Eq. 1 can be written as

$$b_k = A\, b_{k-1} + B\, y_k$$

with $A = \dfrac{(1-BFI_{\max})a}{1-a\,BFI_{\max}}$ and $B = \dfrac{(1-a)BFI_{\max}}{1-a\,BFI_{\max}}$.

$b_k$ is now traced back to the initial value $b_0$:

$$\begin{aligned}
b_k &= A\, b_{k-1} + B\, y_k \\
&= A\,(A\, b_{k-2} + B\, y_{k-1}) + B\, y_k \\
&= A^2\, b_{k-2} + A\, B\, y_{k-1} + B\, y_k \\
&= \ldots \\
&= A^{\mathrm{k}}\, b_0 + B \sum_{i=1}^{k} A^{i-1}\; y_{k-i+1}
\end{aligned}$$

The partial derivative of $b_k$ with respect to $b_0$ is

$$\frac{\partial b_k}{\partial b_0} = A^{\mathrm{k}}$$

The partial derivative of *BFI* with respect to $b_0$ is

$$\begin{aligned}
\frac{\partial BFI}{\partial b_0} &= \frac{\partial}{\partial b_0}\frac{b}{y} \\
&= \frac{1}{y}\sum_{k=1}^{n}\frac{\partial b_k}{\partial b_0} \\
&= \frac{1}{y}\sum_{k=1}^{n} A^k
\end{aligned}$$

For long time series:

$$\frac{\partial BFI}{\partial b_0} \approx \frac{1}{y}\lim_{n\to\infty}\sum_{k=1}^{n} A^k$$

Because of $|\,A\,| < 1$, the limit of the geometric series $\sum_{k=1}^{n} A^k$ for $n \to \infty$ is $\dfrac{A}{1-A}$:

$$\frac{\partial BFI}{\partial b_0} \approx \frac{1}{y}\frac{A}{1-A}$$

$$= \frac{1}{y}\,\frac{(1-BFI_{\max})\,a}{1-a}$$

Small errors in $b_0$ cause an error in *BFI* of

$$\Delta_{b_0} BFI \;=\; \frac{\partial BFI}{\partial b_0}\,\Delta b_0$$

$$\approx \frac{1}{y}\,\frac{(1-BFI_{\max})\,a}{1-a}\,\Delta b_0$$

The sensitivity index for $b_0$ is

$$S(BFI \mid b_0) \;=\; \frac{\Delta_{b_0} BFI}{BFI} \Big/ \frac{\Delta b_0}{b_0}$$

$$\approx \frac{1}{y}\,\frac{(1-BFI_{\max})\,a}{1-a}\,\Delta b_0\,\frac{b_0}{BFI\;\Delta b_0}$$

With $y = b \,/\, BFI$:

$$S(BFI \mid b_0) \;\approx\; \frac{(1-BFI_{\max})\,a}{1-a}\,\frac{b_0}{b}$$

For long time series, $b_0$ is much smaller than $b$ and, hence, $S(BFI \mid b_0) \approx 0$.