Hydrol. Earth Syst. Sci. Discuss., 8, 8793–8830, 2011
www.hydrol-earth-syst-sci-discuss.net/8/8793/2011/
doi:10.5194/hessd-8-8793-2011

Hydrology and Earth System Sciences Discussions

This discussion paper is/has been under review for the journal Hydrology and Earth System Sciences (HESS). Please refer to the corresponding final paper in HESS if available.

A simple water-energy balance framework to predict the sensitivity of streamflow to climate change

M. Renner[1], **R. Seppelt**[2], **and C. Bernhofer**[1]

[1]Dresden University of Technology, Faculty of Forestry, Geosciences and Hydrosciences – Institute of Hydrology and Meteorology – Department of Meteorology, Pienner Str. 23, 01737 Tharandt, Germany
[2]Helmholtz Centre for Environmental Research – UFZ, Department of Computational Landscape Ecology, Permoserstr. 15, 04318 Leipzig, Germany

Received: 15 September 2011 – Accepted: 20 September 2011
– Published: 27 September 2011

Correspondence to: M. Renner (maik.renner@mailbox.tu-dresden.de)

Published by Copernicus Publications on behalf of the European Geosciences Union.

Abstract

Long term average change in streamflow is a major concern in hydrology and water resources management. Some simple analytical methods exist for the assessment of the sensitivity of streamflow to climatic variations. These are based on the Budyko hypothesis, which assumes that long term average streamflow can be predicted by climate conditions, namely by annual average precipitation and evaporative demand. Recently, Tomer and Schilling (2009) presented an ecohydrological concept to distinguish between effects of climate change and basin characteristics change on streamflow. We provide a theoretical foundation of this concept by showing that it is based on a coupled consideration of the water and energy balance. The concept uses a special condition that the sum of the ratio of annual actual evapotranspiration to precipitation and the ratio of actual to potential evapotranspiration is constant, even when climate conditions are changing.

Here we apply this assumption and derive analytical solutions to the problem of streamflow sensitivity on climate. We show how climate sensitivity is influenced by different climatic conditions and the actual hydrological response of a basin. Finally, the properties and implications of the new method are compared with established Budyko sensitivity methods.

1 Introduction

In this paper we consider the question how variations in climate affect the hydrological response of river basins. Thus, we aim to assess climate sensitivity of basin streamflow Q and evapotranspiration E_T, (Dooge, 1992; Arora, 2002; Yang and Yang, 2011; Roderick and Farquhar, 2011). To do so, we need to consider the concurrent climate itself, because naturally the supply of water and energy is the main controlling factor of evapotranspiration (Budyko, 1974; Zhang et al., 2004; Teuling et al., 2009). Basin characteristics are also of high relevance: two basins with similar climate may have

HESSD

8, 8793–8830, 2011

quite different hydrological responses (Yang et al., 2008). Spatio-temporal patterns of precipitation, soils, topography, vegetation and not least human activities have considerable impacts (Arnell, 2002; Milly, 1994; Gerrits et al., 2009; Zhang et al., 2001; Donohue et al., 2007).

Usually one is attempted to represent such basin characteristics by conceptual or physically based hydrological models. However, the uncertainties arising from model structure and calibration may lead to biased and parameter dependent climate sensitivity estimates (Nash and Gleick, 1991; Sankarasubramanian et al., 2001; Zheng et al., 2009).

Considering sufficiently long periods ($\gg$1 year) we can confidently assume that the hydrological response of a given basin is in equilibrium with the climatic conditions. This means that on average, the sum of all relevant processes, which are modified by basin properties generates the average annual streamflow we observe at the outlet of the basin. So, climate conditions and hydrological response form a hydro-climatic state space which is the best representation of a given basin.

A remarkable paper of Tomer and Schilling (2009) introduced a conceptual model to distinguish climate change effects from land-use change effects on streamflow. They utilize two non-dimensional ecohydrologic states representing water and energy balance components, which describe the hydro-climatic state of a basin and carry information of how water and energy fluxes are partitioned at the catchment scale. The central hypothesis of Tomer and Schilling (2009) is that from the observed shift of these states, the type of change can be deduced. Their theory is based on experiments with different agricultural conservation treatments of four small field size experimental watersheds (30–61 ha). They observed that watersheds with different soil conservation treatments also showed different evapotranspiration ratios. Further, the shift within this hydro-climatic state space due to conservation treatments was perpendicular to the shift due to climate changes. They found that over time in their case a climate shift towards increased annual precipitation took place.

HESSD
8, 8793–8830, 2011

The conceptual model posed by Tomer and Schilling (2009) has great scientific appeal and calls for further exploitation of its potential. Here, we aim to employ the framework to address the following research questions:

1. Is there a theoretical justification of the conceptual model?
2. Can this model be used to predict streamflow/evapotranspiration change based on a climate change signal?
3. What are the implications of such a model given the range of possible hydro-climatic states and changes therein?
4. How does it compare to existing climate sensitivity approaches?

This paper is structured as follows. In the methodological section we embed the conceptual model of Tomer and Schilling (2009) into a coupled water and energy balance framework. With that we derive analytical solutions, which can be used to predict the sensitivity of streamflow to climate changes.

We then discuss the properties and implications of the new method. We compare our results with previous studies, namely those which employed the Budyko hypothesis for the assessment of streamflow sensitivity (Dooge, 1992; Arora, 2002; Roderick and Farquhar, 2011). In a second paper (Renner and Bernhofer, 2011), we will address the application of this hydro-climatic framework on a multitude of catchments throughout the contiguous United States.

2 Theory

In this section we aim to derive a general framework for the analysis and estimation of long term average changes in basin evapotranspiration and streamflow. The theory is based on the water and energy balance equations, valid for an area such as a watershed or river basin. We revisit the conceptual framework by Tomer and Schilling

(2009) and employ it to derive analytical solutions for (a) the sensitivity of a given basin to climate changes and (b) the expected changes in basin evapotranspiration and streamflow under a given change in climate.

2.1 Coupled water and energy balance

Actual evapotranspiration E_T is the common variable in the water and the energy balance equations, which both can be applied for a given area, such as a catchment

$$P = E_T + Q + \Delta S_w \tag{1}$$

$$R_n = E_T L + H + \Delta S_e. \tag{2}$$

The water balance equation expresses the partitioning of precipitation P into the water fluxes E_T, streamflow Q expressed as an areal estimate and ΔS_w which is the change in water storage. The energy balance equation describes, how incoming energy expressed as net radiation R_n is divided at the earth surface into the energy fluxes, latent heat flux $E_T \cdot L$, where L denotes the latent heat of vaporization, the sensible heat flux H and the change in energy storage ΔS_e.

As we regard the temporal scale of long term averages and thus consider the integral effect of a range of possible processes involved, we can assume that both, the change in water and in energy storage, are zero. Dividing the energy balance equation by the latent heat of vaporization L both balance equations have the unit of water fluxes, usually expressed as mm per time. Further, the term R_n/L, can also be denoted as potential evapotranspiration E_p, and expresses the typical upper limit of potential evapotranspiration (Budyko, 1974; Arora, 2002). With the above simplification we can write the energy balance equation as:

$$E_p = E_T + H/L. \tag{3}$$

HESSD
8, 8793–8830, 2011

2.2 The ecohydrologic framework for change attribution

In the long term, actual basin evapotranspiration E_T is mainly limited by water supply P and energy supply E_p, which considered together, determine a hydro-climatic state space (P, E_p, E_T).

Regarding long term average changes in the hydrological states, these must be caused whether by a change in climatic conditions or in changes in basin conditions, quietly assuming that our data is homogeneous over time. The conceptual model of Tomer and Schilling (2009) aims to distinguish between both types of causes. They employ two non-dimensional variables, relative excess energy U and relative excess water W, which can be obtained by normalizing, both the water balance and the energy balance by P and E_p, respectively:

$$W = 1 - \frac{E_T}{P} = \frac{Q}{P}, \quad U = 1 - \frac{E_T}{E_p} = \frac{H/L}{E_p}. \tag{4}$$

So, relative excess water W describes the proportion of available water not used by the ecosystem, which is in the case of a catchment the runoff ratio Q/P. Similarly, the remaining proportion of the available energy not used for evapotranspiration is expressed as relative excess energy U. Naturally both terms are within the interval [0, 1], because E_T is generally positive and it cannot be larger than P or E_p, which is also known as the water and energy limits proposed by Budyko (1974). The relation of both terms is essentially a coupled consideration of water and energy balances, to which we will refer to as the UW space. So plotting U versus W in a diagram shows the relative partitioning of water and energy fluxes of a given basin.

A shift in these two variables must be regarded as a disturbance of the equilibrium hydro-climatic state of a basin. Deduced from observations, Tomer and Schilling (2009) proposed that the direction of change in relative excess water and energy (ΔW, ΔU) respectively, can be used to attribute the observed changes, e.g. in streamflow to a change in climate or basin characteristics such as land-use. The conceptual model by Tomer and Schilling (2009) is shown in Fig. 1. It displays shifts in W and U from a reference state. A shift along the negative diagonal indicates effects of only climatic changes, while on the positive diagonal changes in basin characteristics are dominant.

2.2.1 Implications on catchment efficiency (CE)

The significance of the conceptual model of Tomer and Schilling (2009) is probably better understood, when considering the ratio of annual actual evapotranspiration to precipitation E_T/P and the ratio of actual to potential evapotranspiration E_T/E_p on ecosystem level of a river catchment. These non-dimensional terms express the ability of the ecosystem to use the available water and energy for evapotranspiration. Thus the sum of both terms:

$$\mathrm{CE} = \frac{E_T}{P} + \frac{E_T}{E_p} \tag{5}$$

can be regarded as catchment efficiency CE.

Next consider two long-term average hydro-climate state spaces (P_0, $E_{p,0}$, $E_{T,0}$), (P_1, $E_{p,1}$, $E_{T,1}$) of a given basin. Then the changes in relative excess water ΔW and energy ΔU can be expressed by using Eq. (4) as:

$$\Delta W = \frac{E_{T,0}}{P_0} - \frac{E_{,1}}{P_1} \qquad \Delta U = \frac{E_{T,0}}{E_{p,0}} - \frac{E_{T,1}}{E_{p,1}}. \tag{6}$$

Now, considering some change scenario, the question is how does the catchment efficiency change with the change directions given by ΔW and ΔU? To resolve this question let us define the relative catchment efficiency $\mathrm{CE}_1/\mathrm{CE}_0$:

$$\mathrm{CE}_1/\mathrm{CE}_0 = \frac{\frac{E_{T,1}}{P_1} + \frac{E_{T,1}}{E_{p,1}}}{\frac{E_{T,0}}{P_0} + \frac{E_{T,0}}{E_{p,0}}}. \tag{7}$$

HESSD
8, 8793–8830, 2011

A simple water-energy balance framework

M. Renner et al.

By substitution of Eq. (4) into Eq. (7) and rearranging we yield a term which relates the change in catchment efficiency to W, U and the change signals therein:

$$\mathrm{CE}_1/\mathrm{CE}_0 = 1 - \frac{\Delta W + \Delta U}{2 - W_0 - U_0}. \tag{8}$$

With Eq. (8) we can now interpret the conceptual model of Tomer and Schilling (2009) with regard to a change in catchment efficiency. The first consequence of Eq. (8) is that when the sum of ΔW and ΔU is zero, than there is no change in catchment efficiency. This case, represented by the negative diagonal in Fig. 1, means that even when climate becomes more arid (upper arrow) or more humid (lower arrow), CE remains constant.

The second consequence is that if we observe that the sum of changes in relative excess water and energy is positive ($\Delta U + \Delta W > 0$), then there is a decline in the catchment efficiency. Contrarily, a negative sum means that the efficiency is improving. In Fig. 1 the change in catchment efficiency is shown by coloured isolines, where green means improving, yellow constant and red declining catchment efficiencies. Note, that these isolines are parallel to the climate change diagonal and that the strongest gradient in catchment efficiency change is perpendicular to these lines, i.e. on the basin change direction. Last, the strength of catchment efficiency change is also dependent on U and W, i.e. dependent on the hydro-climatic state space (P, E_p, E_T).

2.2.2 Practical implications

Above we have shown that the conceptual framework of Tomer and Schilling (2009) can be derived from the coupled nature of water and energy balances. Depending on the direction of change in the hydro-climate state space, three major hypotheses relevant for streamflow sensitivity can be deduced:

HESSD
8, 8793–8830, 2011

1. climate change impact hypothesis (abbreviated as CCUW), i.e. constant catchment efficiency under climate change:

$$\Delta U/\Delta W = -1 \tag{9}$$

2. basin characteristics change impact hypothesis (BCUW): $\Delta U/\Delta W = 1$

3. a combination of both effects, where the change direction α can be computed from the observed change signals of U and W:

$$\alpha = \arctan\frac{\Delta U}{\Delta W}. \tag{10}$$

2.3 Applying the climate change hypothesis to predict changes in basin evapotranspiration and streamflow

The prediction of changes in streamflow or evapotranspiration based on climate changes requires some strong assumptions. Previous attempts to this problem generally involved some more or less empirically derived functional form of the relationship of E_T, Q to P, E_p. Examples are Dooge et al. (1999); Arora (2002) using Budyko functions, Sankarasubramanian et al. (2001); Zheng et al. (2009) using a statistical approach or Schaake (1990) and many more using hydrologic models.

Here, we approach this problem by posing the strict assumption that climate change impacts on E_T and Q do not change the catchment efficiency (defined above in Eq. 5). This is different to the previous attempts, because our assumption is theoretically based and does not require any empirical parameterisations.

The derivation of an analytical expression for prediction of streamflow or evapotranspiration given a climatic change signal is straightforward. Again, consider the two long-term average hydro-climate states introduced above. Applying the CCUW hypothesis (Eq. 9) to the definitions of W and U (Eq. 6), we reconfirm that the sum of

HESSD
8, 8793–8830, 2011

E_T/P and E_T/E_p of a given basin is constant and thus invariant for different climatic conditions:

$$\frac{E_{T,0}}{P_0} + \frac{E_{T,0}}{E_{p,0}} = \frac{E_{T,1}}{P_1} + \frac{E_{T,1}}{E_{p,1}} = \mathrm{CE} = \mathrm{const.} \tag{11}$$

Finally rearranging yields an expression to compute the evapotranspiration of the new state ($E_{T,1}$):

$$E_{T,1} = E_{T,0} \frac{\frac{1}{P_0} + \frac{1}{E_{p,0}}}{\frac{1}{P_1} + \frac{1}{E_{p,1}}}. \tag{12}$$

By applying the long term water balance equation with $P = E_T + Q$ the expected new state in streamflow Q_1 can also be predicted:

$$Q_1 = \frac{\frac{Q_0}{P_0} - \frac{P_0 - Q_0}{E_{p,0}} + \frac{P_1}{E_{p,1}}}{\frac{1}{P_1} + \frac{1}{E_{p,1}}}. \tag{13}$$

So, given a reference long term hydro-climatic state space of a basin (P_0, $E_{p,0}$, $E_{T,0}$) or (P_0, $E_{p,0}$, Q_0) and changes in the climate state (P_1, $E_{p,1}$), the resulting hydrologic states Q_1 or $E_{T,1}$ can be predicted.

2.4 Derivation of climatic sensitivity using the CCUW hypothesis

To assess the sensitivity of a basin at a given hydro-climatic state space (P, E_p, E_T) to changes in climate, we derive the first derivatives of W and U. The result is the tangent at a given hydro-climatic state space. First W and U are expressed as functions of E_T and E_p and P, respectively:

$$W = w(P, E_T) = 1 - \frac{E_T}{P} \quad U = u\left(E_p, E_T\right) = 1 - \frac{E_T}{E_p}. \tag{14}$$

HESSD
8, 8793–8830, 2011

Then their first total derivatives and solutions of the partial differentials are:

$$\mathrm{d}W = w'(P, E_\mathrm{T}) = \frac{\partial u}{\partial P}\,\mathrm{d}P + \frac{\partial u}{\partial E_\mathrm{T}}\,\mathrm{d}E_\mathrm{T} \tag{15}$$

$$\mathrm{d}U = u'\left(E_\mathrm{p}, E_\mathrm{T}\right) = \frac{\partial u}{\partial E_\mathrm{p}}\,\mathrm{d}E_\mathrm{p} + \frac{\partial u}{\partial E_\mathrm{T}}\,\mathrm{d}E_\mathrm{T} \tag{16}$$

$$\frac{\partial w}{\partial P} = \frac{E_\mathrm{T}}{P^2}, \; \frac{\partial w}{\partial E_\mathrm{T}} = -\frac{1}{P}, \; \frac{\partial u}{\partial E_\mathrm{p}} = \frac{E_\mathrm{T}}{E_\mathrm{p}^2}, \; \frac{\partial u}{\partial E_\mathrm{T}} = -\frac{1}{E_\mathrm{p}}. \tag{17}$$

Combining Eqs. (16), (15) by the CCUW hypothesis Eq. (9) yields an expression for changes in E_T:

$$\mathrm{d}E_\mathrm{T} = \frac{-\frac{\partial u}{\partial E_\mathrm{p}}\,\mathrm{d}E_\mathrm{p} - \frac{\partial w}{\partial P}\,\mathrm{d}P}{\frac{\partial u}{\partial E_\mathrm{T}} + \frac{\partial w}{\partial E_\mathrm{T}}}. \tag{18}$$

Finally, dividing by E_T (i.e. the long term average) and term expansions we yield an expression for the relative sensitivity of E_T to relative changes in P and E_p, in which the partial solutions of relative excess water and energy Eq. (17) are applied to gain a numerical solution:

$$\frac{\mathrm{d}E_\mathrm{T}}{E_\mathrm{T}} = \left[\frac{E_\mathrm{p}}{E_\mathrm{T}}\,\frac{-\frac{\partial u}{\partial E_\mathrm{p}}}{\frac{\partial u}{\partial E_\mathrm{T}} + \frac{\partial w}{\partial E_\mathrm{T}}}\right]\frac{\mathrm{d}E_\mathrm{p}}{E_\mathrm{p}} + \left[\frac{P}{E_\mathrm{T}}\,\frac{-\frac{\partial w}{\partial P}}{\frac{\partial u}{\partial E_\mathrm{T}} + \frac{\partial w}{\partial E_\mathrm{T}}}\right]\frac{\mathrm{d}P}{P} \tag{19}$$

$$\frac{\mathrm{d}E_\mathrm{T}}{E_\mathrm{T}} = \left[\frac{P}{E_\mathrm{p} + P}\right]\frac{\mathrm{d}E_\mathrm{p}}{E_\mathrm{p}} + \left[\frac{E_\mathrm{p}}{E_\mathrm{p} + P}\right]\frac{\mathrm{d}P}{P}. \tag{20}$$

By Eq. (20) we derived an analytical expression of the relative sensitivity of basin E_T to changes in climate. The terms in brackets are sensitivity coefficients, also referred to

HESSD
8, 8793–8830, 2011

as climate elasticity coefficients (Roderick and Farquhar, 2011; Yang and Yang, 2011). They express the proportional change in E_T or Q due to changes in climatic variables. Further, it can be seen from Eq. (20), that the relative sensitivity of E_T to climatic changes is only dependent on the aridity (E_p/P).

The sensitivities of streamflow to climate can be derived by applying the long term water balance equation $dQ = dP - dE_T$ to Eq. (20):

$$\frac{dQ}{Q} = \left[\frac{P\,(P - Q)}{Q\,(E_p + P)}\right]\frac{dE_p}{E_p} + \left[\frac{P}{Q} - \frac{(P - Q)\,E_p}{Q\,(E_p + P)}\right]\frac{dP}{P}. \tag{21}$$

So, besides of being dependent on aridity, streamflow sensitivity itself is also dependent on the long term average streamflow. Again the bracketed terms denote elasticity coefficients.

2.5 The Budyko hypothesis and derived sensitivities

The relation of climate and streamflow has been empirically described already in the early 20th century. In the long term it has been found that annual average evapotranspiration is a function of P and E_p. This is also known as the Budyko hypothesis. Two functional forms (Eqs. 25, 26) are reported in Table 1. However, actual E_T is often different from the functional non-parametric Budyko forms. To account for the manifold effects of basin characteristics on E_T, various functional forms have been proposed, which introduce an additional catchment parameter to improve the prediction of E_T. Widely applied is Eq. (27) established by Bagrov (1953) and Mezentsev (1955), which can be regarded as a generalisation of the function proposed by Pike (1964), (Choudhury, 1999; Roderick and Farquhar, 2011). Another form has been suggested by Fu (1981), where Yang et al. (2008) showed that the catchments parameter has a deterministic relationship with the parameter in Mezentsev's equation.

So more generally the Budyko functions express E_T as a function of climate and a catchment parameter n: $E_T = f(E_p, P, n)$. Once the functional type of f is known,

HESSD
8, 8793–8830, 2011
A simple water-energy balance framework
M. Renner et al.

climate changes causing a change in E_T ($\mathrm{d}E_\mathrm{T}$) from its long-term average can be computed (Dooge et al., 1999). Usually the first total derivative of f is being used (Arora, 2002; Roderick and Farquhar, 2011):

$$\mathrm{d}E_\mathrm{T} = \frac{\partial E_\mathrm{T}}{\partial P}\mathrm{d}P + \frac{\partial E_T}{\partial E_\mathrm{p}}\mathrm{d}E_\mathrm{p} + \frac{\partial E_\mathrm{T}}{\partial n}\mathrm{d}n. \tag{22}$$

Next, by employing the long term water balance equation $\mathrm{d}Q = \mathrm{d}P - \mathrm{d}E_\mathrm{T}$ to Eq. (22) an expression for the change in streamflow ($\mathrm{d}Q$) is gained (Roderick and Farquhar, 2011):

$$\mathrm{d}Q = \left(1 - \frac{\partial E_\mathrm{T}}{\partial P}\right)\mathrm{d}P - \frac{\partial E_\mathrm{T}}{\partial E_\mathrm{p}}\mathrm{d}E_\mathrm{p} - \frac{\partial E_\mathrm{T}}{\partial n}\mathrm{d}n. \tag{23}$$

With Eqs. (22), (23) and solutions of the respective partial differentials being dependent on the type of Budyko function used, we have analytical solutions for evapotranspiration and streamflow changes due to variations in climate conditions (Roderick and Farquhar, 2011). In the case of the non-parametric Budyko functions, the last term in Eqs. (22), (23) can be omitted.

Climatic elasticities ($\mathrm{d}E_\mathrm{T}/E_\mathrm{T}$ and $\mathrm{d}Q/Q$) can easily be obtained from Eqs. (22), (23) by dividing with E_T or Q and term expansions on the right side (Roderick and Farquhar, 2011):

$$\frac{\mathrm{d}E_\mathrm{T}}{E_\mathrm{T}} = \left[\frac{P}{E_\mathrm{T}}\frac{\partial E_\mathrm{T}}{\partial P}\right]\frac{\mathrm{d}P}{P} + \left[\frac{E_\mathrm{p}}{E_\mathrm{T}}\frac{\partial E_\mathrm{T}}{\partial E_\mathrm{p}}\right]\frac{\mathrm{d}E_\mathrm{p}}{E_\mathrm{p}} + \left[\frac{n}{E_\mathrm{T}}\frac{\partial E_\mathrm{T}}{\partial n}\right]\frac{\mathrm{d}n}{n} \tag{24}$$

$$\frac{\mathrm{d}Q}{Q} = \left[\frac{P}{Q}\left(1 - \frac{\partial E_\mathrm{T}}{\partial P}\right)\right]\frac{\mathrm{d}P}{P} + \left[\frac{E_\mathrm{p}}{Q}\frac{\partial E_\mathrm{T}}{\partial E_\mathrm{p}}\right]\frac{\mathrm{d}E_\mathrm{p}}{E_\mathrm{p}} + \left[\frac{n}{Q}\frac{\partial E_\mathrm{T}}{\partial n}\right]\frac{\mathrm{d}n}{n}. \tag{25}$$

As in the previous subsection, the bracketed terms denote the elasticity coefficients for P, E_p and n. For the computation of $\mathrm{d}E_\mathrm{T}$, $\mathrm{d}Q$ and the elasticity coefficients, we only

HESSD
8, 8793–8830, 2011

need to enter the respective partial differentials. These are given for the Schreiber, Oldekop and Mezentsev functions in Table A1, to be found in the appendix.

It is important to add here, that for prediction of absolute changes in streamflow as proposed by Eq. (23), we inherently assume, that the observed streamflow or evapotranspiration value is on the respective Budyko function. However, especially when using non-parametric Budyko functions this assumption is often violated. A practical workaround can be gained by multiplying Eqs. (24), (25) with E_T, Q, respectively. So, for predicting streamflow changes (ΔQ) this would be (Zheng et al., 2009):

$$\Delta Q = Q\left(\varepsilon_{Q,P}\frac{\Delta P}{P} + \varepsilon_{Q,E_p}\frac{\Delta E_p}{E_p}\right). \tag{26}$$

Practically one would first compute the elasticity coefficients $\varepsilon_{Q,P}$ and ε_{Q,E_p}, which are the bracketed terms of Eq. (25) and then apply these to Eq. (26). For the non-parametric Budyko functions the sensitivity term of the catchment parameter has no relevance.

3 Sensitivity analysis

In the previous section we introduced a method to predict streamflow/evapotranspiration changes based on the hypothesis that the catchment efficiency (CE) to use the available water and energy for evapotranspiration remains constant. In this section the properties and implications of the new method are discussed and compared with the established Budyko streamflow sensitivity approaches.

HESSD
8, 8793–8830, 2011

3.1 Mapping of the Budyko functions into UW space

The variables (P, E_p, E_T) used by the Budyko and the CCUW hypothesis are identical and can be easily related between both diagrams (spaces):

$$W = 1 - f\left(E_\mathrm{p}, P, n\right), \quad U = 1 - \frac{f\left(E_\mathrm{p}, P, n\right) P}{E_\mathrm{p}}. \tag{27}$$

Figure 2 illustrates the functional behaviour of the introduced Budyko functions in UW space. The non-parametric Budyko functions of Schreiber and Oldekop follow curves with relatively low values of both, relative excess water and energy. The parametric Budyko functions describe curves in the UW space which also allow higher values of W and U. Also note that for $n = 1$ the Mezentsev function Eq. (27) follows the negative diagonal of the climate change hypothesis, cf. Fig. 1.

More important for streamflow change assessment is that the Budyko functions display curves in the UW space. Generally, the derived climate sensitivity is a tangent at some aridity value of a Budyko function. Meaning that there are different change directions of ΔU and ΔW, depending on the aridity of a basin. So, under humid conditions climatic changes are more sensitive on relative excess water (larger change in runoff ratio than in excess energy), while under arid conditions changes are more sensitive to relative excess energy. Note that the CCUW model per definition assumes an equal, but opposite change in relative excess water and energy, independent of the aridity.

3.2 Mapping CCUW into Budyko space

For comparison of the CCUW hypothesis with the established Budyko functions we map the CCUW hypothesis into Budyko space and visualise the differences. For the purpose of mapping we come back to Eq. (11), which is a consequence of the climate change impact hypothesis in UW space: the catchment efficiency (CE) is a constant basin characteristic. With that we can rearrange Eq. (11) to achieve a mapping to Budyko space:

$$\frac{E_\mathrm{T}}{P} = \frac{\mathrm{CE} \cdot E_\mathrm{p}}{P + E_\mathrm{p}}. \tag{28}$$

The actual value of the catchment efficiency CE determines the asymptote for $E_\mathrm{p}/P \to \infty$. This makes a distinction from the Budyko hypothesis, which employs the water limit $E_\mathrm{T}/P = 1$ as asymptote for $E_\mathrm{p}/P \to \infty$. However, note, that Eq. (28) is not intended to be used for prediction of E_T from climate data, but for estimating the effect of climate changes on E_T.

Figure 3 illustrates the functional form of change predictions of the CCUW hypothesis for different values of CE. These can be compared with the functional forms of the three Budyko functions introduced above. The curves of the CCUW hypothesis are strongly determined by the catchment efficiency, similar to the effect of different values for the catchment parameter n in the parametrized Budyko model of Mezentsev (1955). By recollecting Eqs. (26) and (27) we can see, that for $n = 1$ and $\mathrm{CE} = 1$ both functions are identical.

As mentioned above, there is a different asymptotic behaviour of the CCUW hypothesis compared to the Budyko hypothesis. Especially under more arid climatic conditions the differences in climatic sensitivity are apparent. When $\mathrm{CE} > 1$, the slopes of the CCUW function are steeper than those of the Budyko functions and if $\mathrm{CE} < 1$ the slopes are more levelled. A more detailed discussion on this follows in the next subsection.

Moreover, let us consider the case of increasing aridity and a basin on the curve for $\mathrm{CE} = 1.3$. Then at some point the water limit will be reached, which implies no streamflow as well as not enough water to sustain E_T. So, in reality we would expect that the catchment efficiency decreases and thus we would expect a decline of the ecosystem status. However, such coupled climate-vegetation effects are per definition not covered by the CCUW method for streamflow change estimation.

HESSD
8, 8793–8830, 2011

3.3 Climatic sensitivity of basin evapotranspiration and streamflow

In the theoretical section of this paper we derived analytical equations (i) for predicting the absolute hydrological response for variations in climate and (ii) for estimating the climatic sensitivity, i.e. the proportional change in E_T or Q by a proportional change in climate.

Figure 4 illustrates the general behaviour of the CCUW hypothesis under changes in precipitation or potential evapotranspiration, which is expressed by Eqs. (12), (13). The left panels of Fig. 4 show the relative change of streamflow to P (upper) and E_p (lower panel). From Eq. (13) follows that climatic sensitivity of streamflow is regulated by runoff ratio $W = Q/P$ and aridity E_p/P. We find that the smaller the runoff ratio, the larger the climatic effect on streamflow. The slopes of curves depicting the relative change of streamflow are modulated by aridity, with more arid (humid) basins having a smaller (larger) sensitivity. In the right panels of Fig. 4 the relative changes in E_T due to relative changes in P (upper panel) and in E_p (lower panel) are shown. The figures highlight that the magnitude of relative change is dependent on the aridity of the given basin. So the more arid the climate, the larger are changes in E_T due to changes in P, while changes in E_p show the opposite behaviour.

In addition, the curves shown in Fig. 4 display substantial nonlinear behaviour to changes whether in P or E_p. Considering the rainfall-runoff relation, this means that the relative change in streamflow is not proportional to the change in precipitation, but also depends on the magnitude of change in precipitation. In general positive precipitation changes result in stronger changes in streamflow, than negative precipitation changes. Such features have e.g. been reported by Risbey and Entekhabi (1996), analysing the response of the Sacramento River basin (US) to precipitation changes. While Risbey and Entekhabi (1996) argue that hydrological memory effects are related to this nonlinear behaviour, our analysis suggests that the coupled nature of water and energy balances is the primary cause of the nonlinear response of streamflow to climate.

Next, we discuss and compare climate elasticities derived by the CCUW and the Budyko sensitivity approaches. Kuhnel et al. (1991) showed that $\varepsilon_P + \varepsilon_{E_\mathrm{p}} = 1$. Therefore, we only discuss the elasticity to precipitation. Figure 5 displays the elasticity of E_T ($\varepsilon_{E_\mathrm{T},P}$) as a function of aridity. In more humid or semi-arid conditions ($E_\mathrm{p}/P < 2$), the differences between the Budyko function elasticities and the ones derived by the CCUW hypothesis are small. In each case the sensitivity increases with aridity. In more arid conditions larger differences of the CCUW hypothesis to the Budyko sensitivity functions become apparent. Thereby the non-parametric Budyko functions and the parametric Budyko type with $n > 1$ approach the upper limit ($\varepsilon_{E_\mathrm{T},P} = 1$) distinctly faster than the CCUW method.

So for example, a precipitation decrease of 10 % in an arid basin with $E_\mathrm{p}/P = 4$ results in an estimated change of E_T by 8 %, when the CCUW hypothesis is applied. However, applying Budyko hypothesis with the Oldekop function, E_T changes by 9.6 %. Even though this seems to be a small difference, in absolute values such changes are large, when considering the fact that in such arid basins annual E_T is almost as large as annual precipitation.

Regarding the elasticity of streamflow, the picture gets more complicated. First, the sensitivity of streamflow is also dependent on the streamflow itself (cf. Eqs. 21, 25). Secondly, in arid conditions, streamflow is typically very small compared to all other variables considered here. So even small absolute changes in Q may results in very large elasticity coefficients. In Fig. 6 we show $\varepsilon_{Q,P}$ as a function of aridity. Because of the dependency to streamflow, or rather to catchment efficiency, we plot $\varepsilon_{Q,P}$ as computed by CCUW for different values of CE. The effect of CE on streamflow is shown in the left panel of Fig. 6, where we plot the runoff ratio Q/P as function of aridity. The streamflow elasticities derived by the CCUW method clearly show for arid conditions, that the larger CE (and thus smaller Q) the larger gets $\varepsilon_{Q,P}$. In contrast the Budyko functions converge to a maximal level of $\varepsilon_{Q,P}$, with the exception of Schreiber's equation which is increasing linearly.

3.4 Catchment efficiency and the Budyko hypothesis

As noted above, the sensitivity functions based on the Budyko hypothesis show to have higher sensitivity to E_T and lower sensitivity to Q when aridity increases. This is due to the fact that the Budyko functions asymptotically approach the water limit. Thus, by definition of the Budyko functions it is assumed that catchment efficiency itself is a function of climate. In Fig. 7 catchment efficiency is plotted as function of aridity for the three Budyko function types. This feature implies, when using the Budyko framework for climate sensitivity that the catchment efficiency of a basin is inherently changed when climate is changing. Or phrased differently, we imply some predefined feedback effect of the climate to the basins ecosystem.

The problem however is, that this predefined feedback effect is established by drawing a curve through the long term averages of P, E_p, Q of different, independent basins. That means that the specific characteristics of the basin under investigation are neglected by the Budyko sensitivity functions.

4 Application: three case studies

To demonstrate the applicability of the newly derived streamflow sensitivity method we selected data of three different large river basins. We compare the climate sensitivities and absolute streamflow change predictions with the Budyko approaches.

For the case studies we selected the Murray-Darling Basin (MDB) in Australia (Roderick and Farquhar, 2011), the headwaters of the Yellow River basin (HYRB) in China (Zheng et al., 2009), and the Mississippi River Basin (MRB) in North America (Milly and Dunne, 2001). These large basins differ in climate and include arid (MDB), cold and semi-humid (HYRB) and warm, humid (MRB) climates. All basins have already been subject to climate sensitivity studies. Using hydro-climate data from the above references we derived climate sensitivity coefficients and compute the change in streamflow given the published trends in climate. All data and computations can be found in Table 2.

4.1 Mississippi River Basin (MRB)

The largest observed trend in climate of the three basins is found for the Mississippi River Basin (upstream of Vicksburg). In the period from 1949–1997, we find a marked trend towards a more humid climate with an increasing trend in P and a decreasing trend in evaporative demand (E_p). As one would expect, the observed streamflow increased by 48.9 mm and all predictions are around that magnitude, thus providing evidence that climatic variations explain most of the observed change in runoff. However, the CCUW method as well as Oldekop's function yield somewhat larger sensitivities $\varepsilon_{Q,P}$, and thus predict a larger change in streamflow given the climatic changes. Thus, assuming that the given data are correct, the difference in observed ΔQ and predicted ΔQ_ccuw, which is about 7 mm, must be attributed to some change in basin characteristics. This is consistent with the change direction $\alpha = 304^\circ$, which indicates an increase of 1 % in catchment efficiency. There is some evidence that the increased CE and thus E_T is a result of the activities within a soil and water conservation program established in the 1930s (Kochendorfer and Hubbart, 2010). Note, that the numbers given for changes in human water use (e.g. dam management, groundwater harvesting) as given by Milly and Dunne (2001), do not significantly change the magnitude in observed and predicted changes.

4.2 Headwaters of the Yellow River Basin (HYRB)

The headwaters of the Yellow River basin are highly elevated (above 3480 m a.s.l.) and thus relatively cold and receive seasonal monsoon precipitation (Zheng et al., 2009). This basin is also different to the others considered here, as the observed decrease in streamflow (−36.2 mm) comparing the periods 1960–1990 and 1990–2000 cannot be explained by the long term average changes in precipitation and potential

HESSD
8, 8793–8830, 2011

evapotranspiration, which almost neutralise each other. As a result, the methods considered here can attribute only 24 to 30 % of the observed change to climate variations. Further, the change direction in UW space with $\alpha = 210^{\circ}$ shows that the main direction of the observed change is in basin change direction. In this case the catchment efficiency has been improved by 9 %. The data reported on changes in land cover fractions before and after 1990 in Zheng et al. (2009) support the landcover change hypothesis. Especially the increase in cultivated and forested land (above 120 %) on the cost of grassland supports this direction of change towards higher catchment efficiency.

4.3 Murray-Darling River Basin (MDB)

For a more detailed discussion of the case studies, the MDB has been selected. It has the driest climate ($E_p/P = 3.5$) of all three basins considered. Also the climatic sensitivity coefficients are largest and climate effects on streamflow are expected to be large. We concentrate on the CCUW hypothesis and the parametrised Budyko function approach, a framework which was presented by Roderick and Farquhar (2011), especially for the MDB. We find, that in the MDB the influence of the catchment parameter on climatic sensitivity assessment is rather large. First, the sensitivity of streamflow to the catchment parameter n given by $\varepsilon_{Q,n} = \frac{n}{Q}\frac{\partial E_T}{\partial n}$ (Roderick and Farquhar, 2011) for the MDB is with $\varepsilon_{Q,n} = 2.97$ larger than the sensitivity to precipitation $\varepsilon_{Q,P;\mathrm{mez}} = 2.6$. That means, a small change in n can result in very large changes in streamflow. Secondly, we estimated n using data of the longer period (1895–2006) as Roderick and Farquhar (2011) did with $n = 1.74$. However, for the shorter period (1997–2006) we find $n = 1.81$. That means, that the hydro-climatic state of the shorter period slightly deviates from the fitted Budyko function. So, whether the calibration of n is problematic or the assumption of $\mathrm{d}n = 0$ for estimating climatic sensitivity is not correct.

In contrast, the CCUW hypothesis, does not need to be calibrated and thus there is no sensitivity to some parameter. We find that $\alpha = 135^{\circ}$, i.e. the observed change is in climate change direction, with increased aridity resulting in increased W and reduced U, meaning less runoff and increased surface heating. Further, as we see from

Table 2, the largest sensitivity of streamflow to precipitation is predicted by CCUW. This is because the sensitivity is mainly determined by the inverse of the runoff ratio, which is very large for the MDB ($P/Q = 16.7$) and only secondly by aridity. In effect, CCUW predicts smaller changes in E_T than the Budyko approaches under arid conditions. This damping effect may be caused by the implicit nature of E_T, which is through the coupling to soil moisture storage dependent on itself (Yang et al., 2008).

Figure 8 illustrates the differences between the parametrised Budyko and the CCUW method on climate sensitivity. A diagram, which may be practically considered for the assessment of future hydrological impacts of predicted changes in precipitation and evaporative demand (E_p) (Roderick and Farquhar, 2011). We see that the contour lines of the estimates by the CCUW method are about two times more dense compared to the contours of Roderick and Farquhar's approach. This is due to the fact, that the sensitivity to precipitation is almost twice as large. Thus the CCUW hypothesis provides some evidence, that climate change impacts on runoff in the MDB are probably larger than expected by Budyko frameworks (here with the exception of Schreiber's equation).

5 Conclusions

This paper is based on a conceptual framework which links shifts in ecohydrological states of river basins to shifts in climate and basin characteristics. Here, we utilize this concept of climate effects on streamflow to derive analytical solutions (i) to predict the impact of climate variations on evapotranspiration and streamflow and (ii) to assess the climatic sensitivity of river basins. Both issues are of great practical and scientific concern.

5.1 Justification of the ecohydrological change concept

The original concept published by Tomer and Schilling (2009) is based on the observation that climate impacts on streamflow produce shifts in the ecohydrological states of relative excess water and relative excess energy, which are orthogonal to shifts induced by land-use or land management changes. Here, we show that these changes can be understood by considering the sum of the ratio of annual actual evapotranspiration to precipitation and the ratio of actual to potential evapotranspiration. Considering changes in this term, to which we refer to as catchment efficiency, provides a theoretical explanation of Tomer and Schilling's concept. It is derived by coupling the long term water and energy balance equations. We show that catchment efficiency is a catchment specific parameter, which is constant if we consider climatic change effects only.

5.2 Constant catchment efficiency and climate sensitivity of streamflow

The key findings of this paper are the derived analytical solutions for absolute change prediction as well as climate sensitivity estimation. These are derived by utilizing the assumption that catchment efficiency is constant, denoted as climate change hypothesis (CCUW). This is the key to predict variations in evapotranspiration and streamflow based on a climate change signal. So, for example, consider a shift in climate to more arid conditions. Applying the climate change hypothesis, the increase in the ratio of actual evapotranspiration to precipitation must balance with the decrease in the ratio of actual to potential evapotranspiration to keep the sum of both terms, i.e. the catchment efficiency, constant.

5.3 Properties of the climate change hypothesis

To understand the properties and implications of the new method for estimating climate sensitivity we provide a thorough discussion of its properties for different climates, expressed by aridity and different possible hydrological responses. These findings are supported by case studies of three large river basins. Thereby, we compare the new method with sensitivity estimates derived from the Budyko hypothesis.

HESSD
8, 8793–8830, 2011

First of all, the method can be applied to any reasonable hydro-climatological state. This is important, because sensitivity functions based on non-parametric Budyko functions can yield non-reasonable estimates, when the observed evapotranspiration is far off the predicted one. Secondly, the method does not require calibration. Any catchment specific property is ideally encoded in the observed catchment response (streamflow or evapotranspiration), which is included in the sensitivity equations. Therefore, any ambiguity in (a) finding a catchment parameter and (b) assuming how this parameter changes with climate is avoided. A third advantage over the Budyko sensitivity approaches is the clear definition of climate – ecosystem feedbacks. The CCUW hypothesis allows for changes in the partitioning of water and energy fluxes at the surface, but does not assume that the catchment efficiency is changed with climate as well. The Budyko hypothesis as such, proposes that ecosystem efficiencies are changing with aridity. But it remains to question if we can infer such a feedback relation for a specific river basin.

In addition to these theoretical advantages over the Budyko sensitivity approaches, both approaches are relatively similar under humid conditions ($E_p/P < 2$). Under more arid conditions the differences are shown to be more significant. This is due to the different asymptotic behaviours of both approaches when aridity increases. So the Budyko approaches tend to have an upper level of streamflow sensitivity to precipitation. In contrast, the CCUW hypothesis predicts exponentially increasing sensitivities to precipitation when the ratio of actual evapotranspiration to precipitation approaches unity.

5.4 Outlook

The case studies demonstrate the applicability of the new method and the value for interpreting changes in streamflow using the ecohydrological concept of Tomer and Schilling (2009). Thus, in a companion paper (Renner and Bernhofer, 2011) we assess hydro-climatic changes within the contiguous United States by employing a large and long term hydro-climatic dataset of more than 400 basins. The results reported are encouraging.

Acknowledgement. This work was kindly supported by Helmholtz Impulse and Networking Fund through Helmholtz Interdisciplinary Graduate School for Environmental Research (HIGRADE) (Bissinger and Kolditz, 2008). The first Author wants to thank Kai Schwärzel (TU Dresden) for bringing the paper of Tomer and Schilling to his attention. Also the lively discussions with Martin Volk and Ralf Seppelt (UFZ – Leipzig) encouraged M. R. to develop the theoretical basis of this paper. Nadine Große (Uni Leipzig) is credited for clarifying some mathematical operations. Kristina Brust (TU Dresden) is gratefully acknowledged for reading and correcting the manuscript.

References

Arnell, N.: Hydrology and Global Environmental Change, Prentice Hall, 2002. 8795

Arora, V.: The use of the aridity index to assess climate change effect on annual runoff, J. Hydrol., 265, 164–177, 2002. 8794, 8796, 8797, 8801, 8805

Bagrov, N.: O srednem mnogoletnem isparenii s poverchnosti susi (on multi-year average of evapotranspiration from land surface), Meteorog. i Gridrolog. (Russ.), 10, 1953. 8804

Bissinger, V. and Kolditz, O.: Helmholtz Interdisciplinary Graduate School for Environmental Research (HIGRADE), GAIA-Ecological Perspectives for Science and Society, 17, 71–73, 2008. 8817

Budyko, M.: Climate and life, Academic press, New York, USA, 1974. 8794, 8797, 8798

Choudhury, B.: Evaluation of an empirical equation for annual evaporation using field observations and results from a biophysical model, J. Hydrol., 216, 99–110, 1999. 8804

Donohue, R. J., Roderick, M. L., and McVicar, T. R.: On the importance of including vegetation dynamics in Budyko's hydrological model, Hydrol. Earth Syst. Sci., 11, 983–995, doi:10.5194/hess-11-983-2007, 2007. 8795

Dooge, J.: Sensitivity of runoff to climate change: A hortonian approach, B. Am. Meteorol. Soc. USA, 73, 2013–2024, 1992. 8794, 8796

Dooge, J., Bruen, M., and Parmentier, B.: A simple model for estimating the sensitivity of runoff to long-term changes in precipitation without a change in vegetation, Adv. Water Resour. 23, 153–163, 1999. 8801, 8805

Fu, B.: On the calculation of the evaporation from land surface, Scientia Atmospherica Sinica, 5, 23–31, 1981. 8804

Gerrits, A., Savenije, H., Veling, E., and Pfister, L.: Analytical derivation of the budyko curve based on rainfall characteristics and a simple evaporation model, Water Resour. Res., 45, W04403, doi:10.1029/2008WR007308, 2009. 8795

Kochendorfer, J. and Hubbart, J.: The roles of precipitation increases and rural land-use changes in streamflow trends in the upper mississippi river basin, Earth Interact., 4, 1–12, 2010. 8812

Kuhnel, V., Dooge, J., O'Kane, J., and Romanowicz, R.: Partial analysis applied to scale problems in surface moisture fluxes, Surv. Geophys., 12, 221–247, 1991. 8810

Mezentsev, V.: Eshche raz o raschete srednego summarnogo ispareniya (another remark on the problem of calculating the mean total evaporation), Meteorologiya i Gidrologiya, 1955. 8804, 8808, 8820, 8821, 8825, 8830

Milly, P.: Climate, soil water storage, and the average annual water balance, Water Resour. Res., 30, 2143–2156, 1994. 8795

Milly, P. and Dunne, K.: Trends in evaporation and surface cooling in the mississippi river basin, Geophys. Res. Lett., 28, 1219–1222, 2001. 8811, 8812

Nash, L. and Gleick, P.: Sensitivity of streamflow in the Colorado Basin to climatic changes, Journal of Hydrol. (Amsterdam), 125, 221–241, 1991. 8795

Ol'Dekop, E.: On evaporation from the surface of river basins, Transactions on meteorological observations, 4, 1944. 8820, 8821, 8825

Pike, J.: The estimation of annual run-off from meteorological data in a tropical climate, J. Hydrol., 2, 116–123, 1964. 8804

Renner, M. and Bernhofer, C.: Applying a simple water-energy balance framework to predict the climate sensitivity of streamflow over the contiguous US, in preparation, 2011. 8796, 8816

Risbey, J. and Entekhabi, D.: Observed Sacramento Basin streamflow response to precipitation and temperature changes and its relevance to climate impact studies, J. Hydrol., 184, 209–223, 1996. 8809

Roderick, M. and Farquhar, G.: A simple framework for relating variations in runoff to variations in climatic conditions and catchment properties, Water Resour. Res., 47, W00G07, doi:10.1029/2010WR009826, 2011. 8794, 8796, 8804, 8805, 8811, 8813, 8814, 8822, 8830

Sankarasubramanian, A., Vogel, R. and Limbrunner, J.: Climate elasticity of streamflow in the United States, Water Resour. Res., 37, 1771–1781, 2001. 8795, 8801
Schaake, J.: From climate to flow, in: Climate change and US water resources, edited by: Waggoner, P. E., 177–206, 1990. 8801
Schreiber, P.: Über die Beziehungen zwischen dem Niederschlag und der Wasserführung der Flüsse in Mitteleuropa, Z. Meteorol., 21, 441–452, 1904. 8820, 8821, 8825
Teuling, A., Hirschi, M., Ohmura, A., Wild, M., Reichstein, M., Ciais, P., Buchmann, N., Ammann, C., Montagnani, L., Richardson, A., Wohlfahrt, G., and Seneviratne, S. I.: A regional perspective on trends in continental evaporation, Geophys. Res. Lett., 36, L02404, doi:10.1029/2008GL036584, 2009. 8794
Tomer, M. and Schilling, K.: A simple approach to distinguish land-use and climate-change effects on watershed hydrology, J. Hydrol., 376, 24–33, 2009. 8794, 8795, 8796, 8798, 8799, 8800, 8814, 8815, 8816, 8817, 8823
Yang, H. and Yang, D.: Derivation of climate elasticity of runoff to assess the effects of climate change on annual runoff, Water Resour. Res., 47, W07526, doi:10.1029/2010WR009287, 2011. 8794, 8804
Yang, H., Yang, D., Lei, Z., and Sun, F.: New analytical derivation of the mean annual water-energy balance equation, Water Resour. Res., 44, W03410, doi:10.1029/2007WR006135, 2008. 8795, 8804, 8814
Zhang, L., Dawes, W., and Walker, G.: Response of mean annual evapotranspiration to vegetation changes at catchment scale, Water Resour. Res., 37, 701–708, 2001. 8795
Zhang, L., Hickel, K., Dawes, W., Chiew, F., Western, A., and Briggs, P.: A rational function approach for estimating mean annual evapotranspiration, Water Resour. Res., 40, W02502, doi:10.1029/2003WR002710, 2004. 8794
Zheng, H., Zhang, L., Zhu, R., Liu, C., Sato, Y., and Fukushima, Y.: Responses of streamflow to climate and land surface change in the headwaters of the yellow river basin, Water Resour. Res., 45, W00A19, doi:10.1029/2007WR006665, 2009. 8795, 8801, 8806, 8811, 8812, 8813

HESSD
8, 8793–8830, 2011

A simple water-energy balance framework

M. Renner et al.

Table 1. Budyko functions table.

Equation	References	
$E_T = P \cdot (1 - \exp^{-E_p/P})$	Schreiber (1904)	(25)
$E_T = E_p \cdot \tanh(P/E_p)$	Ol'Dekop (1911)	(26)
$E_T = E_p \cdot P/(P^n + E_p^n)^n$	Mezentsev (1955)	(27)

HESSD
8, 8793–8830, 2011
A simple water-energy balance framework
M. Renner et al.

Table 2. Observations and predictions of streamflow change of the three case-study river basins. Data are taken from the respective reference publications. For prediction of streamflow change we compared the CCUW method Eq. (13) with the the Budyko approaches. Equation (23) has been applied for the parametric function of Mezentsev (1955) (ΔQ_{mez}). For the non-parametric Budyko functions of Ol'Dekop (1911) (ΔQ_{old}) and Schreiber (1904) (ΔQ_{sch}) we employed Eq. (26). Change direction in *UW* space α corresponding with Fig. 1 is computed by Eq. (10).

	unit	MDB	HYRB	MRB
area	km^2	1.1e+06	1.2e+05	3.0e+06
P	mm	457.0	511.6	835.0
E_P	mm	1590.8	773.6	1027.0
Q	mm	27.3	179.3	187.0
E_P/P	–	3.5	1.5	1.2
Q/P	–	0.06	0.35	0.22
ΔP	mm	−17.0	−21.0	85.4
ΔE_P	mm	21.0	−23.0	−17.8
ΔQ	mm	−5.6	−36.2	48.9
n	–	1.74	1.13	2.00
$\varepsilon_{Q,P;\mathrm{old}}$	–	2.94	2.70	2.58
$\varepsilon_{Q,P;\mathrm{sch}}$	–	4.48	2.51	2.23
$\varepsilon_{Q,P;\mathrm{mez}}$	–	2.60	1.71	2.38
$\varepsilon_{Q,P;\mathrm{ccuw}}$	–	4.51	1.74	2.55
ΔQ_{old}	mm	−3.7	−10.8	54.6
ΔQ_{sch}	mm	−5.8	−10.4	46.6
ΔQ_{mez}	mm	−3.2	−8.8	50.0
ΔQ_{ccuw}	mm	−5.7	−8.8	56.1
α	°	135	210	304

HESSD
8, 8793–8830, 2011

A simple water-energy balance framework

M. Renner et al.

Table A1. Partial differentials of the Budyko functions used in the text.

type	$\frac{\partial E_T}{\partial P}$	$\frac{\partial E_T}{\partial E_p}$
Schreiber	$1 - \exp^{-E_p/P} - \frac{E_p}{P}\exp^{-E_p/P}$	$\exp^{-E_p/P}$
Oldekop	$1 - \tanh^2\left(\frac{P}{E_p}\right)$	$-\frac{P}{E_p}\left(1 - \tanh^2\left(\frac{P}{E_p}\right)\right) + \tanh\left(\frac{P}{E_p}\right)$
Mezentsev	$\frac{E_T}{P}\left(\frac{E_p^n}{P^n + E_p^n}\right)$	$\frac{E_T}{E_p}\left(\frac{P^n}{P^n + E_p^n}\right)$
Roderick and Farquhar (2011)	$\frac{\partial E_T}{\partial n} = \frac{E_T}{n}\left(\frac{\ln\left(P^n + E_p^n\right)}{n} - \frac{\left(P^n \ln(P) + E_p^n \ln(E_p)\right)}{P^n + E_p^n}\right)$	

HESSD
8, 8793–8830, 2011
A simple water-energy balance framework
M. Renner et al.

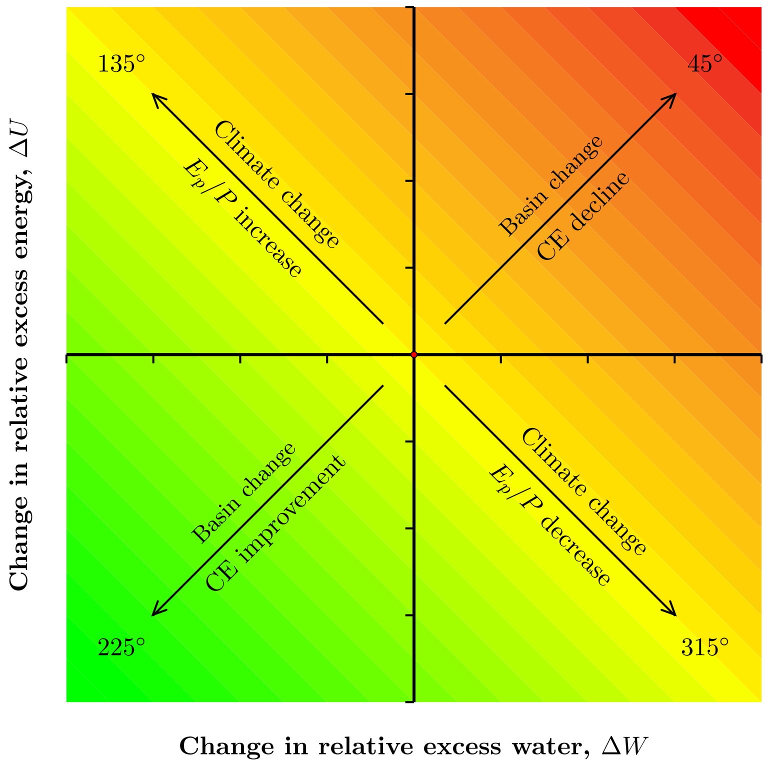

Fig. 1. Illustration of change attribution framework established by Tomer and Schilling (2009, after their Fig. 2). Considering climatic change effects, a change in either precipitation or potential evapotranspiration, will result in a change of both, relative excess water and energy but in opposite direction (change along the negative diagonal). Basin change effects, such as a change in vegetation or soils lead to a change in the evapotranspiration efficiency of the catchment (CE) and thus a deviation from the negative diagonal. Relative changes of CE are shown as filled contourlines, where red implies a decline, yellow constant and green improving CE.

HESSD
8, 8793–8830, 2011

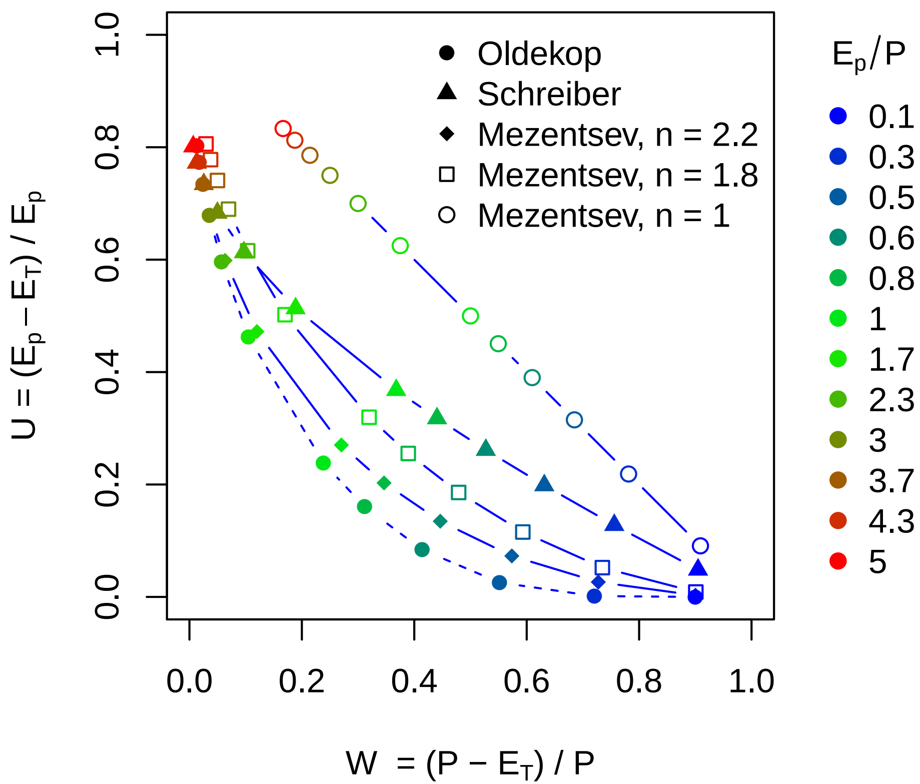

Fig. 2. Budyko function mapped into *UW* space using Eq. (27). The colours depict certain aridity (E_p/P) values indicated by the legend in the right.

HESSD
8, 8793–8830, 2011

A simple water-energy balance framework

M. Renner et al.

Title Page
Abstract
Introduction
Conclusions
References
Tables
Figures
◀
▶
Back
Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

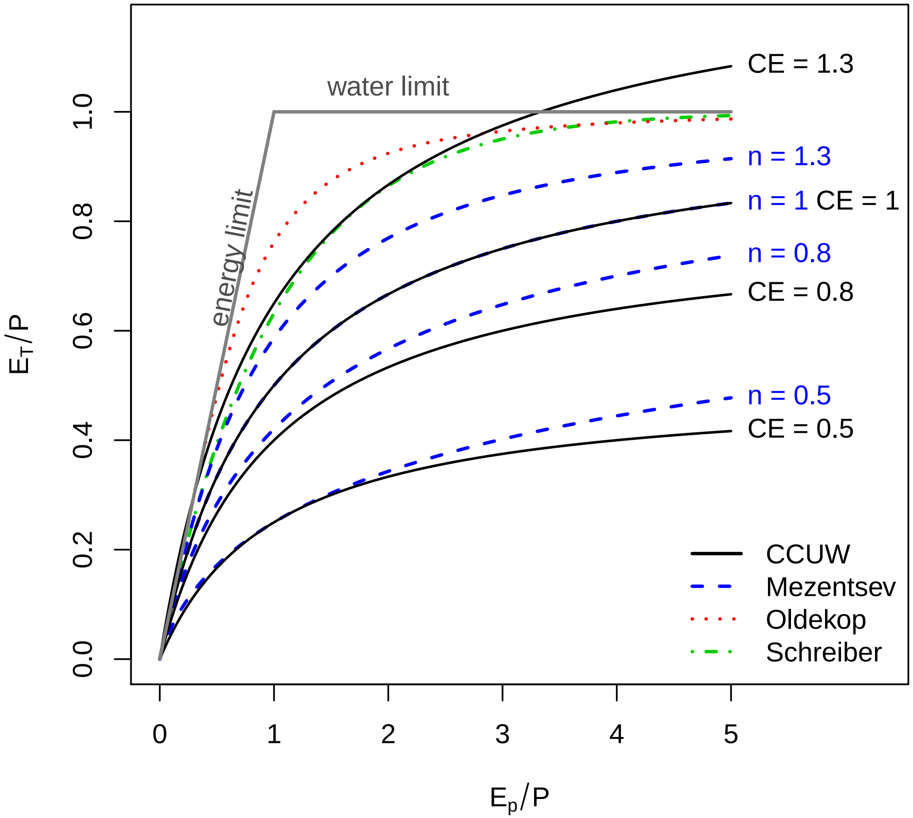

Fig. 3. Mapping of CCUW hypothesis into Budyko space for different values of catchment efficiency (CE) using Eq. (28). For comparison the Budyko functions of Schreiber (1904); Ol'Dekop (1911); Mezentsev (1955) are also shown. The grey lines depict the theoretical limits for water and energy.

HESSD
8, 8793–8830, 2011

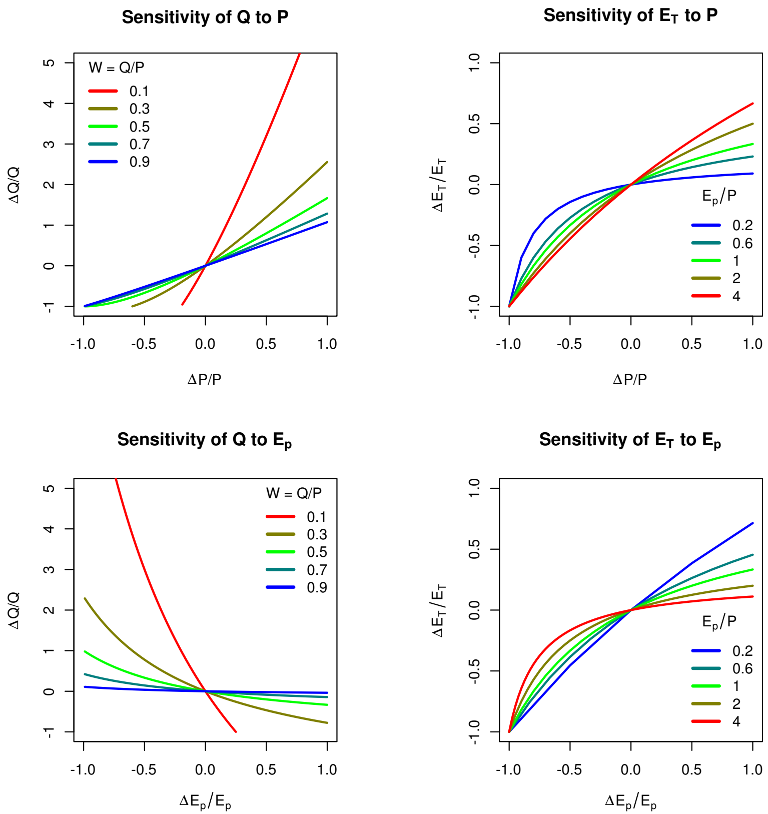

Fig. 4. Relative change in response to relative changes in P (top panels) and in E_p (bottom panels) of Q (left panels) and E_T (right panels) as predicted by the CCUW hypothesis. Changes in Q are dependent on runoff ratio W and on aridity E_p/P and are coloured with respect to the respective runoff ratio and shown for a aridity of $E_p/P = 1$. Relative changes in E_T are dependent on aridity only and lines are coloured with respect to different aridities. Note, that changes of $\Delta Q/Q$ smaller than -1 are not realistic.

HESSD
8, 8793–8830, 2011

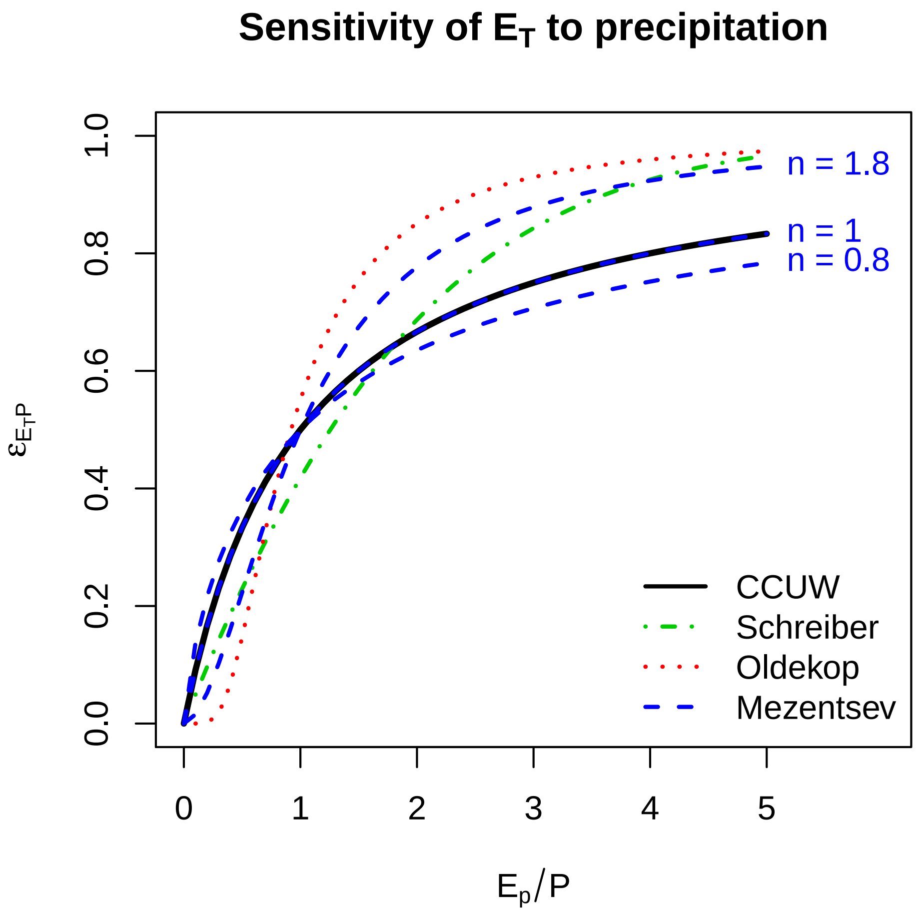

Fig. 5. Sensitivity (elasticity) of basin evapotranspiration with respect to changes in precipitation ($\varepsilon_{E_T,P}$), cf. Eq. (20). This corresponds with the slope of the curves shown in the top right panel of Fig. 4.

HESSD
8, 8793–8830, 2011

A simple water-energy balance framework

M. Renner et al.

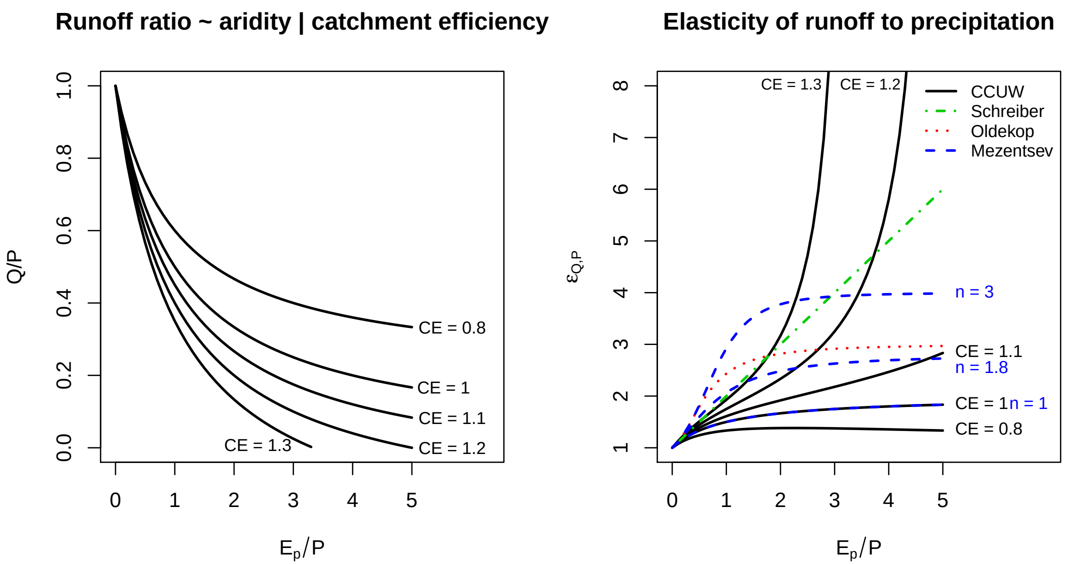

Fig. 6. Left panel: runoff ratio as function of aridity for different, but fixed values of catchment efficiency (CE) using Eq. (28). Right panel: elasticity coefficient of streamflow to precipitation $\varepsilon_{Q,P}$ as function of aridity. Displayed are the elasticities derived from the CCUW hypothesis (black for different values of CE), and the elasticities derived from different Budyko functions using Eq. (25).

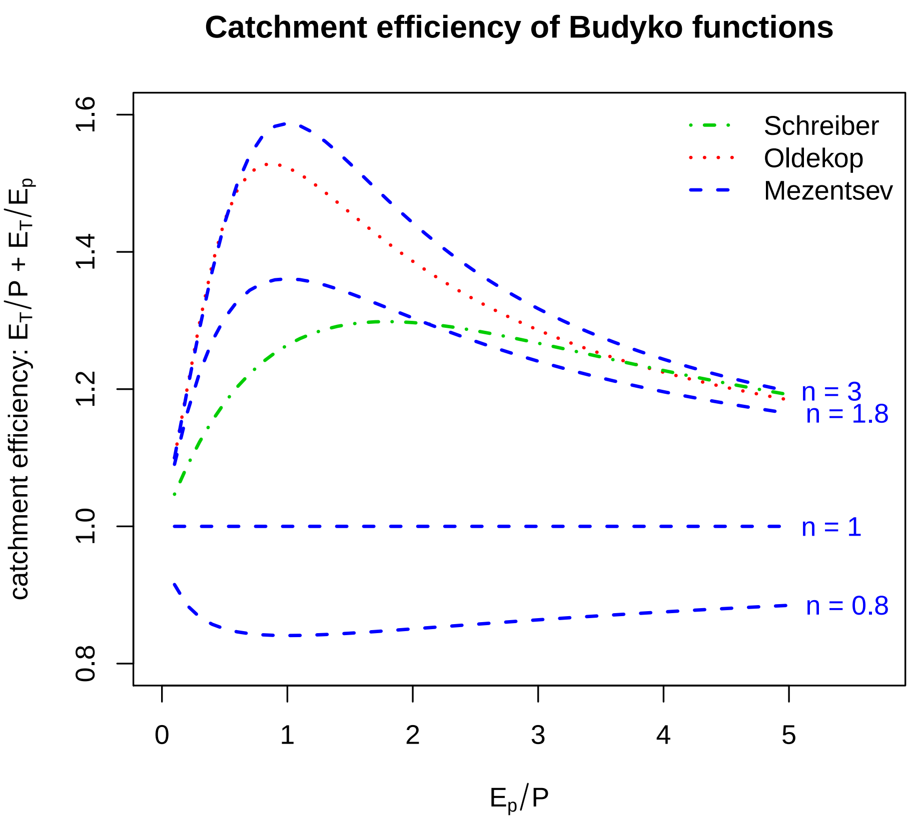

Fig. 7. Catchment efficiency as function of aridity as predicted by Budyko functions.

HESSD
8, 8793–8830, 2011

A simple water-energy balance framework

M. Renner et al.

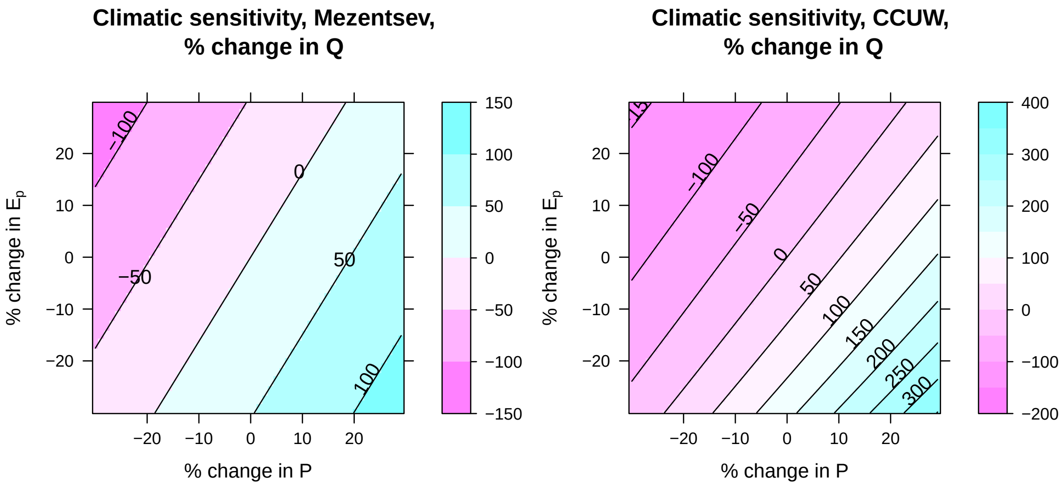

Fig. 8. Sensitivity plots of streamflow to percent changes of precipitation and E_p, estimated for the long term hydro-climatic states of the Murray-Darling Basin (as given in Table 2). Contour lines depict the percent change in streamflow. Right panel: The Budyko framework using the Mezentsev (1955) function and Eq. (25) in accordance to Roderick and Farquhar (2011, Fig. 4). Left panel, sensitivity estimation by the CCUW framework (Eq. 13).

HESSD
8, 8793–8830, 2011