“*Natural laws of precipitation, great cycle, infiltration overland and groundwater runoff with a new formulas*”

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Replay to Anonymous Referees

We fully appreciate your comments as they are very important for the further work of the proposed subject which may be important. We would especially like to thank to the last author of the comment, who gave an opinion of the importance and the clarity of the mathematical formula, though we have understood all the criticism, including the negative one, as good-willed. The method is fully based upon the Lvovitch six-component method, which is truly broken into a three-component one. Lvovitch himself emphasized his remarks in the Russian edition of Global Water Resources and Their Future published in 1974. The first formula observed is the relation between precipitation, overland runoff and changeable infiltration, formula 1.

Formula

$$P = R + E \pm \Delta W$$

is original Lvovitch formula from Global Water Resources and Their Future, p.74.

The formula has shown a trend in infiltration which may be positive or negative (Global Water Resources and Their Future, the Russian edition page 75). That means, the infiltration may show various trends depending on the terrain and the type of the basin or the kind of water being observed. Because of the hydrological stations erected along the river basins in Serbia assisted by the Republican Hydro- Meteorological Association (RHMZS), the only data available to cover was for the river basin, the next formula is the one by means of which we define the relation between the total runoff and evaporation. (Water Resources and Their Future, the Russian edition, Moscow 1974, p. 75.)

The formulas:

$$P = S + U + E, \quad S + U = R, \quad W = P - S = U + E, \quad Ku = \frac{U}{W}, Ke = 1 - Ke = \frac{E}{W}$$

are the original Lvovitch formulas, taken from Water Resources and Their Future, the Russian edition, Moscow, p. 76.

It follows in the formula that primarily territories of smaller areas are defined, yet it is successfully applied to larger area territories. Two sums emerge from the formula, though two quotients are

$$W = P - S \Rightarrow W + S = P \wedge S = P - W$$

the quotients of precipitation infiltration and the quotients of overland runoff. What matters in defending the proof is that the equation should equal 1, because 1 is the total amount of water balance. This is just a quality feature not a quantity one. The most important factors are the four quotients: Ku, Ke, Ks and Kw.

Lvovitch equation has the following form:

$$Kw + Ke = 1 \Rightarrow Ke = 1 - Kw$$

By multiplying the coefficients KwKe we work out $KwKe = \frac{W}{P} \cdot \frac{E}{W} = \frac{E}{P}$ the coefficient we call the evaporation coefficient in regard to the precipitation. By dividing the coefficients in the cartodiagram-nomogram, we work out the coefficient E/P.

$$\frac{Kw}{Ke} = \frac{\frac{W}{P}}{\frac{E}{W}} = \frac{W^2}{PE}$$

It further follows $W^2 = PE \Rightarrow W = \sqrt{PE}$ which can be graphically and mathematically proven. It is interesting to notice that by dividing the equation KuKe

$$\frac{Ku}{Ke} = \frac{\frac{U}{W}}{\frac{E}{W}} = \frac{UW}{EW} = \frac{U}{E} = 1$$

So that, if there is a total sum of water balance, we can calculate the subterranean flow, as the formula proposed should be further elaborated and mathematically proved. So the formula $W = \sqrt{PE}$ is proven in the mathematical sense and confirmed by the cartodiagram. In order to achieve the permeability of the terrain, we need to know certain laws of the relation between the infiltration and the groundwater runoff. It is defined in such a way that with an increase of infiltration, it is the value of the groundwater runoff that increases too. By multiplying the coefficients KwKu we work out

$$Kw \cdot Ku = \frac{W}{P} \cdot \frac{U}{W} = \frac{U}{P} \ .$$

U/P is the coefficient of the groundwater runoff. Since it emerges from the Lvovitch formula it follows as:

$$Kw + Ke = 1 \Rightarrow Ke = 1 - Kw$$

$$K_w K_e = \frac{E}{P} \quad \Longrightarrow \quad K_w(1-K_w) = \frac{E}{P} \quad \Longrightarrow \quad K_w - K_w^2 = \frac{E}{P}$$

$$K_w^2 - K_w + \frac{E}{P} = 0 \quad \Longrightarrow \quad K_{w_{1,2}} = \frac{1 \pm \sqrt{1-\frac{4E}{P}}}{2} \ .$$

The coefficient of infiltration exists and shows permeable terrains if and only if for values $K_{w_1}$and K_{w_2} holds

$$1 - \frac{4E}{P} \geq 0 \quad \Longrightarrow \quad \frac{4E}{P} \leq 1 \quad \Longrightarrow \quad \frac{E}{P} \leq \frac{1}{4} = 0{,}25.$$

or in the other words if $\frac{E}{P} \leq 0{,}25$. For the other, impermeable terrains, holds opposite inequality. This is derived from the mathematical fact that quadratic equation has two real solutions if and only if the discriminant (the value under the root) is greater then zero. This mathematical definition for permeable and impermeable terrains holds in correlation with known definitions in physical sense, in which is observed porosity, capillarity as well as the type of soil and type of rocks. So, we can conclude that there is an obvious connection between mathematical shape and definition in physical sense.

Also, the solutions of the square equation are

$$K_{w_1} = \frac{1+\sqrt{1-\frac{4E}{P}}}{2} \geq \frac{1+0}{2} = \frac{1}{2} \qquad\qquad K_{w_2} = \frac{1-\sqrt{1-\frac{4E}{P}}}{2} \leq \frac{1-0}{2} = \frac{1}{2}$$

or in other words, the first solution is greater then 0,50 and shows high permeability, but the second solution is less then 0,50 and this formula should be used with some other definition for determining is the terrain permeable or not.

Finally we know how to write name Lvovitch, there is correct. (Global Water Resources and Their Future, p. 2)