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An analytical model for soil-atmosphere feedback

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Abstract

Soil-atmosphere feedback is a key for understanding the hydrological cycle and the direction of potential system changes. This paper presents an analytical framework to study the interplay between soil and atmospheric moisture, using as input only the boundary conditions at the upstream end of an atmospheric moisture stream line. The underlying Eulerian-Langrangean approach assumes advective moisture transport with average wind speed along the stream line and vertical moisture exchange with the soil compartment of uniform vertical properties. Precipitation, evaporation from interception and runoff are assumed to depend through simple functional relationships on the soil moisture or the atmospheric moisture. Evaporation from soil moisture (including transpiration) depends on both state variables, which introduces a nonlinear relationship between the two compartments. This nonlinear relationship can explain some apparently paradoxical phenomena such as a local decrease of precipitation accompanied by a runoff increase.

The solutions of the resulting water balance equations correspond to two different moisture regimes along a stream line, either monotonically increasing or decreasing when traveling inland, depending on boundary conditions and parameters. The paper discusses how different model parameters (e.g. time scales of precipitation, evaporation or runoff) influence these regimes and how they can create regime switches. Such an analysis has potential to anticipate the range of possible land use and climate changes or to interpret the results of complex land-atmosphere interaction models. Based on derived analytical expressions for the Horton index, the Budyko curve and a precipitation recycling ratio, the analytical framework opens new perspectives for the classification of hydrological systems.

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1 Introduction

Feedback processes between the land surface and the atmosphere have long been recognized as being key to understanding the hydrological cycle, e.g. for local and regional variability of precipitation (Tuinenburg et al., 2011; Eltahir, 1998; DeAngelis et al., 2010) or for the study of different sources of precipitation at continental scales, i.e. for moisture recycling studies (Burde and Zangvil, 2001; Eltahir and Bras, 1994; Trenberth, 1998). Recent results in this field demonstrate that on large continental areas, moisture recycling can be a dominant mechanism to sustain precipitation (e.g. Van der Ent et al., 2010).

Nevertheless, explicit representation or assessment of moisture recycling receives limited attention in classical meteorological or hydrological models. From a meteorological perspective, this is not surprising since advective moisture fluxes are generally an order of magnitude larger than evaporative fluxes (e.g. Schär et al., 1999). Moreover the calculation of evaporation is complex as it depends in a non-trivial way on soil moisture, atmospheric moisture, land roughness, energy exchange, and indirectly on topography, soil properties and land use, all of which are highly heterogeneous and variable in time. From a hydrological perspective, climate is generally considered as an exogenous forcing in terms of precipitation and potential evaporation. This viewpoint is a natural choice when analyzing individual catchments of up to few thousand square kilometers. However, if we model the hydrologic cycle at continental scales, if we analyze climate or land use change impacts or if we try to classify catchments across hydroclimatic regions (Wagener et al., 2007), we can only benefit from understanding the coupled soil-atmosphere system and moisture recycling.

Such insights can be obtained by methods ranging from analyzing the isotopical origin of precipitation (Tian et al., 2007) to different numerical techniques or analytical studies (see a discussion in Dominguez et al., 2006). Numerical studies commonly use e.g. month-long integration of regional or global coupled atmosphere-land surface models to analyze moisture feedbacks by varying soil and vegetation parameters and

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boundary conditions (Schär et al., 1999; Dirmeyer et al., 2006; Kunstmann and Jung, 2007). Such studies give valuable insights into these feedbacks. The results are, however, at least partially influenced by the model sensitivity, which could be addressed by multi-model studies (Koster et al., 2004). Furthermore, it is difficult to trace back how a parameter change modifies, directly or indirectly, a system output such as evaporation. This is, in contrast, the strength of analytical recycling models that quantify e.g. the contribution of local evaporation to total precipitation based on a set of simple balance equations used to compute water budgets based on observed or reanalysis data of evaporation and precipitation (e.g. Burde and Zangvil, 2001; Dominguez et al., 2006).

In this paper, we present a different type of analytical model: it describes the hydrologic cycle at points along an atmospheric stream line (Eulerian-Langrangean approach) using only the atmospheric storage at the upstream boundary (at the coast) as input. Atmospheric moisture is transported along the stream line with advection and exchanged with the soil through precipitation and evaporation that are formulated as functions of atmospheric and soil moisture. Evaporation from transpiration and intercepted water are quantified separately and the model also accounts for runoff. It may be considered a "toy model" that can be used to analyze moisture regimes and their sensitivity to interception, advected moisture, soil moisture and runoff and evaporation time scales.

In the following, we first present our coupled model, its analytical solutions and the possible moisture regimes along a flow path (Sect. 2). To illustrate the use of the model, we present three different types of analyses (Sect. 3): (i) the effect of parameter changes on moisture profiles along an atmospheric moisture flow path, (ii) the relationship between atmospheric moisture and the Horton index and the Budyko curve, which are used to describe the hydrologic behavior of a system (Troch et al., 2009), and (iii) the relationship between the key parameters and precipitation recycling. Before summarizing our main conclusions (Sect. 5), we briefly discuss the potentialities and limitations of the proposed model (Sect. 4).

2 Method

We adopt a Eulerian-Lagrangean modelling scheme (e.g. Huang et al., 1994) to simulate moisture transport along atmospheric stream lines where physical Langrangean quantities (atmospheric moisture, particle paths, dispersion and advection) are computed with Eulerian fluxes (rainfall, evaporation). This approach presents analytical advantages since the Langrangean trajectories, which can be obtained from data (e.g. Dominguez and Kumar, 2008; Van der Ent et al., 2010), contain considerably more information than what we would have in a purely Eulerian description using only the velocity fields.

An atmospheric moisture stream line starts at the coast, the positive x-direction is pointed inland. At a given location x, we assume uniform vertical properties of the atmosphere and model the exchange of moisture with a vertically uniform soil compartment by encoding the vertical fluxes between the two compartments (precipitation and evaporation, Fig. 1). Lateral transport through advection and turbulent diffusion is modelled only for atmospheric moisture and the only influx of water to the soil compartment is precipitation; the outfluxes are runoff, groundwater recharge and total evaporation (evaporation from the soil surface and transpiration). The boundary condition of the atmospheric compartment at the upstream boundary of a stream line is given by atmospheric moisture at the coast.

2.1 Modelling framework

Consider the control volume V, a tropospheric column of area $\Delta x b$ [L^2] and of mass $M = VW$, where W [–] is the relative atmospheric moisture filling.

The conservation of mass for M reads as (see also Fig. 1):

$$\frac{\partial (VW)}{\partial t} = -u_x \frac{\partial (VW)}{\partial x} + D_x \frac{\partial^2 (VW)}{\partial x^2} - \Delta x b (P - E_\mathrm{T} - E_\mathrm{I}) \quad (1)$$

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where P [LT^{-1}] is the precipitation, E_T [LT^{-1}] is the evaporative flux from the soil moisture compartment to the atmosphere mostly due to the transpiration of vegetation (but it also includes soil evaporation) and E_I is the evaporative flux from water intercepted on vegetation, forest floor or bare surface. D_x [$\mathrm{L}^2\mathrm{T}^{-1}$] is the dispersion coefficient and u_x [LT^{-1}] the wind speed in the flow direction. Note that all state and flux variables depend on space and time but for reasons of readability, we use the short forms $W = W(x,t)$ where appropriate.

Horizontal atmospheric mixing rates in the troposphere are typically in the order of magnitude of $10^4\ \mathrm{m}^2\mathrm{s}^{-1}$ (e.g. Pisso et al., 2009). Given the very small horizontal concentration gradients for atmospheric moisture C [L] (a few mm per 100 km, i.e. a gradient of $10^{-7}\mathrm{m\ m}^{-1}$, (e.g. Randel et al., 1996)), the dispersive flux $F_d = D_x \frac{\mathrm{d}c}{\mathrm{d}x}$, has an order of magnitude of $10^{-3}\ \mathrm{m}^2\ \mathrm{s}^{-1}$. Assuming average horizontal wind speeds of the order of $10\ \mathrm{m\ s}^{-1}$ and atmospheric moisture storage in the troposphere of the order of 10^{-2} m, it is readily apparent that the advective flux $F_a = u_x C \gg F_d$. We, therefore, neglect dispersion at the spatio-temporal scales considered here.

Expressing the control volume height in terms of the water holding capacity c_m [L] of the tropospheric column and assuming that c_m is constant in time, the left-hand term reads as

$$\frac{\partial(VW)}{\partial t} = W\frac{\partial V}{\partial t} + V\frac{\partial W}{\partial t} = \Delta x c_\mathrm{m} W \frac{\partial b}{\partial t} + \Delta x b c_\mathrm{m}\frac{\partial W}{\partial t}, \tag{2}$$

and we can re-write Eq. (1) as

$$\frac{\partial W}{\partial t} + u_x\frac{\partial W}{\partial x} = -\frac{1}{c_\mathrm{m}}(P - E_\mathrm{T} - E_\mathrm{I}) - \frac{1}{b}W\left(\frac{\partial b}{\partial t} + u_x\frac{\partial b}{\partial x}\right) \tag{3}$$

Equation (3) can be written in a Lagrangean framework using the substantial or Lagrangean derivative (Trenberth, 2009):

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + u_x\frac{\partial f}{\partial x} = u_x\frac{\mathrm{d}f}{\mathrm{d}x}, \tag{4}$$

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$$\frac{dW}{dt} = u_x \frac{dW}{dx} = -\frac{1}{c_m}(P - E_T - E_I) - u_x W \frac{1}{b}\frac{db}{dx}. \quad (5)$$

The last term in the above equation encodes the change of the streamline shape (of its width) along x (see Fig. 1), corresponding either to a convergence ($\frac{db}{dx} < 0$) or divergence ($\frac{db}{dx} > 0$). In the case of convergence, the narrowing of the control width results in an increased concentration of water in the control volume, which results in an apparent inflow of moisture. For simplicity, this inflow due to convergence is termed relative lateral inflow, $I = -u_x W \frac{1}{b}\frac{db}{dx}$ [T^{-1}].

For soil moisture, we assume absence of lateral transport and of volume change; the conservation of mass becomes

$$\frac{\partial S}{\partial t} = P - E_I - E_T - R, \quad (6)$$

where S [L] is the soil moisture and R [LT^{-1}] represents all water that is lost from the soil compartment through other processes than evaporation, i.e. it includes slow and rapid discharge processes and groundwater recharge. In the following, we refer to R as runoff. We assume a simple linear relationship to S through a residence time τ_q:

$$R = \frac{1}{\tau_q} S. \quad (7)$$

This corresponds to the frequently used assumption of a linear relationship between slow discharge or recharge processes and soil moisture (e.g., Fenicia et al., 2006). τ_q [T] is the time scale of the sum of these processes. We assume here that rapid discharge processes (e.g. surface runoff) are negligible.

Precipitation on a daily timescale can be assumed to depend (linearly) on the atmospheric moisture above a certain threshold (e.g. Trenberth et al., 2003; Savenije, 1995b):

$$P = \max\left(0, \frac{1}{\tau_c}(C - c_t)\right) = \max\left(0, \frac{c_m}{\tau_c}(W - w_t)\right) \quad (8)$$

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where τ_c [T] is the time scale of the precipitation process. On longer timescales, a squared relationship between P and W appears to capture their relationship reasonably well (see supplement, Fig. S1).

$$P = \frac{c_\mathrm{m}}{\tau_p} W^2, \tag{9}$$

where τ_p [T] is a corrected "effective" residence time for precipitation.

Interception is generally also assumed to be a threshold process at an hourly to daily time scale (e.g. Gerrits et al., 2010). de Groen and Savenije (2006) derived an expression for monthly interception as a function of monthly precipitation and number of rain days. However, to be able to derive analytical solutions, we retain here the simple linear relationship between interception and precipitation with the interception parameter α:

$$E_\mathrm{I} = \alpha P. \tag{10}$$

Following classical transpiration formulations in rainfall-runoff models (e.g. Clark et al., 2008), E_T is modeled as a function of potential evaporation E_P and the degree of soil saturation S/s_m

$$E_\mathrm{T} = E_\mathrm{P} \frac{S}{s_\mathrm{m}}, \tag{11}$$

where $S = S(x,t)$ [L] is the actual soil moisture storage and s_m [L] the maximum soil moisture storage [L]. E_P [L] depends on the actual meteorological conditions, most prominently on the available energy and the atmospheric moisture deficit $(1-W)$ (see, e.g, the Penmann-Monteith formulation, Monteith, 1965). We assume that the available energy is typical for a given climate and a given time of the year, i.e. that it is constant for our modeling purposes, and that its limiting effect on transpiration can be expressed in terms of a maximum amount of water, e_m [L], that could be transpired

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over the time scale of evaporation, τ_e, if W was not limiting. Accordingly, E_P is parameterized as:

$$E_\mathrm{P} = \frac{1}{\tau_e} e_\mathrm{m}(1-W). \tag{12}$$

Combining the above two equations, the complete equation for E_T, becomes

$$E_\mathrm{T} = \frac{1}{\tau_e}\frac{e_\mathrm{m}}{s_\mathrm{m}}(1-W)S. \tag{13}$$

τ_e represents the time scale of transpiration of the vegetation, i.e. the amount of time that the vegetation would require to transpire e_m if neither atmospheric moisture nor soil moisture was limiting ($W=0, S=s_\mathrm{m}$); this value is characteristic for a given vegetation-soil system.

Evaporation as parameterized in Eq. (13) is limited by the available soil moisture as well as by the capacity of the atmosphere to receive water and, thus, couples the two compartments.

Combining the above equations, the coupled water balance model becomes

$$u_x \frac{\mathrm{d}W}{\mathrm{d}x} = -\frac{1}{c_\mathrm{m}}\left((1-\alpha)\frac{c_\mathrm{m}}{\tau_p}W^2 - \frac{1}{\tau_e}\frac{e_\mathrm{m}}{s_\mathrm{m}}(1-W)S\right) + I \tag{14}$$

$$\frac{\partial S}{\partial t} = (1-\alpha)\frac{c_\mathrm{m}}{\tau_p}W^2 - \frac{1}{\tau_e}\frac{e_\mathrm{m}}{s_\mathrm{m}}(1-W)S - \frac{1}{\tau_q}S. \tag{15}$$

Recall that in the above equations we use the short notation W, S and I for $W(t,x)$, $S(t,x)$ and $I(t,x)$.

2.2 Analytical solution

Soil moisture is well-known to undergo a seasonal cycle of gradual filling and emptying, depending on the seasonality of precipitation and of vegetation growth. We, thus,

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assume that for sufficiently small time steps (weeks to months), the change of soil moisture $\frac{\partial S}{\partial t}$ can be approximated with a constant rate of change ξ_s [LT^{-1}].

Equation (15) can be re-written as:

$$\frac{S}{c_\mathrm{m}} = \left((1-\alpha)\frac{\tau_q}{\tau_p}W^2 - \frac{\tau_q}{c_\mathrm{m}}\xi_s\right)\frac{1}{1+\kappa(1-W)}, \tag{16}$$

where we have introduced $\kappa = \frac{e_\mathrm{m}\tau_q}{s_\mathrm{m}\tau_e}$. This parameter κ corresponds to the ratio of maximum potential evaporation e_m/τ_e to maximum runoff s_m/τ_q and is a parameter which controls the spatial dynamics.

Substituting the above S/c_m into Eq. (14) yields a second order ordinary differential equation for W:

$$u_x\frac{\mathrm{d}W}{\mathrm{d}x} = -\frac{(1-\alpha)}{\tau_p}\frac{1}{1+\kappa(1-W)}\left(W^2 + \frac{\tau_p\kappa\xi_s(1-W)}{(1-\alpha)c_\mathrm{m}}\right) + I \tag{17}$$

If we assume that $u_x, \tau_p, \tau_q, \tau_e, e_\mathrm{m}, s_\mathrm{m}, I$ are all constant in space, the solution of Eq. (17) is

$$\frac{x}{L} = -\log\left[\left(\frac{W(x)-W_1}{W_0-W_1}\right)^{A^*}\left(\frac{W(x)-W_2}{W_0-W_2}\right)^{B^*}\right] \tag{18}$$

where $L = u_x\frac{\tau_p}{1-\alpha}$ is the horizontal length scale for this solution, W_0 is the atmospheric moisture content at $x=0$ and W_1 and W_2 are the two equilibrium points of Eq. (17) that correspond to the solutions in the special case that $\mathrm{d}W/\mathrm{d}x = 0$. They are given by

$$W_{1,2} = \frac{1}{2}\left(\kappa(D^*-I^*) \pm \sqrt{\kappa^2(D^*-I^*)^2 - 4\kappa(D^*-I^*) + 4I^*}\right) \tag{19}$$

where we have used the scaled moisture convergence I^*

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$$I^* = \frac{\tau_p}{1-\alpha} I \quad (20)$$

and the scaled soil moisture variation D^*

$$D^* = \frac{\tau_p}{(1-\alpha)c_\mathrm{m}} \xi_s. \quad (21)$$

The dimensionless quantity D^* relates the soil moisture variation ξ_s to the maximum precipitation input to the soil $(1-\alpha)c_\mathrm{m}/\tau_p$.

The exponents A^* and B^* in Eq. (18) are

$$A^* = \frac{1+\kappa-\kappa W_1}{W_1 - W_2} \quad (22)$$

$$B^* = \frac{-1-\kappa+\kappa W_2}{W_1 - W_2} \quad (23)$$

It holds that $A^* + B^* = -\kappa$, $W_1 > W_2$ and $W_1 + W_2 = \kappa(D^* - I^*)$.

The behavior of Eq. (18) and the shape of $W(x)$ is further discussed hereafter. The corresponding soil moisture content is given in Eq. (16).

2.3 Behavior of the analytical solution

The implicit solution of Eq. (18) shows that $W(x)$ is either monotonically increasing along x or decreasing, depending on the model parameters and the boundary condition W_0. W_1 is the equilibrium moisture for $x \to +\infty$ and W_2 the equilibrium moisture for $x \to -\infty$. It follows that if $W_0 > W_2$ then W will converge to W_1 as $x \to +\infty$, either from above or below, depending on whether $W_0 > W_1$ or $W_0 < W_1$. If $W_0 < W_2$ then W will reach 0 at a finite positive value of x, and the mathematical solution is not physically realistic beyond that x.

Given that it has to hold that $0 \leq W(x) \leq 1$, a physical solution only exists if there is real equilibrium moisture $W_1 \in [0,1]$. The conditions on the model parameters for such

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a W_1 to exist are summarized in Table 1. This table also summarizes the conditions for $W_2 \in [0,1]$, which are relevant for the occurrence of the above special situation $W_0 < W_2$.

In the following, we only discuss the physically possible situation where $W(x)$ starts at the upstream boundary condition W_0 and then either increases or decreases to reach the equilibrium point W_1. These two regimes are illustrated in Fig. 2.

The soil moisture profile always shows the same regime as the atmospheric moisture profile. Hereafter, we first present the solutions for some special cases before discussing in detail the behavior of the coupled system in Sect. 3.

2.3.1 Case 1a: no moisture convergence, stationary soil moisture

If $I = 0$ and $\xi_s = 0$, then the solution of Eq. (17) is

$$\frac{x}{L} = (1+\kappa)\left(\frac{1}{W} - \frac{1}{W_0}\right) + \kappa \log\left(\frac{W}{W_0}\right) \tag{24}$$

We have that $\frac{x}{L} > 0$ for $W < W_0$ and $\frac{x}{L} < 0$ for $W > W_0$, which implies $W < W_0$, i.e. the relative atmospheric moisture can only decrease if traveling inland. The equilibrium moisture for $\frac{\mathrm{d}W}{\mathrm{d}x} = 0$ is $W = 0$.

2.3.2 Case 1b: no moisture convergence, non-stationary soil moisture

If $I = 0$ and $\xi_s \neq 0$, we have physical solutions (there is a physical equilibrium point W_1), if and only if $D^* < 0$, which only holds if $\xi_s < 0$.

From Eq. (17) it can be seen that $\mathrm{d}W/\mathrm{d}x > 0$ for $D^* < -W^2/(\kappa(1-W))$. It also holds that $-W^2/(\kappa(1-W)) \le 0$ for all W. Accordingly, if soil moisture depletion is strong ($\xi_s \ll 0$, i.e. $D^* \ll 0$) or W_0 is low, then $W(x)$ increases inland, otherwise $W(x)$ decreases as in the limiting case of $\xi_s = 0$ (see Sect. 2.3.1).

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2.3.3 Case 2: scaled moisture convergence = 1, stationary soil moisture

If $I^* = 1$ and $\xi_s = 0$, Eq. (17) has the special solution $W_1 = 1$, which implies that independent of the other parameter values, the atmospheric moisture can only increase if traveling inland.

2.3.4 Case 3: Slow soil evaporation, rapid discharge

If evaporation from the soil compartment is very slow and discharge very rapid so that κ tends to zero, then $W_{1,2} = \pm\sqrt{I^*}$, $A^* = (W_1 - W_2)^{-1} = (2\sqrt{I^*})^{-1}$, $B^* = -A^*$ and Eq. (18) becomes

$$\frac{x}{L} = -\log\left[\left(\frac{W(x) - \sqrt{I^*}}{W_0 - \sqrt{I^*}}\right)^{\frac{1}{2\sqrt{I^*}}}\left(\frac{W(x) + \sqrt{I^*}}{W_0 + \sqrt{I^*}}\right)^{-\frac{1}{2\sqrt{I^*}}}\right] \tag{25}$$

The above equation has an explicit solution:

$$W(x) = \sqrt{I^*}\frac{1+B}{1-B} \tag{26}$$

with

$$B = \frac{W_0 - \sqrt{I^*}}{W_0 + \sqrt{I^*}}\left(e^{-\frac{x}{L}}\right)^{2\sqrt{I^*}} \tag{27}$$

Since no moisture is returned from the soil, the moisture decay process is only driven by the precipitation of moisture from the atmosphere, convergence and interception. If in addition, $I = 0$, Eq. (24) applies and it simplifies to

$$W = \frac{W_0}{1 + W_0\frac{x}{L}}, \tag{28}$$

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which goes the faster to zero, the smaller interception is (recall $L = u_x \frac{\tau_p}{1-\alpha}$). For the same slope in $W(x=0)$, it goes to zero more slowly than the often assumed exponential decay (e.g. Savenije, 1995a).

2.3.5 Case 4: Rapid evaporation, slow discharge

If evaporation is very rapid and discharge very slow so that κ tends to infinity (almost all precipitation is returned to the atmosphere), then Eq. (17) reduces to

$$\frac{\mathrm{d}W(x)}{\mathrm{d}x} = \frac{1}{L}\left(-\frac{\tau_p \xi_s}{(1-\alpha)c_\mathrm{m}} + I^*\right) \tag{29}$$

and the solution is

$$W(x) = \left(-\frac{\xi_s}{u_x c_\mathrm{m}} + \frac{I}{u_x}\right)x + W_0 \tag{30}$$

In this case, the moisture profile along x depends only on the variation of soil moisture ξ_s and the climatic factors u_x, c_m and I. In a climate where convergence is dominant ($I > 0$), the increasing regime will prevail during the soil moisture depletion (dry) season and a switch to a decreasing regime during the wet season is only possible if convergence is low or soil moisture accumulation is very fast (short wet season). If in addition $I = 0$, the regime only depends on ξ_s and an increasing regime occurs during the dry season, a decreasing regime during the wet season.

The assumptions behind the above solution will break down at large x, because the atmospheric moisture content W cannot exceed unity.

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3 System behavior

3.1 Plausible parameter values

The water holding capacity c_m can be estimated based on the average amount of precipitable water in the atmosphere, which corresponds to around 50 to 80 mm near the equator and around 10 times less at the poles (Randel et al., 1996). The time scale of precipitation is of the order of magnitude of a few days (see, e.g. Trenberth, 1998). The time scale of transpiration can vary considerably depending on vegetation and climate; it is of the order of a few weeks to months. Runoff processes are generally slow and have a time scale much higher than evaporation (months to years). The amount of interception depends on vegetation and the rainfall regime and is of the order of magnitude of 10% up to 50% of the rainfall (de Groen and Savenije, 2006). Some authors do not treat the slow E_T and the fast E_I separately (see also Savenije, 2004), which leads to low total evaporation time scales (e.g. Trenberth, 1998).

s_m is of the order of magnitude of a few hundred mm (Brutsaert, 2005) and can be obtained based on a porosity estimate multiplied with the root zone depth (delimiting the zone from which vegetation can extract water). An order of magnitude of ξ_s can be obtained by dividing s_m by the length of the wet season, respectively of the dry season. The potentially evaporable water in a year, ranges from a few hundred mm up to 2500 mm depending on the climate (e.g. Matsoukas et al., 2011).

Lateral convergence l can be positive or negative (divergence). It corresponds to a relative humidity flux and has an absolute order of magnitude between 0 and 10 month^{-1} (l=5 month^{-1} with c_m=20 mm corresponds to a lateral influx of 100 mm). Possible values of l for physical solutions of the system have to be studied for different settings of the values of κ and D^* (see Table 1).

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Finally, to ensure physical soil moisture values ($S \in [0, s_\mathrm{m}]$) for W_0 and for W_1, it has to hold that (see Eq. 16)

$$\frac{\xi_s}{c_\mathrm{m}} \leq \frac{1-\alpha}{\tau_p} W_j^2 \tag{31}$$

and

$$\frac{\xi_s}{c_\mathrm{m}} \geq \frac{(1-\alpha)\frac{\tau_q}{\tau_p} W_j^2 - \frac{\tau_q}{c_\mathrm{m}}}{1+\kappa(1-W_j)}, \tag{32}$$

where W_j stands for either W_0 or W_1.

The above values and the order of magnitude of the climatic parameters discussed in Sect. 2 are summarized in Table 2. If nothing else is stated, we use the reference parameter values of Fig. 2, which illustrate how the atmospheric and soil moisture contents vary with distance downwind for a given set of parameters. Fig. 3 shows the corresponding fluxes.

3.2 Relationship between W and S

The relationship between the two state variables depends on all hydroclimatic parameters. For plausible parameter values, the soil moisture increases slower than the atmospheric moisture for low values but goes faster to its maximum. Figure 4 shows a dimensionless plot of $S/\max(S)$ against $W/\max(W)$) for different parameter values, for the two cases of $\xi_s = 0$ and $\xi_s = 10$ mm month^{-1}. The figures also show the case of no coupling term $(1-W)$ in Eq. (13); in this case the degree of soil filling for a given relative atmospheric moisture would be overestimated with respect to the case with coupling. This overestimation would be even stronger if precipitation was parameterized as a linear function of W (Fig. 4). If, in addition, $\xi_s = 0$, then the soil storage would behave exactly like the atmospheric storage (Fig. 4, left); in all other cases, the relative filling of the soil is lower than the relative filling of the atmospheric storage.

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The functional relationship between W and S represents a valuable tool to derive first order estimates of the effect of process modifications on both compartments. Since this relationship is nonlinear, a parameter modification will have a rather different effect on the profile of W and of S and on the related fluxes. If the evaporation process becomes faster (an assumed effect of increasing temperature), the atmospheric moisture and, thus, precipitation increases along the entire stream line (see Fig. 5a), which is a commonly assumed and observed effect (see Trenberth, 1998, and references therein). For runoff and storage, the effect depends on the location along x; it decreases close to the coast and increases inland, as visible in the linearly related runoff profile (Fig. 5c). The related evaporation increase shows a maximum at a certain distance from the coast.

3.3 Regime switches

A given hydroclimatologic parameter set Θ corresponds to a particular moisture profile (in the atmosphere and in the soil) that is characterized by the equilibrium moisture W_1 and the length scale L. If the parameters change to a new value Θ' at a given point x' of the flow path, three situations can occur in the increasing regime: i) if $W(x'|\Theta) < W_1(\Theta') < W_1(\Theta)$ the rate of moisture increase slows down, ii) if $W_1(\Theta') > W_1(\Theta))$ the rate of increase accelerates, iii) if $W(x'|\Theta) > W_1(\Theta')$ the moisture starts decreasing in x'. We call this last situation, where the slope of the moisture profile changes sign, a regime switch. For the decreasing regime, a regime switch occurs if $W(x'|\theta) < W_1(\Theta')$.

In mathematical terms, if a parameter Θ_i varies along x, a regime switch occurs in x' if and only if it holds

$$\left[W_1(\Theta') - W(x'|\Theta)\right] \left.\frac{\mathrm{d}W(x|\Theta)}{\mathrm{d}x}\right|_{x=x'} > 0 \tag{33}$$

The susceptibility for a regime change, thus, depends on $W(x)$ and on the sensitivity of W_1 with respect to a parameter change. Since there is no explicit solution $W(x)$ of Eq. (17), a qualitative analysis of this susceptibility has to be completed for individual

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parameter sets. A special case is the situation where $I^* = 1$: it holds that $W_1(I^* = 1) = 1$ and no regime switch is possible.

In nature, a sudden variation of the hydrometerological parameters can occur e.g. due to topography. Mountain ridges can decrease the precipitation time scale, modify lateral convergence or induce very different evaporation time scales. Particularly interesting are potential regime switches due to land use changes. A common question is to anticipate the impact of a modification of the evaporation process on runoff. Considering the feedback system rather than the isolated hydrologic system suggests that the expected response depends on the moisture regime and on the lateral convergence. For example, a decrease of the evaporation time scale could cause a regime switch further downstream if an increasing moisture regime is dominating close to the coast (Fig. 6). This means that an increase of τ_e could either lead to an increase of soil moisture and atmospheric moisture and related fluxes further downstream (as in the example of Fig. 6), it could slow down the increasing regime or lead to a regime switch, depending on the values of all other parameter values and on the location of the land use change (see Fig. 6 where a modification of τ_e in two different locations is illustrated).

3.4 The role of interception

For given climatic parameters I and τ_p, an increase in interception always leads to an increase of the atmospheric equilibrium moisture ($\mathrm{d}W_1/\mathrm{d}\alpha > 0\,\forall\,\Theta$). Accordingly, in the decreasing regime, even a small increase of α can cause a regime switch. A regime switch induced by a change in α is illustrated in Fig. 6.

Furthermore, an increase of α leads to a decrease of soil moisture at any location x ($\mathrm{d}S/\mathrm{d}\alpha > 0\,\forall\,\Theta$). This results in a decreased runoff coefficient c_R defined as

$$c_R = \frac{R}{P}. \tag{34}$$

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Using Eq. (7), Eq. (9) and substituting $\frac{S}{c_\mathrm{m}}$ with Eq. (16) shows the direct relationship between the interception parameter α and c_R:

$$c_R = \frac{1-\alpha}{1+\kappa(1-W)}\left(1-\frac{D^*}{W^2}\right). \tag{35}$$

If we consider the runoff coefficient for an entire year $\overline{c_R} = c_R(\xi_s = 0)$, it can easily be seen that $\frac{\mathrm{d}\overline{c_R}}{\mathrm{d}\alpha} < 0 \, \forall \, \Theta$, i.e. any increase of interception will decrease the runoff coefficient.

Interception also determines the length scale of the feedback system, $L = \tau_p u_x/(1-\alpha)$; for higher α, the equilibrium moisture is reached further inland and the same relative moisture is reached at a shorter distance inland. In an increasing regime, this results in an increase of atmospheric moisture at a given x, in a decreasing regime, this results in decreasing moisture at a given x.

3.5 Horton index

From a hydrological point of view, the system can be characterized by the so-called Horton index (see, e.g. Troch et al., 2009), defined as the ratio between the average amount of water leaving the hydrologic system (i.e. the soil) through evaporation and total water entering the soil compartment, i.e.

$$H_I = \frac{E_\mathrm{T}}{P-E_\mathrm{I}}. \tag{36}$$

Replacing P with Eq. (9), E_T with Eq. (13), E_I with Eq. (10) and substituting S/s_m with Eq. (16) yields

$$H_I = \frac{\kappa(1-W)}{1+\kappa(1-W)}\left(1-\frac{D^*}{W^2}\right). \tag{37}$$

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Given that the Horton index is often defined for complete hydrological years, i.e. $\xi_s = 0$; $\overline{H_l} = H_l(\xi_s = 0)$ simplifies to

$$\overline{H_l} = \frac{\kappa(1-W)}{1+\kappa(1-W)}. \tag{38}$$

This relationship only depends on the parameters of the hydrologic system (s_m, τ_p, τ_e) and the climatic parameter e_m and is independent of the functional relationship between P and W. It summarizes the assumptions about $E_\mathrm{T}(S,W)$ and $R(S)$. For the increasing moisture regime, the Horton index is decreasing inland, for the decreasing moisture regime, the Horton index is increasing.

$\overline{H_l}$ is an increasing function of $(1-W)$ and has the form of the the well-known Langmuir equation (Langmuir, 1916) that expresses the equilibrium between adsorption to a solid surface and the concentration in the surrounding medium, with a constant corresponding to the ratio between rate of adsorption and desorption. This analogy is interesting: the relative outflux from the soil surface (i.e. $\overline{H_l}$) is a function of available storage in the atmosphere $(1-W)$. The shape of this function is given by the ratio κ of maximum evaporation to maximum runoff. $\overline{H_l}$ has the limit $\overline{H_l}(W \rightarrow 0) = \kappa(1+\kappa)^{-1}$, which corresponds to the relationship that we would obtain if there was no feedback term $(1-W)$ in Eq. (13).

In a recent empirical study, Troch et al. (2009) suggested that the Horton index could be some linear decreasing function of the humidity index, the ratio between annual precipitation and potential evaporation. For plausible parameter values, our analytical model reproduces this almost linear relationship (Fig. 7), with slopes very similar to the ones found by Troch et al. (2009). Voepel et al. (2011), on the other hand, found a power-law-like relationship between the Horton index and the aridity index ϕ, the inverse of the humidity index. For our model, ϕ equals:

$$\phi = \frac{E_\mathrm{P}+E_\mathrm{I}}{P} = \frac{s_\mathrm{m}\tau_p}{c_\mathrm{m}\tau_q}\frac{\kappa(1-W)}{W^2} + \alpha. \tag{39}$$

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Expressing $\kappa(1-W)$ as a function of H_l, Eq. (38), and as a function of ϕ, Eq. (39), we find

$$\phi = \alpha + \frac{H_l}{\psi(1-H_l)}, \tag{40}$$

where $\psi = \frac{\tau_q}{s_\mathrm{m}} \frac{c_\mathrm{m} W^2}{\tau_p}$ represents the ratio of precipitation to maximum runoff. This relationship represents well the type of relationship found by Voepel et al. (2011) (see their Fig. 3c; note that they did not consider interception losses in their analysis). As postulated by Voepel et al. (2011), it summarizes how the climate interacts with landscape properties.

3.6 Budyko curve

Closely related to the Horton index, but more well-known, is the Budyko curve (Budyko, 1984; Gerrits et al., 2009), relating the ratio of annual evaporation to annual precipitation to the aridity index. Following the same derivation as for the Horton index, we obtain for $B_u = E/P$

$$B_u = \frac{E}{P} = \frac{\alpha + \kappa(1-W)}{1 + \kappa(1-W)}, \tag{41}$$

with $E = E_\mathrm{T} + E_\mathrm{I}$.

Expressing $\kappa(1-W)$ as a function of B_u and as a function of ϕ, we find

$$B_u = 1 - \frac{1-\alpha}{1+\psi(\phi-\alpha)}. \tag{42}$$

The equation gives a reasonable approximation of the relationships proposed by previous authors (see a collection in Gerrits et al., 2009) and namely of the simplest model, B_u =1-exp($-\phi$), proposed by Schreiber (1904). It has the main advantage of explicitly highlighting the role of interception. Note, however, that the model only holds for $\phi > \alpha$ and that certain parameter values lead to non-physical solutions.

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3.7 Recycling ratio

A fundamental property of the hydroclimatic feedback system is the recycling of water originally evaporated over the ocean through multiple cycles of evaporation and precipitation over the continent along the streamline (e.g. Dirmeyer et al., 2009; Van der Ent et al., 2010; Worden et al., 2007).

There are different methods to characterize this recycling (see Van der Ent et al., 2010, for a discussion); we retain here the scale-independent formulation of precipitation recycling $\rho(x)$ as a function of the distance x traveled along a streamline, proposed by van der Ent and Savenije (2011) based on the work of Dominguez et al. (2006):

$$\rho(x) = 1 - \exp\left(-\int_{x_0}^{x} \frac{E(x')}{C(x')u_x} dx'\right) = 1 - \exp\left(-\frac{x}{\lambda(x)}\right), \tag{43}$$

where E is the total evaporation and $C = c_{\mathrm{m}}W$ the atmospheric moisture storage. $\lambda(x)$ is the length scale of precipitation recycling, which characterizes the process. It holds that $\frac{\mathrm{d}\rho}{\mathrm{d}x} > 0$ and $\rho(x \to \infty) = 1$.

In the above formulation, $\rho(x)$ is the recycling ratio defined in x, whereas $\lambda(x)$ is an integrated value over $x_0 \to x$. Accordingly, there is no analytical expression for $\lambda(x)$, and $\rho(x)$ can only be approximated numerically. Using the discretization $x_i = x_{i-1} + \delta_x$, we re-write

$$\rho(x_i, \lambda_i) = 1 - \exp\left(-\frac{x_i}{\lambda(x_i)}\right) = 1 - \exp\left(-\frac{x_{i-1} + \delta_x}{\lambda(x_{i-1} + \delta_x)}\right). \tag{44}$$

Given that λ varies gradually along x, we assume that $\lambda(x_{i-1} + \delta_x) \simeq \lambda(x_{i-1}) = \lambda_{i-1}$. The above can then be decomposed as follows:

$$\begin{aligned}\rho(x_i, \lambda_i) &\simeq 1 - \exp\left(-\frac{x_{i-1} + \delta_x}{\lambda_{i-1}}\right) = 1 - \exp\left(-\frac{x_{i-1}}{\lambda_{i-1}}\right) \exp\left(-\frac{\delta_x}{\lambda_{i-1}}\right) \\ &= 1 - \exp\left(-\frac{x_{i-1}}{\lambda_{i-1}}\right) \left[1 - \rho(\delta_x, \lambda_{i-1})\right]\end{aligned}$$

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$$= 1 - \exp\left(-\frac{x_{i-1}}{\lambda_{i-1}}\right) + \exp\left(-\frac{x_{i-1}}{\lambda_{i-1}}\right)\rho(\delta_x, \lambda_{i-1})$$
$$= \rho(x_{i-1}) + \left[1 - \rho(x_{i-1})\right]\rho(\delta_x, \lambda_{i-1}). \tag{45}$$

The last term of the above expression can be estimated following van der Ent and Savenije (2011), who showed that, choosing a sufficiently fine discretization, the recycling length scale $\lambda_{\delta x} = \lambda(\delta_x)$ can be approximated as

$$\lambda(\delta_x) = u_x \frac{\overline{C}(x_{i-1}, x_i)}{\overline{E}(x_{i-1}, x_i)} = u_x c_\mathrm{m} \frac{\overline{W}(x_{i-1}, x_i)}{\overline{E}(x_{i-1}, x_i)}, \tag{46}$$

where $\overline{E}(x_{i-1}, x_i)$ is the average total evaporation in the interval $[x_{i-1}, x_i]$.

Using Eq. (46) and assuming $\rho(\delta_x, \lambda_{i-1}) \simeq \rho(\delta_x, \lambda_{\delta x})$, we can iteratively compute $\rho(x_i, \lambda_i)$ with Eq. (45), starting in $x = 0$. An example is illustrated in Fig. 8 for the default parameter values and the increasing and the decreasing regime. Since the stream line starts at the coast, $\rho(x_i, \lambda_i)$ gives an estimate of continental precipitation recycling (Van der Ent et al., 2010).

Considering an entire year ($\xi_s = 0$) and assuming $\overline{W}(x_{i-1}, x_i) \simeq W(x_i)$, we can further analyze the behavior of $\lambda(\delta_x)$ as a function of the model parameters:

$$\lambda(\delta_x) \simeq u_x c_\mathrm{m} \frac{W(x_i)}{E_\mathrm{T}(x_i) + E_\mathrm{I}(x_i)} = u_x \tau_p \frac{1}{W} \frac{1 + \kappa(1 - W)}{\alpha + \kappa(1 - W)} = u_x \tau_p \frac{1}{W} \frac{1}{B_u}. \tag{47}$$

As expected, the wind speed as well as the precipitation time scale directly influence the recycling length scale, modulated by a factor depending on W, α and κ, just as the length scale of the moisture regime, $L = \frac{u_x \tau_p}{1-\alpha}$, is modulated by a factor depending only on interception. L is longer than λ for high values of W and of α. Equation (47) also shows that if the Budyko value B_u increases (i.e. for increasing aridity), the recycling length scale decreases.

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4 Discussion

The presented analytical, fully coupled model of the soil-precipitation feedback system allows direct insights into the nonlinear relationship between soil and atmospheric moisture and the emerging effects on precipitation, evaporation and runoff and it can distinguish between interception (fast feedback of moisture) and delayed feedback through the soil by way of transpiration and soil evaporation. These are two major advantages over existing analytical approaches that only consider the atmospheric moisture explicitly and make simplifying assumptions about fluxes that depend on the soil moisture (Savenije, 1995a, 1996).

We see three types of applications of the analytical framework. First of all, the resulting nonlinear relationship between soil and atmospheric moisture can explain why there is no simple answer to questions of the type "what happens if rainfall increases?". We presented only a generic example but we anticipate that a detailed analysis for seasonally dominant moisture stream lines on different continents could give valuable indications on how different the effect of climate or land use changes can be in different moisture recycling hotspots (Koster et al., 2004; Van der Ent et al., 2010). Furthermore, the analytic framework reveals how the different parameter values could influence the seasonal moisture regimes and what types of parameter modifications could create regime switches. Such a regime switch at a given location would cause a major modification of the hydrologic cycle further downstream, possibly resulting from some minor local change of process time scales e.g. due to vegetation change.

In summary, this suggests that studies that analyze and try to anticipate climate or land use changes (Seneviratne et al., 2006) could profit from a preliminary analysis of the relationship between W and S along the dominant stream line for dry and wet seasons, focusing on: (1) the moisture regime (decreasing or increasing inland), (2) how close the actual processes are to a potential regime switch, and (3) which system characteristics could cause it. As discussed for interception, such a preliminary analysis could e.g. show that even a parameter with a priori minor importance could

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be decisive for a regime switch. A next step would be to analyze the dynamics of the system, to show, e.g. how long it takes for a step change in moisture at the coast to propagate to some distance inland, but this is left for future research.

Finally, the analytic framework could also be useful to quantify hydrologic similarity. Such an analysis aims at understanding how the basic hydrologic functions "partitioning", "storage" and "release" of water (see Wagener et al., 2007) are related to physiographic characteristics and climate, especially for the prediction of future hydrologic behavior. An example of how to make use, hereby, of purely analytical tools is the work of Woods (2009), who presented an analytic seasonal snow cover model to understand the interplay of the temperature regime, meteorological seasonality and precipitation rates.

For the present analytical model, the Horton index shows nicely what we can gain from analytical modeling of soil and atmospheric moisture for understanding hydrologic similarity: potential relationships between how the hydrologic system partitions water between runoff and evaporation and climate are not "blurred" by some exogenous forcing of which we do not know how representative they are for the behavior of the system. The precipitation recycling ratio has been derived for the same purpose of understanding how different time scales "conspire" to increase or decrease moisture recycling along a stream line.

5 Conclusions

We presented a soil-atmosphere feedback model and derived its analytical solutions in an Eulerian-Lagrangean framework, yielding functional relationships between moisture profiles along a dominant stream line starting at the coast and hydroclimatic parameters. The key features of the model are the nonlinear coupling between the atmospheric and the soil moisture stores resulting from the functional dependance of evaporation on both moisture storages and the separate treatment of soil evaporation (mostly through transpiration) and evaporation from intercepted water. The model considers only water

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fluxes; energy constraints are incorporated in the form of parameters in the potential evaporation formulation.

This analytical model, although it might be qualified as a “toy model” given the overwhelming complexity of underlying natural processes, allows first order analyses of the nonlinear relationship between the states of soil moisture and of atmospheric moisture as a function of process parameters characterizing a given hydroclimatic behavior, in particular the time scales of evaporative fluxes, precipitation and runoff, but independent of observed meteorological time series. For hydrology, this represents a perspective change: precipitation and potential evaporation are no longer exogenous forcing variables. Hydrologic behavior and its sensitivity to changes can be analyzed in terms of local moisture exchanges as well as upstream climate or moisture regimes.

While we presented only generic examples here, we hope that the analytical framework will be of use in future work to explore the range of potential impacts of climate and land use change on different continents. We also look forward to an expansion of the framework to e.g. include feedback between state variables and the time scale of dominant processes (e.g. soil wetness on precipitation), to explicitly account for topographic or temperature effects (e.g. through a variation of the atmospheric water holding capacity) or to include surface runoff.

Supplementary material related to this article is available online at: http://www.hydrol-earth-syst-sci-discuss.net/8/8315/2011/hessd-8-8315-2011-supplement.pdf.

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Table 1. Conditions on the parameters κ, D^*, I^* for the existence of $W_1 \in [0,1]$ or $W_2 \in [0,1]$; for simplification, we use $\chi = \kappa(D^* - I^*)$. If situations 1 and 5 occur jointly with $W_0 < W_2$ then the solution is not physically realistic for all $x > 0$, see Sect. 2.3.

Situation number	Condition 1	Condition 2	Conclusion
1	$\chi < 0$	$\chi < I^* < 1$	$W_1 \in [0,1]$
2	$0 < \chi < 2$	$\frac{1}{4}\chi(4-\chi) < I^* < 1$	$W_1 \in [0,1]$
3	$\chi > 2$	–	$W_1 \notin [0,1]$
4	$\chi < 0$	–	$W_2 \notin [0,1]$
5	$0 < \chi < 2$	$\frac{1}{4}\chi(4-\chi) < I^* < \chi$	$W_2 \in [0,1]$
6	$\chi > 2$	$1 < I^* < \chi$	$W_2 \in [0,1]$

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Table 2. Order of magnitude of parameter values used for numerical applications and default values used if nothing else is stated. The value for e_m is obtained based on estimates of the maximum annual potential evaporation E_pm as $e_\mathrm{m} = \tau_e E_\mathrm{pm}$.

Parameter	Unit	Min. val.	Max. val.	Def. val.
W_0	–	0	1	0.8
τ_p	days	5	20	10
τ_e	months	0.5	4	1
τ_q	months	4	24	12
α	–	0	0.5	0.2
s_m	mm	0	1000	300
ξ_s	mm months^{-1}	−300	300	10
E_pm	mm year^{-1}	100	2400	1200
e_m	mm	$f(\tau_e, E_\mathrm{pm})$	$f(\tau_e, E_\mathrm{pm})$	100
I	[month^{-1}]	−10	10	1.2
u_x	m/s	0.5	10	5
c_m	mm	10	80	20

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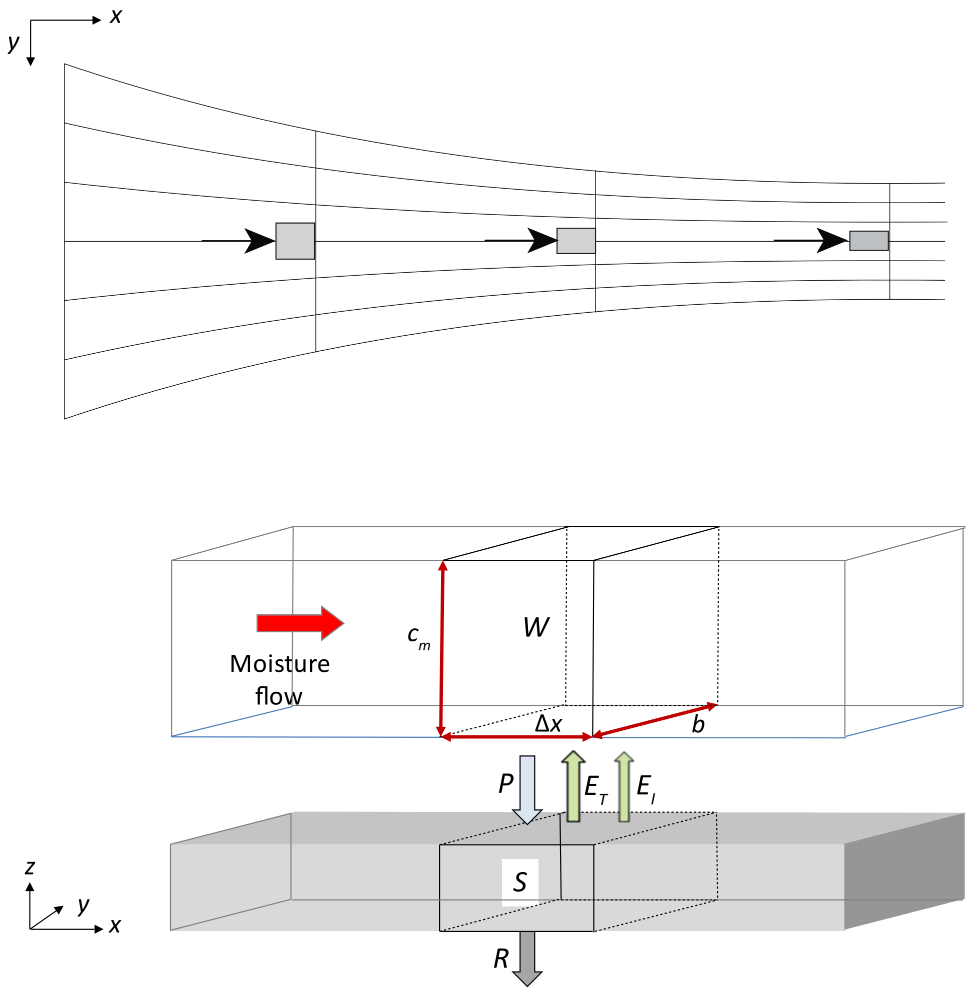

Fig. 1. Sketch of the model (top view and side view).

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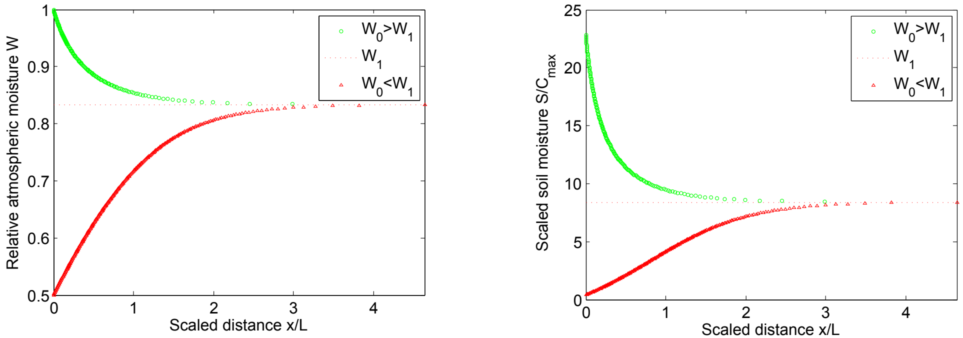

Fig. 2. Moisture profiles obtained with default parameter values (Table 2, $L = 5400$ km) for $W_0 = 0.5$ (increasing regime) and $W_0 = 1.0$ (decreasing regime); the left plot shows atmospheric moisture, the right plot soil moisture.

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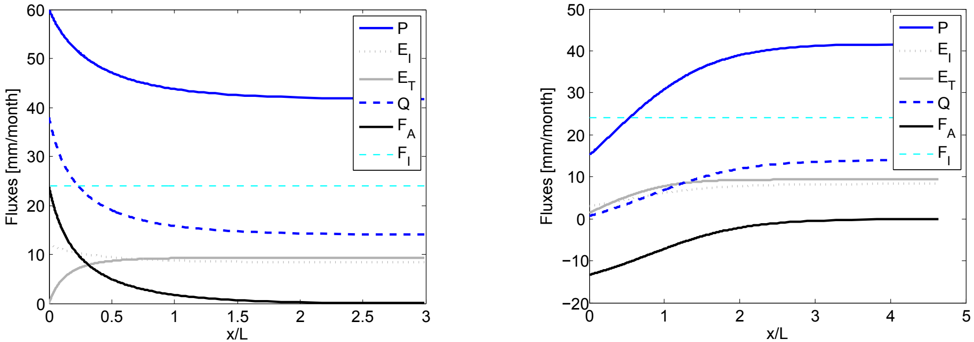

Fig. 3. Left: Point scale fluxes along x corresponding to the two regimes of Fig. 2; left; $W_0 = 1.0$, right: $W_0 = 0.5$; F_A stands for the advective flux, F_l for the lateral influx.

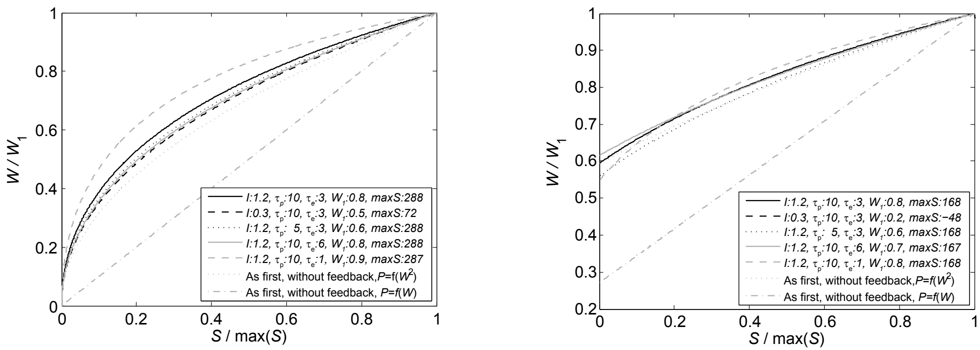

Fig. 4. Relative atmospheric moisture plotted against relative soil moisture filling, left: for $\xi_s = 0$, right: for $\xi_s = 10$ mm month^{-1} (note the y-axis scale); τ_p is given in days and τ_e in months. The value max(S) corresponding to these parameter values is given for information. The last two cases correspond to a model without the coupling term (1-W) in Eq. (13) and with P given as a linear function of W. In the case of $\xi_s \neq 0$, one of the parameter sets is physically not possible (negative max(S)).

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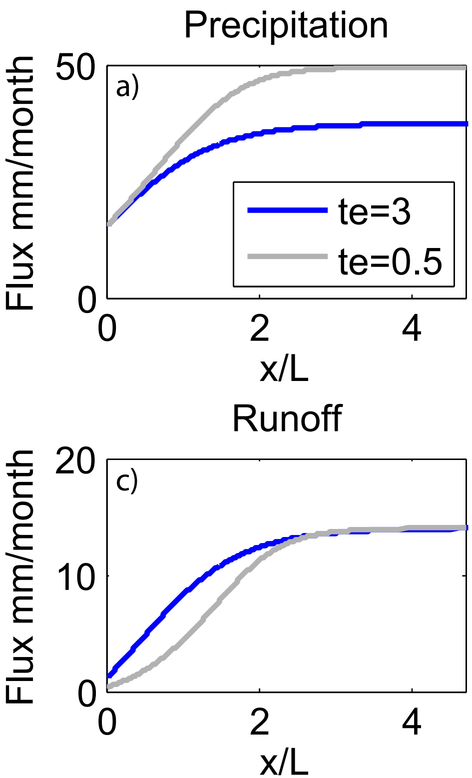

Fig. 5. Effect of faster evaporation on fluxes (default parameter values with $W_0 = 0.5$ and $s_m = 200$ mm)

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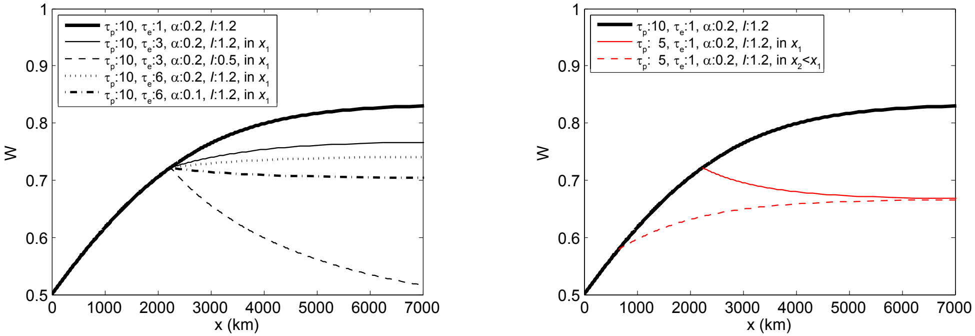

Fig. 6. Regime switches due to parameter changes (parameter units as in Table 2), left: for changes of τ_e, α, l in x_1, right for a change of τ_p in two different locations x_1 and x_2; note the role of α in the switch induced with the parameter set τ_p=10 days, τ_e=6 months, $l = 1.2\,\mathrm{mm}^{-1}$ (left figure).

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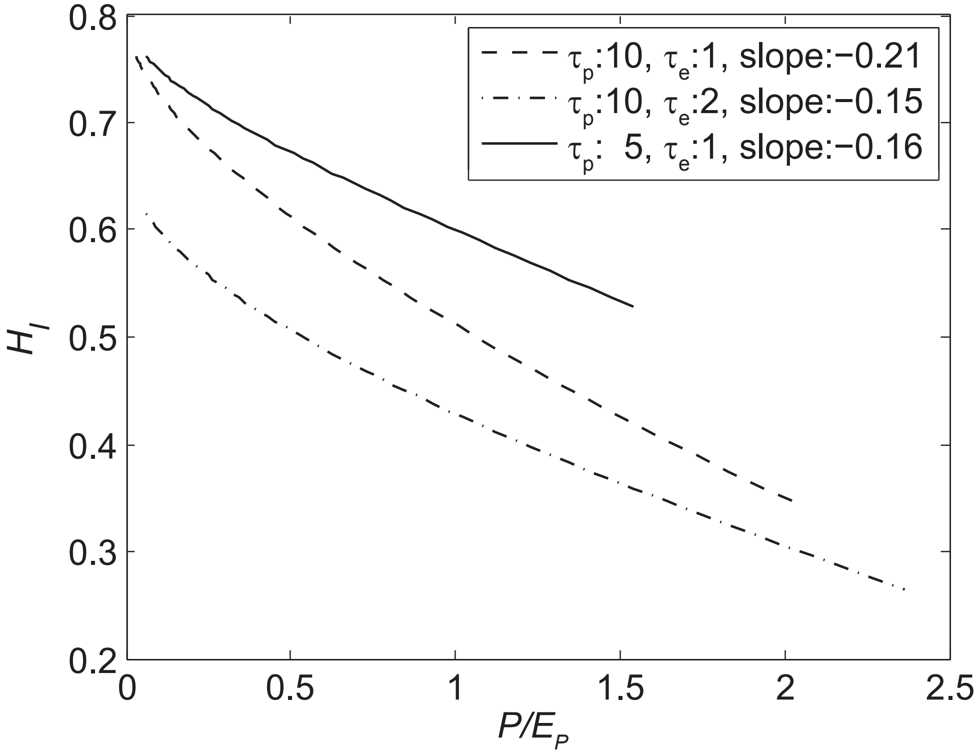

Fig. 7. Horton index $\overline{H_I}$ as a function of humidity index P/E_P computed for $W \in [0.2,1]$; for each parameter set, only part of the humidity index domain is covered by the possible model outcomes.

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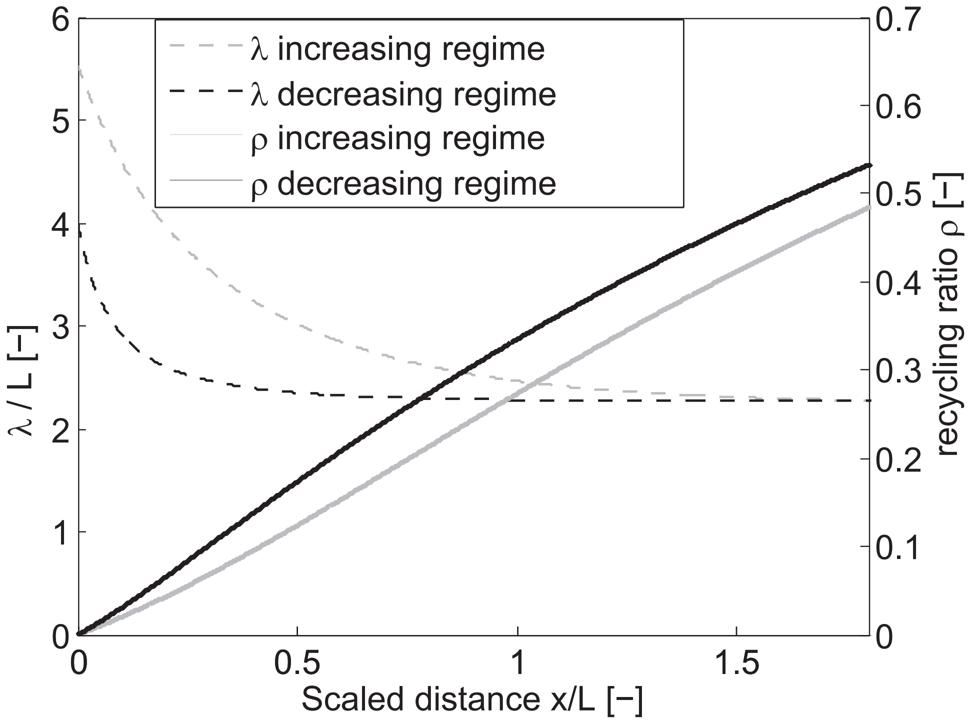

Fig. 8. Continental recycling ratio and recycling length scale (relative to moisture regime length scale L) along x for the two regimes of Fig. 2 ($W_0 = 0.5$ resp. $W_0 = 1.0$, $W_1 = 0.83$, $L = 5400$ km) with $\delta_x = 5$ km.