Spatial moments of catchment rainfall:

rainfall spatial organisation, basin morphology, and flood response

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We thank Ross Woods for his comments, which helped to clarify some points and improve the revised version of the paper. The reviewer's comments are quoted above the authors responses.

Main Comment 1

"In this paper, the rainfall spatial organization is analysed with respect to the flow distance, i.e. the distance along the runoff flow path from a given point to the outlet." The authors need to make explicit what is the rationale for this choice – i.e., the assumption that distance is a useful surrogate for travel time (variations in celerity can be neglected), and that this travel time (and its variation) is an important determinant of hydrograph response (hydrodynamic dispersion can be neglected)

Response

We agree with the reviewer. The text has been modified as follows "Runoff routing through branched channel networks imposes an effective averaging of spatial rainfall excess across locations with equal routing time, in spite of the inherent spatial variability. The flow distance coordinate may be used as a surrogate for travel time, when the hydrograph response is determined mainly by the distribution of travel times, neglecting hydrodynamic dispersion, and variations in runoff propagation celerities may be disregarded. This implies that rainfall spatial organisation measured along the river network by using the flow distance coordinate may be a significant property of rainfall spatial variability when considering flood response modelling."

Main Comment 2

"the river network geometry plays a central role in the structure of the catchment dampening properties" This statement is unclear to me; a reader might infer that the network is always a dominant factor in determining the damping. The work of Woods and Sivapalan (1999) and Viglione et al (2010) (hereafter V2010) both show that network geometry dominates only in some cases.

Response

We agree with the reviewer. The text has been modified as follows "Observational and modelling studies have shown that the river network geometry plays a central role in the structure of the catchment dampening properties, particularly for cases of extreme floods when the impact of land properties heterogeneity on runoff generation is less significant with respect to moderate floods and the stream network extends to previously unchanneled topographic elements, hence increasing drainage density and flow response rate of hillslopes."

Main Comment 3

Equations (3) & (5). The positive and negative signs of covariance terms of V2010 contain much the same information as scaled rainfall moments being greater or less than unity. Equations (3) & (5). Do the variables δ_1 and δ_2 or Δ_1 and Δ_2 correspond to variables in V2010? Which ones? Are they generalisations of components of the moments of network travel time described in Woods and Sivapalan (1999; Appendix C3)? Zoccatelli et al. (2011) (hereafter Z2011) need to make clear the significance of Equations (3) & (5) in relation to what was already done.

Response

The storm averaged first spatial moment of the catchment rainfall (P_1, Eqs 1&4) corresponds to the mean network travel time of Equation (19) of V2010, which is valid under the hypothesis of constant flow velocity. Equation (19) of V2010 is a sum of two parts, En_1 + En_2, where En_2 contains the covariance term $[cov_{xy}(D,R_t]$, which accounts for the additional travel time caused by the spatial variability of rainfall relative to a rainfall event uniform in space.

Analogously, the storm averaged spatial moment of the catchment rainfall (P_2, Eqs 1&4) is Equation (23) of V2010, which defines the mean network travel time under the hypothesis of constant flow velocity. Also in this case, the equation in V2010 is additive (Vn_1 + Vn_2, where Vn_2 includes the covariance terms).

The statistics P_1 and P_2 are normalised by the moments of the flow distance (g_n, Equation 2) to derive the scaled moments Δ_1 and Δ_2. Therefore values greater than 1 of Δ_2, for example, correspond to positive values of Vn_2 in V2010 and vice-versa.

Following the Reviewer's suggestion, we added the following sentence at the beginning of Section 2 "Spatial moment of catchment rainfall: definitions":

"The spatial moments of catchment rainfalls are defined after rearranging some of the covariance terms employed in Viglione et al. (2010a) to represent the mean and the variance of the network travel time, under the hypothesis of constant flow velocity (Appendix)."

The formal derivation of the Equations is reported in the Appendix.

Main Comment 4

"*The computation of the catchment-scale storm velocity …*" It appears that a time derivative of δ_1 is needed for this computation. Plate 1 indicates that δ_1 includes significant noise which would need to be smoothed before a useful derivative could be computed. A comment on this might be useful in the paper.

Response

We do not perform any explicit derivative of δ_1 to obtain the catchment scale storm velocity. Equation (6) has been introduced only to formally represent the concept of storm velocity and how this relates to the first scaled moment δ_1.

It is true that δ_1 includes significant noise and an average value of the storm velocity cannot be derived directly from the derivative of δ_1.

As anticipated in lines 1-2 of page 5819 of section 2 and illustrated in section 3 page 5824, we suggest a simplified approach (derived from V2010) to provide the average storm velocity. The storm velocity is defined by the slope of space-time linear regressions. Thus, we provide an estimate of the average storm velocity, based on a linear estimator. We think that this aspect is well illustrated in pages 5823-5824.

We modified test in Section1.1 as follows:

"The concept of the catchment-scale storm velocity as defined by Eq. 6 takes into account the role of relative catchment orientation and morphology with respect to storm motion and kinematics. For instance, for the same storm kinematics, the same elongated basin will be subject to different catchment scale storm velocities with varying its orientation with respect to that of the storm motion. In this work, we will not perform any explicit derivative of δ_1 to obtain the catchment scale storm velocity. Equation (6) has been introduced only to formally represent the concept of storm velocity and how this relates to the first scaled moment δ_1. A simple way to derive the mean value of V_s, derived from the methodology introduced by Viglione et al. (2010a), is reported in the next sections."

Main Comment 5

Equation (9). V2010's Equation (19) is an equivalent expression for the same quantity given in Equation (9), but V2010 separates the term into additive contributions from network geometry and rainfall variability, while Z2011 provide a separation into multiplicative factors. What is the advantage of writing the expression in the form of Equation (9) instead? The interpretations, in terms of the effects on timing of rain falling near to, or far from, the catchment outlet, seem to be exactly the same.

What do the authors consider to be the advantages and disadvantages of taking a spatial moment approach as opposed to the mean-covariance approach of V2010? Perhaps this should be addressed in the Discussion or Conclusions.

Response

The Appendix introduced in the revised version clarifies how the equations in Z2011 are related to those in V2010, including Eq. (10).

In the Introduction of the revised paper, we specify that the main purposes for introducing the Spatial Moments are as follows: i) to provide a theoretical foundation for various measures of rainfall spatial variability based on the flow distance coordinate, which have been reported in the literature in the last decade (Smith et al., 2002, 2005 ; Syed et al., 2003; Sangati et al., 2009); ii) to allows for the introduction of the concept of catchment scale storm velocity; and, iii) to extend to the case of runoff propagation under condition of spatial rainfall variability the concept of Spatial Moments used for analysis of solute transport in porous media (Goltz and Roberts, 1987). The development of this similarity, which is not pursued in this paper but is subject of current investigation, aims to order theoretical results appeared in disparate fields into a coherent theoretical framework for both hydrologic flow and transport, as shown by Rinaldo et al. (2006).

The multiplicative approach we have used in this paper is related to the mathematical structure of the Spatial Moments. The advantages of using the Spatial Moments with respect to the approach used in V2010 are as follows: i) a simplified and more readable structure of the Equation for the representation of $Var(T_c)$ and $Cov(T_r,T_c)$; ii) a linkage with existing statistics reported in the literature in the last ten years; iii) a linkage with the Spatial Moments used for for analysis of solute transport in porous media. Of course, the additive structure was a requirement in V2010 because of the need to express the individual contribution of the various elements to the mean and variance terms of T_q.

Main Comment 6

"One should note that the storm velocity has no influence on E(Tq)." This statement needs a little qualification before it can be accepted. It depends on many of the assumptions made previously (e.g. the neglect of runoff generation processes, the assumed time-invariant celerity).

Response

We modified this section as follows: "One should note that the storm velocity has no influence on E(T_q). This is a direct consequence of the hypotheses beyond the method: the catchment response is described as fully kinematic, therefore it is influenced by the averaged spatial organization of the rainfall and not by the variability of the spatial organization within the storm, and the routing is linear."

Main Comment 7

"The role of catchment scale storm velocity is represented by the term of Cov(Tr, Tc)." The authors needed to provide some justification for, or elaboration of, this statement.

Response

This was discussed in V2010. We changed the sentence into: "As discussed in V2010a, eq. (25), the role of catchment scale storm velocity is represented by the term $Cov(T_r,T_c)$."

General Comment 8

Equation (15) It seems that the authors make use of a connection between g1 D1 and some covariance terms. What are these relationships? This would assist readers trying to understand how this work relates to V2010.

Response

Analogously to the comment GC7, the Appendix in the revised paper clarifies this point.

Main Comment 9

"Details about the application of the model to the individual events, its calibration and its verification are reported in the relevant papers" A very brief summary of the adequacy of the model verification is needed here; e.g. did it work equally well on all 5 catchments?

Response

We introduced the following text in the revised version. "The model parameters were estimated over the catchments available for each event by means of a combination of manual and automatic calibration to minimize either the Nash−Sutcliffe efficiency index over the flood hydrographs (for the gauged catchments) or the mean square error over the flood peak and the timing data (rise, peak and recession) (for catchments where runoff data were provided from post-event surveys). Details about the application of the model to the individual events, its calibration and its verification are reported in the relevant papers (*Sangati et al*., 2009; *Zoccatelli et al*., 2010; *Zanon et al*., 2010). In general, the model simulations of the flood hydrographs were closer to observations for the smaller basins where the linear routing approach implemented in the model provides a better description of the actual runoff propagation processes."

Main Comment 10

Plate1 and Plate 2. The time series for δ_1, δ_2 and velocity include fluctuations of several different magnitudes and timescales. It is not clear which fluctuations are significant, and which are a consequence of measurement uncertainty when using radar rainfall. Some kind of uncertainty analysis would be very helpful. For example, if these time series for d1, d2 and velocity were computed 50 times, each time with a different realisation of "noise" added to each radar rainfall field, which features of the time series in Plates 1 and 2 would remain?

Response.

We revised Plates 1 and 2 by removing the time periods before and after the main precipitation events. This reduces some wide fluctuations which were visible in the original submission. Anyway, some fluctuations remained. Analysis of the impact of radar rainfall estimation uncertainty on temporal variability of spatial moments and velocity is part of ongoing investigations. This also combine with a current focus on uncertainty assessment in the radar hydrology literature, where methods very similar to that mentioned by the Reviewer Ross Woods are used to assess the impact of radar based estimates of rainfall.

Examination of the mathematical structure of the spatial moments shows that these statistics should not be affected by time-constant bias in radar rainfall estimates. This shows that these statistics could be reliably derived also from biased radar-based rainfall estimates.

Main Comment 11

"it seems that the intriguing overlapping between the theoretical analysis represented by Eq. (19) and the empirical results represented by Eq. (21) needs to be substantiated"

The theory of V2010 provides guidance on how to explore counter-intuitive results of this type, and I think it should be applied here. For example, is there a correlation between hillslope residence time and flow distance? Is it large enough to explain the timing shifts?

I think that the authors need to provide more justification to support the results, since they are somewhat unexpected. Can the authors confirm that if the hillslope residence time is reduced to very near zero, the slope of the line in Figure 4a changes from 0.33 to a value near 1.0? The 0.91 slope in Figure 4b was quite unexpected for me, in the light of the 0.33 slope for the previous case. The authors could assist the reader by providing a discussion of the spatial and temporal variability of the modelled surface runoff generation, and giving an indication of the relative amounts of runoff generated by surface and subsurface pathways. Again, the theory of V2010 could be usefully applied, rather than leaving the reader with an unexplained conundrum.

Response

We did our best to clarify the results provided in this Section. We reported that these findings do not depend on the correlation between hillslope residence time and flow distance, but only on the correlation between spatial rainfall patterns and flow distance (as captured by the statistic $\Delta 1$).

We have shown that, when the hillslope residence time is reduce to zero, the slope of the line in Figure 4a changes to a value near 1.0.

The shift of the slope of the line from 0.33 to 0.91 when infiltration is accounted in the rainfall-runoff modelling is the result of the non linearity characterizing the rainfall to runoff transformation. This non linearity leads to a magnification of the values of the dT_n statistic with respect to those obtained in the impervious case. Essentially, this means that when rainfall is either focused on the headwaters or on the outlet, the runoff exhibits an even stronger offset towards the periphery of the catchment as a result of the non linear hydrological processes implied in the runoff generation. We have documented this effect in the revised version of the work.

Minor Comments

We also addressed all the Minor Comments by the reviewer.

APPENDIX

In this Appendix we show how Eqs. (10), (13) and (15) may be derived from *V2010*.

Derivation of Equation (10)

Equation (19) in *V2010* provides the average time to route the rainfall excess from the geographical centroid of the rainfall spatial pattern to the catchment outlet. Using the same notation used in the current work, it is written down as follows:

$$E(T_c) = \frac{g_1}{v} + \frac{\mathrm{cov}_{x,y}\left(d(x,y), r_t(x,y)\right)}{vP_0} \tag{A1}$$

where $Cov_{x,y}(\ )$ is the spatial covariance.

Eq. A1 is developed as follows to derive Eq. (10):

$$\begin{aligned} E(T_c) &= \frac{g_1}{v} + \frac{\mathrm{cov}_{x,y}\left(d(x,y), r_t(x,y)\right)}{vP_0} = \\ &= \frac{g_1}{v} + \frac{\int_A d(x,y) r_t(x,y) dA}{vP_0} - \frac{g_1}{v} = \frac{P_1}{P_0 v} = \frac{\Delta_1 g}{v} \end{aligned} \tag{A2}$$

Derivation of Equation (13)

Equation (23) in *V2010* provides the variance of the time to route the rainfall excess to the catchment outlet:

$$Var(T_c) = \frac{g_2 - g_1^2}{v^2} + \frac{\mathrm{cov}(d(x,y)^2, r_t(x,y))}{v^2 P_0} +$$

$$-\frac{\mathrm{cov}_{x,y}\left(d(x,y), r_t(x,y)\right)}{vP_0}\left[2\frac{g_1}{v} + \frac{\mathrm{cov}_{x,y}\left(d(x,y), r_t(x,y)\right)}{vP_0}\right] \tag{A3}$$

Eq. A3 is developed as follows to derive Eq. (13):

$$Var(T_c) = \frac{g_2 - g_1^2}{v^2} + \frac{\int_A d(x,y)^2 r_t(x,y) dA}{Av^2 P_0} - \frac{g_2}{v^2} - \left(\frac{P_1}{P_0 v} - \frac{g_1}{v}\right)\left(\frac{P_1}{P_0 v} + \frac{g_1}{v}\right) =$$

$$\left(\frac{P_2}{P_0} - \frac{P_1^2}{P_0^2}\right)\frac{1}{v^2} = \frac{\Delta_2}{v^2}(g_2 - g_1^2)$$

Derivation of Equation (15)

Equation (25) in *V2010* provides the covariance between the rainfall time and the routing time:

$$Cov(T_r, T_c) = \frac{Cov_t\left[T, Cov_{xy}(d(x,y), r_t(x,y))\right]}{vP_0} - \frac{Cov_t\left[T, p_0(t)\right]}{P_0}\frac{Cov_{xy}\left[d(x,y), r_t(x,y)\right]}{vP_0} \tag{A4}$$

Eq. A4 is developed as follows to derive Eq. (15):

$$Cov(T_r, T_c) = g_1\frac{Cov_t\left[T, \delta_1(t)w(t)\right]}{v} - g_1\frac{Cov_t\left[T, w(t)\right]}{v} - \frac{Cov_t\left[T, w(t)\right]}{v}\frac{Cov_{xy}\left[D, P(x,y)\right]}{P_0} =$$

$$g_1\frac{Cov_t\left[T, \delta_1(t)w(t)\right]}{v} - \frac{Cov_t\left[T, w(t)\right]}{v}\left(g_1 + \Delta_1 g_1 - g_1\right) =$$

$$g_1\left\{\underbrace{\frac{Cov_t\left[T, \delta_1(t)w(t)\right]}{v}}_{term1} - \underbrace{\frac{Cov_t\left[T, w(t)\right]}{v}}_{term2}\Delta_1\right\}$$

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