**Spatial moments of catchment rainfall:**

**rainfall spatial organisation, basin morphology, and flood response**

**by: Davide Zoccatelli, Marco Borga, Alberto Viglione, Giovanni Battista Chirico,**

**Günter Blöschl**

**Manuscript number: hess-2011-186**

We thank the reviewer for her/his comments, which helped to clarify some points and improve the revised version of the paper. The reviewer's comments are quoted above the authors responses.

**Main Comment 1**

The authors mention that their work builds on Woods and Sivapalan (1999) and Viglione et al. (2010). However it is difficult to judge how the proposed statistics are similar/different from these previous works, and what is really new in the proposed framework.

**Response**

We have introduced an Appendix to illustrate how the Spatial Moments may be derived based on the work in Woods and Sivapalan (1999) and Viglione et al. (2010).

**Main Comment 2**

At the beginning, simplifying hypotheses are stated, but not discussed enough. Some of them seem to correspond to simplifications of the Viglione et al. (2010) framework (for instance the authors neglect hill slope travel time whereas Viglione et al. (2010) consider it). The assumption of a constant rainfall coefficient is also a strong one. On the other hand, most of these hypotheses are removed when using the distributed hydrological model. So what can be compared between the analytical framework and the model results?

**Response**

We have introduced a new paragraph in the Introduction of the revised paper to address the comment by the Reviewer.

"The conceptual meaning of the Spatial Moments is illustrated by means of application to five extreme flash floods occurred in various European regions in the period 2002-2007. High resolution, carefully controlled, radar rainfall fields and a spatially distributed hydrologic model are employed to examine the use of these statistics to describe the degree of spatial rainfall organisation which is important for runoff modelling, with a focus on runoff timing. The size of the study catchments ranges between 36 to 982 $km^2$. Hillslope residence time and spatial variability of runoff ratio, which are disregarded in the derivation of the Spatial Moments, are included in the distributed hydrological model. Therefore, contrasting model results with information inferred from the Spatial Moments provides a necessary evaluation of the impact of the working assumptions on the use of these statistics, at least in the context of extreme floods. "

**Main Comment 3**

In addition, the applications show the interest of the approach, but it is unclear how it could be used in practice: what are the required data for the application of this method, which accuracy is required on rainfall descriptions, on the river network description? What can be expected when calculating those statistics?

**Response**

In the introduction of the revised paper, we specify that the main purposes for introducing the Spatial Moments are as follows: i) to provide a theoretical foundation for various measures of rainfall spatial variability based on the flow distance coordinate, which have been reported in the literature in the last decade (Smith et al., 2002, 2005 ; Syed et al., 2003; Sangati et al., 2009); ii) to allows for the introductionof the concept of catchment scale storm velocity; and iii) to extend to the case of runoff propagation under condition of spatial rainfall variability the concept of Spatial Moments used for analysis of solute transport in porous media (Goltz and Roberts, 1987). The development of this similarity, which is not pursued in this paper but is subject of current investigation, aims to order theoretical results appeared in disparate fields into a coherent theoretical framework for both hydrologic flow and transport, as shown by Rinaldo et al. (2006).

Sensitivity analyses of the Spatial Moments to uncertainty in radar rainfall estimates and morphological parameters are on going and will be reported soon.

**Specific Comment 1**

p.5813, lines 20-25. The authors propose to "introduce measures to quantify the catchment filtering effect which, as a function of rainfall organization, basin scale and the heterogeneities embedded in the basin geomorphic structure, control the possible extent of the influence of rainfall spatial organisation on the hydrologic response." To what extend is this objective reached in the paper? This point would require further developments in the discussion in relation with the illustration of the practical use of the approach.

**Response**

The analysis of the case studies reported in the paper suggests that the spatial moments are effective in (i) describing the degree of spatial organisation which is important for runoff modelling and (ii) quantifying the effects of neglecting the spatial rainfall variability on flood modeling. Our analysis is currently focused on the timing error and will be extended to other hydrograph features in future works.

**Specific Comment 2**

p.5814, lines 5-10. The authors should better justify/discuss the hypothesis that "Runoff routing through branched channel networks imposes an effective averaging of spatial rainfall excess at equal flow distance, in spite of the inherent spatial variability."

**Response**

We modified the Introduction as follows: "Runoff routing through branched channel networks imposes an effective averaging of spatial rainfall excess across locations with equal routing time, in spite of the inherent spatial variability. The flow distance coordinate may be used as a surrogate for travel time, when the hydrograph response is determined mainly by the distribution of travel times, neglecting hydrodynamic dispersion, and variations in runoff propagation celerities may be disregarded. This implies that rainfall spatial organisation measured along the river network by using the flow distance coordinate may be a significant property of rainfall spatial variability when considering flood response modelling."

**Specific Comment 3**

p.5815, lines 14-15. The authors say that their work is a generalization of Viglione et al. (2010) approach. But it seems to be more a simplification than a generalization

**Response**

We dropped the term 'generalises' in the revised version.

**Specific Comment 4**

p.5816, lines 5-10. The hypothesis that hill slope travel time cannot be neglected in the analysis is strong. The authors should discuss it a little more, all the more than hill slope travel time is considered in the model used in section 3.

**Response**

For this comment, please see our response to the Main Point 2 above.

**Specific Comment 5**

p.5817, Eq.(2). This equation provides a kind of average distance to the catchment outlet, for n=1. As hill slope travel time is neglected, is the calculation performed only on river network grid points?

**Response**

As hillslope travel time is neglected, all the catchment grid elements are connected to the outlet through the flow paths and only one flow celerity. This is the foundation for using flow distance in place of routing time.

We made clear this concept by introducing the following text in the revised Introduction:

"The flow distance coordinate may be used as a surrogate for travel time, when the hydrograph response is determined mainly by the distribution of travel times, neglecting hydrodynamic dispersion, and variations in runoff propagation celerities may be disregarded."

Moreover, the definition of the Spatial Moments has been revised to make clear this point: "Spatial moments of catchment rainfall provide a description of overall spatial rainfall organisation at a certain time t, as a function of the rainfall field r(x,y,t) (L T-1) value at any position x,y inside the watershed and of the distance d(x,y) (L) between the position x,y and the catchment outlet measured along the flow path."

**Specific Comment 6**

p.5817, lines 12-18: a scheme/figure could help the reader in understanding the physical interpretation of the proposed scaled moments.

**Response**

The extension of the text in this point of the revised paper should suffice to improve understanding of the interpretation of the spatial moments.

**Specific Comment 7**

p.5817 Eq. (5). The definition of the instantaneous δ and temporally integrated statistics Δ are similar. Is it possible to derive analytical relationships between both quantities?

**Response**

The temporally integrated statistics Δare obtained by integration over time of the instantaneous statistics δ(t). We clarified this point in the revised version of the paper.

**Specific Comment 8**

p.5818, Eq.(6). The introduction of the catchment scale velocity requires further development. What are the rationale behind the definition proposed in Eq. (6)?

**Response**

We modified test in this Section of the revised version as follows:

"The concept of the catchment-scale storm velocity as defined by Eq. 6 takes into account the role of relative catchment orientation and morphology with respect to storm motion and kinematics. For instance, for the same storm kinematics, the same elongated basin will be subject to different catchment scale storm velocities with varying its orientation with respect to that of the storm motion. In this work, we will not perform any explicit derivative of $\delta_1$ to obtain the catchment scale storm velocity. Equation (6) has been introduced only to formally represent the concept of storm velocity and how this relates to the first scaled moment $\delta_1$. A simple way to derive the mean value of $V_s$, derived from the methodology introduced by Viglione et al. (2010), is reported in the next sections."

**Specific Comment 9**

p.5820, lines 5-8. The sentence is not clear.

**Response**

We modified text as follows: "The hypothesis of spatially uniform flow velocity is consistent with the results of previous studies, showing that it is always possible to substitute a given pattern of flow velocity with a pattern of uniform flow velocity, without changing the catchment response time. (Robinson et al., 2005; Saco and Kumar, 2002; D'Odorico and Rigon, 2003)".

**Specific Comment 10**

p.5810 Eq.(9) (but also Eq. (13) and (15). An appendix providing more details on the derivation of these equations would be useful to the reader. In addition, such appendix could permit a better explanation of the links/differences between the relationships found in Viglione et al. (2010).

**Response**

The Appendix in the revised version described the mathematical derivations of the equation based on V2010.

**Specific Comment 11**

p.5821, lines 22-25. These sentence seem to be trivial and evident. I guess that the proposed statistics allow the derivation of more quantitative conclusions. It could be better explained in the paper.

**Response**

We modified this section as follows:

"Based on Eq. (10), the statistic $\Delta_1$ is expected to control the hydrograph timing shift related to the position of the rainfall centroid over the catchment. As it will be shown later in the paper, the statistic $\Delta_1$ is related to the normalised mean time difference between the hydrograph obtained by considering the actual rainfall pattern and the hydrograph obtained by neglecting the spatial rainfall pattern (all other factors being equal). The normalising quantity is given by the response time of the catchment. The effect of a less-than-one value of $\Delta_1$ indicates an anticipation of the mean hydrograph time with respect to the case of spatially uniform precipitation. The opposite holds true for the case of a less-than-one value of the statistic. To show this in quantitative terms, this means that a value of $\Delta_1$ equal to 1.5 indicates that the mean time difference between the two hydrographs corresponds to half the catchment response time, with the hydrograph obtained from the spatially distributed rainfall delayed with respect to the one obtained from uniform rainfall. A value of $\Delta_1$ equal to 0.5 indicates the same normalized mean difference, but with the opposite sign (the hydrograph obtained from the spatially distributed rainfall is anticipated with respect to the one obtained from uniform rainfall by half the catchment response time)."

**Specific Comment 12**
p.5822, lines 1-4. The conclusion that the storm velocity is independent of E(Tq), derives directly from the hypotheses stated about the flow velocity. To what extend can this conclusion be generalized?

**Response**
We are currently investigating this aspect based on an analytical model of planar flow, for which the kinematic model can be solved thanks to the characteristic method. This analysis will provide an answer at the very question the reviewer is asking here.

**Specific Comment 13**
p.5822, lines 15-23. A scheme/figure could be useful to visualize what the authors mean here.

**Response**
The extension of the text in this point of the revised paper should suffice to improve understanding of the meaning of $\Delta_1$ and $\Delta_2$.

**Specific Comment 14**
p.5823, Eq.(16). The authors introduce a new storm velocity. How does it relates to the one introduced in Eq.(6)?

**Response**
We thank the reviewer for noticing the error. We modified Eq (6) accordingly.

**Specific Comment 15**
p.5826, lines 13-15. This sentence is not very clear.

**Response**
We modified text as follows:

"The values of catchment scale storm velocity were computed based on Eq. (16) by computing the two velocity terms $V_{s1}$ and $V_{s2}$ . The slope terms in the linear regressions used to provide $V_{s1}$ and $V_{s2}$ were computed by using a moving window with window size equal to the response time of the corresponding catchment. "

**Specific Comment 16**
p.5827, line 11-12. Clarify better what you mean with flow celerity and how it is related to the storm velocity?

**Response**
We modified text as follows: "In the three cases, the values of the storm velocity are relatively small with respect to the flood flows celerity characterizing flash floods, which was quantified as 3 $ms^{-1}$ by *Marchi et al*. (2010) with reference to several flash floods in Europe. Previous work on the impact of storm velocity on hydrograph shape (*Ogden et al.*, 1995) has shown that the effect of

storm velocity is important when its magnitude become comparable to that of flood flow celerity. The significant differences between storm velocity and flood flows celerity suggests that even for these cases the values of storm velocity may be not large enough to influence the flood hydrograph shape.”

**Specific Comment 17**
Section 5. The use of the model to assess the relevance of the proposed spatial scale moments is not clearly presented. In particular, the authors should better explain why they retain a model which does not fulfil the hypotheses they have made in the analytical framework (neglecting the hill slope travel time, a constant runoff coefficient, etc..) 18) p.5832, lines 23-26. These sentences are not very clear. How do the authors conciliate their analytical simplified approach and that of the detailed model?

**Response**
In the revised Introduction, we clarified the purpose of this comparison, as follows:

“Hillslope residence time and spatial variability of runoff ratio, which are disregarded in the derivation of the Spatial Moments, are included in the distributed hydrological model. Therefore, contrasting model results with information inferred from the Spatial Moments provides a necessary evaluation of the impact of the working assumptions on the use of these statistics, at least in the context of extreme floods. “

**Specific Comment 18**
p.5833, lines 18-25. How the method could be used for other catchments? Which data are required? With which accuracy?

**Response**
In our view, these are exactly the new questions which can be answered thanks to the introduction of the spatial moments. We introduced the following new text in the revised version:

“This provides new statistics and criteria both for defining the optimality of raingauge network design in areas where flash floods are expected and for evaluating the accuracy of radar rainfall estimation algorithms and attendant space-time resolution. “

**Specific Comment 19**
p.5834, lines 5-7. This is an interesting perspective for the use of the method. Could the authors elaborate a little more about this possibility to contribute to comparative hydrology?

**Response**
We introduced the following text in the revised version:

“With the use of the spatial moments, the interaction of rainfall forcing and catchment characteristics could be now described not only in terms of mean areal rainfall, but also by considering the rainfall spatial concentration and the storm velocity. For example, this may help to reveal the effect of orography not only on the precipitation accumulation at the catchment scale, but also on the space-time organization of the rainfall patterns. "

**Specific Comment 20**
Plates 1 and 2 are very small and difficult to read. In addition, the authors could use the same scale between catchments for easier comparison.

**Response**
We have modified the plates to improve their readability.

**Specific Comment 21**
Figure 4 and 5: they could be shown with the same scale, so that they can be compared more easily.

**Response**
Using the same scale details are lost in Figure 4. Owing to this reason, we have used the original scale in Figure 4 of the revised version.

**APPENDIX**

In this Appendix we show how Eqs. (10), (13) and (15) may be derived from *V2010*.

**Derivation of Equation (10)**

Equation (19) in *V2010* provides the average time to route the rainfall excess from the geographical centroid of the rainfall spatial pattern to the catchment outlet. Using the same notation used in the current work, it is written down as follows:

$$E(T_c) = \frac{g_1}{v} + \frac{\mathrm{cov}_{x,y}\left(d(x,y), r_t(x,y)\right)}{vP_0} \qquad \text{(A1)}$$

where $Cov_{x,y}(\ )$ is the spatial covariance.

Eq. A1 is developed as follows to derive Eq. (10):

$$E(T_c) = \frac{g_1}{v} + \frac{\mathrm{cov}_{x,y}\left(d(x,y), r_t(x,y)\right)}{vP_0} = $$
$$= \frac{g_1}{v} + \frac{\int_A d(x,y) r_t(x,y) dA}{vP_0} - \frac{g_1}{v} = \frac{P_1}{P_0 v} = \frac{\Delta_1 g}{v} \qquad \text{(A2)}$$

**Derivation of Equation (13)**

Equation (23) in *V2010* provides the variance of the time to route the rainfall excess to the catchment outlet:

$$Var(T_c) = \frac{g_2 - g_1^2}{v^2} + \frac{\mathrm{cov}(d(x,y)^2, r_t(x,y))}{v^2 P_0} +$$
$$- \frac{\mathrm{cov}_{x,y}\left(d(x,y), r_t(x,y)\right)}{vP_0}\left[2\frac{g_1}{v} + \frac{\mathrm{cov}_{x,y}\left(d(x,y), r_t(x,y)\right)}{vP_0}\right] \tag{A3}$$

Eq. A3 is developed as follows to derive Eq. (13):

$$Var(T_c) = \frac{g_2 - g_1^2}{v^2} + \frac{\int_A d(x,y)^2 r_t(x,y)dA}{Av^2 P_0} - \frac{g_2}{v^2} - \left(\frac{P_1}{P_0 v} - \frac{g_1}{v}\right)\left(\frac{P_1}{P_0 v} + \frac{g_1}{v}\right) =$$
$$\left(\frac{P_2}{P_0} - \frac{P_1^2}{P_0^2}\right)\frac{1}{v^2} = \frac{\Delta_2}{v^2}(g_2 - g_1^2)$$

**Derivation of Equation (15)**

Equation (25) in *V2010* provides the covariance between the rainfall time and the routing time:

$$Cov(T_r, T_c) = \frac{Cov_t\left[T, Cov_{xy}(d(x,y), r_t(x,y))\right]}{vP_0} - \frac{Cov_t\left[T, p_0(t)\right]}{P_0}\frac{Cov_{xy}\left[d(x,y), r_t(x,y)\right]}{vP_0} \tag{A4}$$

Eq. A4 is developed as follows to derive Eq. (15):

$$Cov(T_r, T_c) = g_1\frac{Cov_t\left[T, \delta_1(t)w(t)\right]}{v} - g_1\frac{Cov_t\left[T, w(t)\right]}{v} - \frac{Cov_t\left[T, w(t)\right]}{v}\frac{Cov_{xy}\left[D, P(x,y)\right]}{P_0} =$$
$$g_1\frac{Cov_t\left[T, \delta_1(t)w(t)\right]}{v} - \frac{Cov_t\left[T, w(t)\right]}{v}(g_1 + \Delta_1 g_1 - g_1) =$$
$$g_1\left\{\underbrace{\frac{Cov_t\left[T, \delta_1(t)w(t)\right]}{v}}_{term1} - \underbrace{\frac{Cov_t\left[T, w(t)\right]}{v}}_{term2}\Delta_1\right\}$$

--

**References**

D'Odorico, P. and Rigon, R., 2003: Hillslope and channel contributions to the hydrologic response. *Water Resour. Res.* 39 (5), 1113. doi:10.1029/2002WR001708.

Goltz, M. And P.V. Roberts, 1987: Using the Method of Moments to Analyze Three-Dimensional Diffusion-Limited Solute Transport From Temporal and Spatial Perspectives. *Water Resour. Res.*, 23(8), 1575-1585.

Marchi L., M. Borga, E. Preciso and E. Gaume, 2010: Characterisation of selected extreme flash floods in Europe and implications for flood risk management. *J. Hydrol,* 394 (1–2), *118–133.* doi:10.1016/j.jhydrol.2010.07.017.

Ogden, F.L., Richardson, J.R. and Julien, P.Y., 1995: Similarity in catchment response: 2. Moving rainstorms. *Water Resour. Res.,* Vol. 31, No. 6, pp. 1543-1547.

Rinaldo, A., G. Botter, E. Bertuzzo, A. Uccelli, T. Settin, and M. Marani, 2006: Transport at basin scales: 1. Theoretical framework. *Hydrology and Earth System Sciences*, 10, 19–29.

Robinson, J., Sivapalan, M., Snell, J., 1995: On the relative roles of hillslope processes, channel routing, and network geomorphology in the hydrologic response of natural catchments. *Water Resour. Res.*, 31 (12), 3089–3101.

Saco, P., Kumar, P., 2002: Kinematic dispersion in stream networks – 1. Coupling hydraulic and network geometry. *Water Resour. Res.*, 38 (11), 1244. doi:10.1029/2001WR000695.

Sangati, M., M. Borga, D. Rabuffetti and R. Bechini, 2009: Influence of rainfall and soil properties spatial aggregation on extreme flash flood response modelling: an evaluation based on the Sesia river basin, North Western Italy. *Adv. Water. Res*, 32, 7, 1090-1106, 2009.

Smith, J. A., M. L. Baeck, J. E. Morrison, P. L. Sturdevant-Rees, D. F. Turner-Gillespie and P. D. Bates, 2002: The regional hydrology of extreme floods in an urbanizing drainage basin. *J. Hydrometeorol.*, 3(3), 267 – 282.

Smith, J. A., M. L. Baeck, K. L. Meierdiercks, P. A. Nelson, A. J. Miller, E. J. Holland, 2005: Field studies of the storm event hydrologic response in an urbanizing watershed. *Water Resour. Res.*, 41, W10413, doi:10.1029/2004WR003712.

Syed, K., Goodrich, D., Myers, D., Sorooshian, S., 2003: Spatial characteristics of thunderstorm rainfall fields and their relation to runoff. *J. Hydrol.* 271, 1–22.

Viglione A., Chirico G.B., Woods R., Blöschl G., 2010: Generalised synthesis of space–time variability in flood response: An analytical framework. *J. Hydrol*, doi:10.1016/j.jhydrol.2010.05.047.

Woods, R.A. and M. Sivapalan, 1999: A synthesis of space-time variability in storm response: Rainfall, runoff generation and routing. *Water Resour. Res.* , 35(8), 2469-2485.

Zoccatelli, D., M. Borga, A. Viglione, G. Chirico, G Blöschl, 2011. Spatial moments of catchment rainfall: rainfall spatial organisation, basin morphology, and flood response. *Hydrology and Earth System Sciences Discussion*, 8, 5811-5847. doi: 10.5194/hessd-8-5811-2011.