	Concave (37.2)	1

Table 4. Relative surface area precision obtained for each HWF of the des Anglais watershed

Hillslope	Relative surface area precision (%)
1	-0.897
2	0.380
3	-0.138
Total	-0.656

Table 5. Relative surface area precision obtained for each HWF of the BEREV watershed

Hillslope	Relative surface area precision (%)
1	-0.401
2	-0.363
3	-0.256
4	-0.037
5	-0.214
6	-0.006
7	1.019
8	0.079
Total	-0.180

Table 6. Plan shapes and profile curvatures for each hillslope of the des Anglais watershed and associated Dikau form with a 1-m resolution on elevation data

Hillslope	Plan shape (% convexity)	Profile curvature (% convexity)	Dikau form
1	Convergent (17.3)	Concave (33.3)	1
2	Uniform (52.7)	Straight (58.3)	8
3	Uniform (54.5)	Straight (51.6)	5

Table 7. Plan shapes and profile curvatures for each hillslope of the BEREV watershed and associated Dikau form with a 1-m resolution on elevation data

Hillslope	Plan shape (% convexity)	Profile curvature (% convexity)	Dikau form
1	Uniform (53.1)	Straight (49.0)	5
2	Divergent (67.4)	Convex (73.1)	9
3	Divergent (91.6)	Convex (78.7)	9
4	Divergent (65.5)	Convex (63.3)	9
5	Convergent (21.2)	Convex (79.4)	7
6	Uniform (51.8)	Concave (22.4)	2
7	Divergent (63.6)	Concave (34.2)	3
8	Convergent (38.1)	Concave (40.1)	1