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Hydrology and Earth System Sciences Discussions

Interactive comment on "Effects of seasonality on the distribution of hydrological extremes" *by* P. Allamano et al.

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Dr. P. Rasmussen and Dr. A. Evans are gratefully acknowledged for their insightful comments. In the following are reported our responses (plain text) to their highlighted comments (italic).

The specific case of the peaks-over-threshold (POT) model with seasonal variation of parameters was studied in a paper by Rasmussen and Rosbjerg (1991) who – in a manner very similar to Allamano et al. – assessed the impact of pooling inhomogeneous seasonal exponential distributions into a single population that was then fitted by an exponential distribution. In terms of model formulation, the only real difference between the two studies is that Allamano et al. assume a sinusoidal variation of parameters whereas Rasmussen and Rosbjerg's paper assumed seasons with constant parameters. In either case, when ignoring seasonality a bias is introduced in the estimation of quantiles, because the combination of different exponentials is not an exponential.

Thanks a lot for pointing us to this very relevant contribution. The model developed by Rasmussen and Rosbjerg (1991) is indeed similar to ours (and of course it deserves a citation), except for the fact that the parameters of the exponential Poisson model are varied in continuous-time in our contribution. This is itself a relevant difference in our opinion, not only because it allows full analytical tractability but also because it prevents the user from subdividing the year in (supposedly) homogeneous seasons: when one imposes a discrete partition, the question arises of how to select objectively the number of seasons and their respective duration. Moreover the aims of the two papers are rather different, as also discussed in the next point: we adopt a very simple and controllable framework (which we call "toy-model" in Section 2) with the aim of sharpening the questions regarding the possible effects of seasonality on design event under- (or over-) estimation (i.e., we concentrate on the estimation bias), while Rasmussen and Rosbjerg (1991) mainly concentrate their attention on the total prediction uncertainty (bias and variance of estimation). As discussed below, this corresponds to considering a different research question.

However, bias is not the only consideration in assessing the quality of an estimator. Rasmussen and Rosbjerg also assessed the standard deviation and the root-mean-square error (RMSE) of the estimator that ignores seasonality. The RMSE, which combines bias and standard deviation, may be a better measure of whether it is worthwhile taking seasonality into account or not. More specifically, pooling seasonal events into a single sample increases the ratio of the number of data to the number of model parameters which generally implies smaller sampling variability. This gain has to be balanced against the increased bias. The present manuscript looks exclusively at bias, as quantified by the RT ratio in Eq. 9. One should not be surprised to find biases when fitting *an exponential distribution to a population that is not exponential.*

Probably this comment derives from the fact that we have not been clear enough in declaring the scope of our contribution: our aim is not to propose a new distribution for use in POT (Eq. 6) or AM (Eq. 8) analyses, but simply to analyze the magnitude of design event under- (or over-) estimation in the presence of seasonality. We therefore use the seasonal distributions (Eq. 6 and 8) merely as benchmarks (i.e., as the real parent distributions), and not as models to be used in practice. In other words, our aim is to address the question "what is the impact of seasonality on design event estimation" and not the broader one "how can we efficiently estimate the design event under the presence of seasonality". This is the main reason why we concentrated on estimation bias rather than on the mean square error of estimation. We will better clarify the diagnostic nature of our approach in the revised version of the manuscript. We also agree with Dr. Rasmussen regarding the foreseeable presence of a bias "when fitting an exponential distribution to a population that is not exponential". In fact, in our analysis we do not look just at the presence of the bias but we analyze the magnitude of the bias itself and its sensitivity to the typology and strength of seasonality.

Of course, a fundamental question is if nature is really exponential at the seasonal level. The answer is probably no since nature tends to be much more complex than what can be described by a 1-parameter distribution. The results in the paper are only truly applicable in the assumed simplistic world of seasonality. It is not clear how significant the results are in the real world where it could well happen that daily precipitation events pooled over the seasons are roughly exponentially distributed, despite the presence of seasonality. Indeed, the fundamental problem here is not the seasonality itself, but the fact that an incorrect distribution is being fitted to the non-seasonal sample.

This reviewer's comment can be usefully split in two parts in our opinion: first, he argues that the exponential distribution may not be suitable to represent reality at a seasonal scale: we fully agree with him, and we will try to better clarify that the exponential is merely used as an example distribution (rather simple but commonly used) to

demonstrate the possible effects of seasonality on the distribution of extreme events. Other distributions could of course be used, but these would be more parameterized than the exponential, thus complicating the interpretation of the seasonality effects. In the second part of his comment, the reviewer, if we correctly interpret his reasoning, reverts his previous argument by claiming that, even in the presence of seasonality, the pooled events may have a distribution which is roughly exponential. Again, we fully agree with him: in fact, the message that we wanted to convey with this contribution is that the seasonal distribution (Eq. 6) may be almost indistinguishable from the exponential (see Fig. 1 below), but still the bias inherent with using the exponential for extrapolating to high return periods might be rather large.

The type of seasonality assumed in Eq. 2 may not fit real data very well and therefore limits the use of the results in practical applications. It is perhaps acceptable as an exploratory model, but one still wonders if a simpler version of seasonality could have been used. Although the sine assumption is relatively simple, it leads to a quite complicated analytical expression for the marginal distribution of daily precipitation amounts. The derivation of Eq. 6 is a nice analytical result, but the complexity and effort to derive it may obscure the real point the authors want to make.

The sine assumption is taken as a compromise between the need to provide a realistic representation of a periodic time series and the quest for simplicity which drives our research. A complete Fourier series expansion would have been better suitable to represent the seasonal variability, but at the cost of making the reasoning much more complicated, with many nuisance parameters obscuring the effect of seasonality. On the other hand, a more simple representation of seasonality (e.g., through two seasons of constant duration) would not have been realistic enough in our opinion (see the behavior of the data in Fig. 5 of the manuscript for an example).

The reparameterization of Eq. 1 is critical for the paper, but its rationale is not well explained. It would be instructive to provide some examples of how well the densities in Eqs. 1 and 3 can be matched since the reparametrization is only approximate. We *plotted a few cases and found that for some parameters the agreement is very good while for others it is not. It seems that the larger the amplitude of the sine wave, the higher the discrepancy. A more troubling problem is that the discrepancy depends on α_0, with higher α_0-values leading to poorer agreement, even in relative terms. This implies that the agreement between Eqs. 1 and 3 is scale-dependent, or, in other words, that one would get different results depending on whether precipitation is measured in millimetres or in inches.*

Again, the reparameterization in Eq. 3 can be seen as a compromise which allows one to have an analytical solution, at the expense of losing some detail in the representation of the phenomena. In particular, the approximation works very well when large events (i.e., large x to α_0 ratios) are considered, while the quality of the approximation decreases when small events are considered; these latter, however, are of course less relevant for design value estimation. Also note that the ratio in Eq. 5 is not scale-dependent, because the ratio of x to α_0 appears as the argument of the exp function. The value of x here represents the considered extreme value, which in turn scales with α_0 (if $\alpha_0 = 1$, $x = 5$ is an extreme value, if $\alpha_0 = 10$ it is not).

Detailed comments

P4790, L18. "Annual time scales" should be "seasonal time scales".

Right, there is a syntactical error. We will rephrase the period to amend it.

P4790, L25. " . . . could belong to different populations." This is an inaccurate statement and contrary to what the authors argue, this is not a problem for standard frequency analysis. If seasonality of the type considered in the paper is ignored, the population distribution is the one given in Eq. 6. A problem only arises if one tries to fit an incorrect distribution to this population.

We agree with Dr. Rasmussen that this statement can be misleading, and we will rephrase it. What we intended with this sentence can be better explained by resort-

ing to a simpler representation of seasonality, where exponentially distributed extreme events occur in two seasons with the same rate of occurrence but different average intensity. If one applies a POT analysis in this case, he/she will obtain a sample of over-threshold values extracted from two different distributions (both exponential, but with different average value). Still, as correctly noted by Dr. Rasmussen, a single population distribution could be obtained also in this case (analogously to our Eq. 6), which will take the form of a mixed exponential distribution. These interpretations (sample with non-iid exponentially distributed values, or with iid values from a non-exponential distribution) are, in our opinion, two alternative ways to see the same thing: the effect of the presence of seasonality, which, in any case, makes some commonly used assumptions inappropriate (i.e., the iid assumption or the assumption of a standard probability distribution). We are grateful to this reviewer for pointing us to this subtle reasoning.

P4791, L10-12. The statement about the hypotheses of identically distributed random variables for monthly and annual extremes needs to be explained and justified. It is not clear what the authors mean.

See comment above for an explanation of the rationale behind this sentence. The need for identically distributed variables is the main motivation for extracting seasonal/monthly maxima (see Carter and Challenor, 1981 and Buishand and Demaré (p. 90, l. 9-11, 1990). In fact, the “identically distributed” hypothesis better applies to shorter, within-year, time intervals. We will try to better clarify this point in the text.

P4791, L12. The statement regarding estimation uncertainty of design values based on monthly maxima is incorrect. The ratio of the number of parameters to number of data should stay the same, so the modeling of separate seasons should not affect the sampling uncertainty.

The collateral effect of subdividing the year in shorter intervals is an increase in the number of data points, but this does not automatically entail an increase in the information content of the available sample. In fact, extreme value analyses are aimed at representing the upper tails of a distribution, and the 11 additional values per year added to the sample when using monthly maxima will all be lower than the annual maximum, and will therefore provide few information about the behaviour of the upper tail. In a POT framework, it has been demonstrated that a 5-fold increase of the data induces a (less than) 1.5-fold reduction in the estimation error, which in turn corresponds to increasing the AM sample size of less than 1.5 times (e.g., Madsen et al., 1997; Martins and Stedinger, 2001). 100 POT (or monthly maxima) data are therefore far less informative than 100 AM for design event estimation. This reduces the possible favourable effect on uncertainty of using a parametric model with less than 12x parameters. Moreover, the use of a parametric model introduces the need for assessing the uncertainty in the model parameters estimation. It is not easy to demonstrate that the overall uncertainty reduces in this case.

P4792, L16. Use "cycles" instead of "peaks" in this sentence.

Ok, we will rephrase as suggested

P4792, L24. Bayes' theorem is not used to obtain the marginal distribution. The marginal is obtained by simply integrating the conditional distribution.

The marginal distribution in Eq. 6 is obtained by solving the Bayes integral or, more rigorously, the "Law of Total Probability" integral. We will amend this confusion in the revised manuscript.

P4793, L4. The expression for $\alpha'(t)$ is missing the term 2π/365.

Right, amended

P4793, Eq. 3. The dependence of this integral on x should be specified on the left-hand side.

Do you mean Eq. 4? Ok

P4794, Eq. 6. The middle expression should be divided by 365.

Right, thank you

P4794, Eq. 7. How was this expression derived?

The mean was derived (analytically) as the first moment of the distribution in Eq. 6.

P4796, L20. Here the authors argue that statistical tests typically fail to recognize that the marginal distribution of exceedances is not exponential. We have not tried to reproduce the test results, but are curious about the results. For example, in Figure 2, with the case of $\lambda = 50$ and a 20 year record, one would have on average of 1000 observations in a sample. This is a very large sample and goodness-of-fit tests should be able to pick up discrepancies from the assumed distribution. In fact, when samples are very large, statistical tests tend to become "too powerful" and are usually avoided. This is the well-known issue of "statistical significance" versus "practical significance".

Goodness-of-fit tests are not able to recognize the difference between the two distributions because the distributions are indeed very similar, in particular in the central part of the distribution (see for example Fig. 1 of this document). However, in the right tail of the distribution the differences are large enough to produce a 5-fold underestimation of the return period associated to an event. Despite the very large sample size, this is therefore a situation where the differences are practically significant (for example, for design purposes it is important to know that neglecting seasonality could entail underestimation), but they are not statistically significant in a large percentage of cases.

P4799, L6. A value $\lambda = 20$ and a record length of 74 years will give sample of 1500 events. Our guess is that there would be significant benefit in raising the threshold, because the lower the threshold, the more significant the problem of seasonality will be. For estimation of a, say, 100-year event, one is probably better off using only the 100 or 200 largest values.

As suggested by this Referee we have tried to raise the threshold to 50 mm, which corresponds to about $\lambda = 5$ events per year over the whole dataset. $R_T^{(POT)}$ obtained for each station are reported in the Fig. 2 below. The values - as expected - are a bit lower than the ones shown in Fig. 6 (of the manuscript) but still relevant and worth to be accounted for.

P4799, L17. Remove expression in parentheses.

Ok

P4801, L7-9. This is only true if the additional events in the POT model are consistent with the assumed distribution.

Also for the AM there might be a lack of consistency between model and data.

Change "sample dimension" to "sample size".

Ok, thanks.

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 8, 4789, 2011.

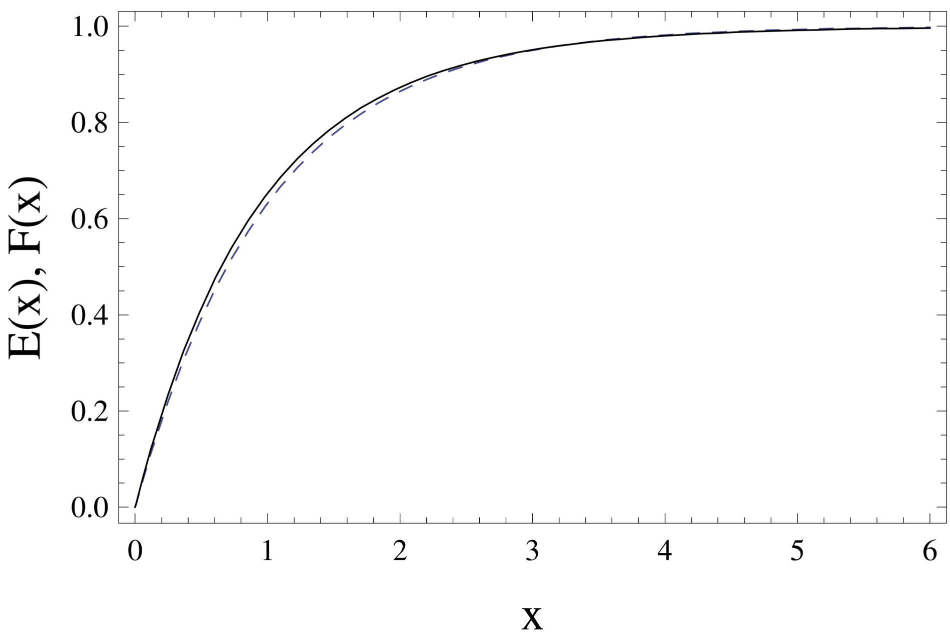

Fig. 1. Exponential (E(x), dashed) versus seasonal (F(x), solid) distribution with a_alpha=0.5, a_lambda=0.5 and delta=pi

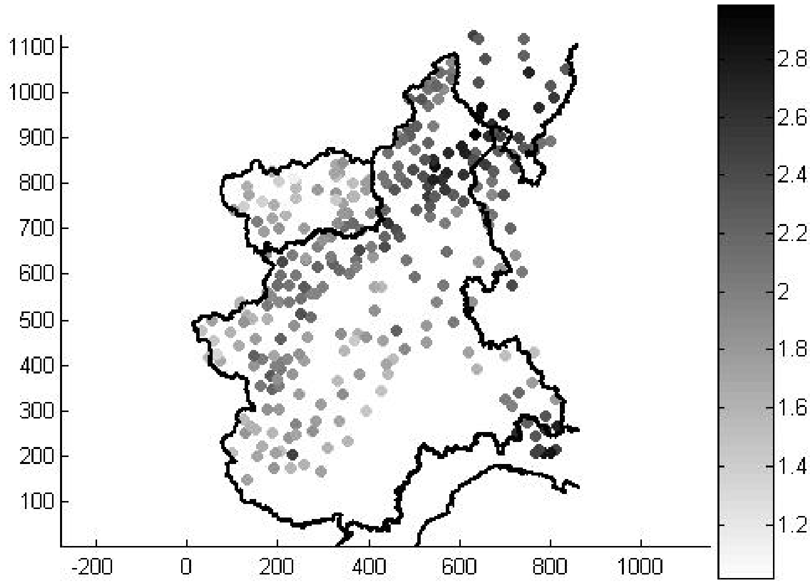

Fig. 2. Return period ratios evaluated for the 294 stations (the color scale refers to the resulting range of variation of R_Tˆ(POT))