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Effects of seasonality on the distribution of hydrological extremes

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Abstract

This paper focuses on the seasonality of hydroclimatic extremes and on the problem of accounting for their non-homogeneous character in determining the design value. To this aim we devise a simple stochastic world in which extremes are produced by a non-homogeneous extreme value generation process. The design values are estimated in closed analytical form both in a peak over threshold framework and by using the standard annual maxima approach. In this completely controlled world of generated hydrological extremes, a statistical measure of the error associated to the adoption of a homogeneous model is introduced. The sensitivity of this measure, assumed in terms of return period ratio, to the typology and strength of seasonality is investigated. We find that seasonality induces a downward bias in design value estimators. The magnitude of the bias may be large when the peak over threshold approach is adopted, while the return period distortion is limited when the annual maxima are considered. An application to a prealpine to alpine transition region located in North-Western Italy is presented to better clarify the effects of disregarding seasonality in a real case.

1 Introduction

Hydrological variables, like precipitation depth and river discharge, often exhibit marked periodic variability on annual time scales (Black and Werritty, 1996; Sivapalan et al., 2005; Rust et al., 2009; Viglione et al., 2010; Villarini et al., 2011). As a consequence, also the corresponding extreme values, i.e. the precipitation or discharge values associated to relevant and unfrequent events, will likely be dependent on the non-homogeneity of their respective date of occurrence. The seasonal variability of extreme events represents an issue for the standard applications of the frequency analysis, because in the presence of seasonality the sample values available for design value estimation could belong to different populations. The manner how these date-dependent sample values should be treated represents an open problem in statistical hydrology, tackled in different ways.

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The standard procedures for design value estimation are based on the analysis of series of annual maxima (AM). Estimation of the distribution of annual maxima from data with seasonal variability has received attention in the literature. Attempts to obtain the annual maximum distribution by fitting different distributions to the maxima in separate seasons – and to derive the expressions for the bias and variance of the resulting annual quantile estimators – were done by Lamberti and Pilati (1985) and Buishand and Demaré (1990). The basic idea is to have, in each season, random variables that satisfy the "identically distributed" hypothesis, which is essential in standard statistical inference. The use of monthly maxima has also been advocated (e.g., Carter and Challenor, 1981; Ettrick et al., 1987; Rust et al., 2009) because the hypothesis of identically distributed random variables in classical extreme-value theory is better satisfied for monthly than for annual maxima. The problem with these approaches is that the number of parameters to be estimated increases, which in turn inflates the estimation uncertainty of design values. Moreover, the application of these methods requires non-standard data (maxima in separate seasons) which are seldom available for long time periods.

An alternative is offered by the peak over threshold (POT) or partial duration series approach. This approach considers several over-threshold values instead of a unique extreme value per year (Madsen et al., 1997; Lang et al., 1999; Claps and Laio, 2003). The strongest motivation for adopting the POT approach is that monthly or seasonal maxima constitute precious additional information about the upper tail of the distribution, which is very important in hydrological applications (Revfeim, 1991; Katz et al., 2002). The POT approach naturally lends itself to being used with a rate of occurrence of the exceedances (λ) and an exceedances distribution having an annual cycle (Todorovic and Zelenhasic, 1970). However, standard applications of the POT approach do not consider time-dependent parameters (e.g., Madsen et al., 1997; Claps and Laio, 2003). The aim of this paper is to provide indications about the effects of neglecting seasonality within standard POT ar AM approaches.

2 A "toy-model" for understanding seasonality effects on the distributions of extremes

A simplified analytical model of the distribution of precipitation extremes is devised, in which precipitation is assumed to follow a non-homogeneous Poisson process of storm arrivals in time with rate λ, the depth x of each storm being distributed according to an exponential function

$$F(x|t) = 1 - \exp[-x/\alpha(t)] \tag{1}$$

where the mean $\alpha(t)$ of the distribution is dependent on time. Despite its simplicity, the model is widely adopted in hydrology, as demonstrated by the numerous applications that can be found in the literature (e.g., Eagleson, 1978; Rodriguez Iturbe et al., 1987; Rasmussen and Rosbjerg, 1989; Laio et al., 2001). To mimic the effects of seasonality on the precipitation regime, a sinusoidal (time-dependent) variation is imposed on the parameters of the Poisson process. We write $\alpha(t)$ and $\lambda(t)$ as

$$\begin{aligned} \alpha(t) &= \alpha_0\left(1 + a_\alpha \sin\left(\tfrac{2\pi}{365} nt\right)\right) = \alpha_0 \cdot h(t), \\ \lambda(t) &= \lambda_0\left(1 + a_\lambda \sin\left(\tfrac{2\pi}{365} nt + \delta\right)\right) = \lambda_0 \cdot k(t), \end{aligned} \tag{2}$$

where α_0 and λ_0 are the average rainfall intensity and rainfall rate values; t is time in days ($t \in [0;365]$); n is the number of peaks per year; $h(t)$ is the non-dimensional α regime and $k(t)$ the non-dimensional λ regime; a_α and a_λ are, respectively, the amplitudes of the sinusoids $h(t)$ and $k(t)$; and δ is the temporal shift between the two. Note that, to constrain $h(t)$ and $k(t)$ above zero, a_α and a_λ can vary in the range (0,1). Observe also that the initial time $t = 0$ is taken at any point where $\alpha(t) = \alpha_0$ and $d\alpha(t)/dt > 0$. An example for the $h(t)$ and $k(t)$ curves is reported in Fig. 1.

The distribution in Eq. (1) is the conditioned distribution $F(x|t)$ of the rainfall depths x on the date of occurrence t. The marginal distribution of precipitation exceedances $F(x)$ is then obtained by applying the Bayes theorem. To solve the integral analytically, some further manipulation of Eq. (1) is needed. To this end we approximate $F(x|t)$ in Eq. (1) by

$$F'(x|t) = 1 - \exp[-x \cdot \alpha'(t)] \quad (3)$$

with $\alpha'(t) = \alpha'_0 \cdot h'(t) = \alpha'_0(1 - a'_\alpha \sin(nt))$. The minus sign before the sine assures that the maxima and minima of α'_0 and a'_α are in phase. The values of the parameters α'_0 and a'_α should be found which minimize the distance between $F'(x|t)$ and $F(x|t)$. A possible strategy could be to calculate the total squared distance

$$\mathrm{TSD}(\alpha'_0, a'_\alpha) = \int_0^{365} [F(x|t) - F'(x|t)]^2 \mathrm{d}t = \int_0^{365} e^{-2x/\alpha(t)} \left[1 - e^{-x(\alpha'(t) - 1/\alpha(t))}\right]^2 \mathrm{d}t \quad (4)$$

and to find the values of α'_0 and a'_α which minimize $\mathrm{TSD}(\alpha'_0, a'_\alpha)$. The solution to this equation does not provide an analytical result, but one can observe that the integral on the right hand side of Eq. (4) can be seen as a weighted average of the distance between the curves, where the weighting factor is $w(t) = e^{-2x/\alpha(t)}$. It is clear that this weighting factor is maximum where $\alpha(t)$ is largest, i.e. in $t' = 365(\pi/2n + 2\pi/n)/2\pi$ and minimum in $t'' = 365(3\pi/2n + 2\pi/n)/2\pi$. The ratio between the minimum and maximum weighting factors reads

$$r = \frac{e^{-\frac{2x}{\alpha_0(1-a_\alpha)}}}{e^{-\frac{2x}{\alpha_0(1+a_\alpha)}}} = e^{-\frac{4xa_\alpha}{\alpha_0(1-a_\alpha^2)}} \quad (5)$$

and rapidly converges to zero for large values of x, which are those of interest in this paper. The differences between $F'(x|t)$ and $F(x|t)$ are therefore negligible when $\alpha(t)$ is small, and it is sensible to determine α'_0 and a'_0 when $\alpha(t)$ attains its maximum, i.e. in t'. By setting t' for t in $F(x|t)$ and $F'(x|t)$ and equating the two expressions one easily obtains $\alpha'_0 = 1/\alpha_0$ and $a'_\alpha = a_\alpha/(1 + a_\alpha)$.

Using $F'(x|t)$ instead of $F(x|t)$ guarantees the analytical tractability of the Bayes integral, and leads to an expression of the distribution of exceedances in closed analytical

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form:

$$F(x) = \int_0^{365} F'(x|t)k(t)\mathrm{d}t = 1 - e^{-\frac{x}{\alpha_0}} \left\{ \Phi\left[0, \frac{a_\alpha}{\alpha_0(1+a_\alpha)}x\right] + a_\lambda \Phi\left[1, \frac{a_\alpha}{\alpha_0(1+a_\alpha)}x\right] \cos(\delta) \right\} \quad (6)$$

where $\Phi[\cdot]$ is the modified Bessel function of the first kind (Abramowitz and Stegun, 1965, ch.9) and $k(t)$ is as defined in Eq. (2). An important property of $F(x)$ is that its expression does not change by changing the oscillation frequency (parameter n in Eq. (2)). Observe also that the mean of $F(x)$ can be expressed analytically as

$$\mu_x = \frac{\alpha_0 \left(a_\alpha - (1+a_\alpha) \left(\sqrt{\frac{1+2a_\alpha}{(1+a_\alpha)^2}} - 1 \right) a_\lambda \cos(\delta) \right)}{a_\alpha \sqrt{\frac{1+2a_\alpha}{(1+a_\alpha)^2}}}. \quad (7)$$

Moreover, the distribution in Eq. (6) tends to an exponential distribution (also called "base POT distribution") in the homogeneous case, i.e. when $a_\alpha = a_\lambda = 0$.

The distribution of exceedances Eq. (6) can then be transposed into the correspondent annual maximum distribution. In fact, despite the non-homogeneity of the process, the standard relation $F_{\mathrm{AM}}(x) = e^{-\lambda_0(1-F(x))}$ (relating the distribution of exceedances to the distribution of annual maxima, $F_{\mathrm{AM}}(x)$) still holds (e.g., Coles, 2001, p. 131). On this premise, the following expression for the distribution of annual maxima is obtained

$$F_{\mathrm{AM}}(x) = \exp\left[-\exp\left[-\frac{x}{\alpha_0} \right] \lambda_0 \left(\Phi\left[0, \frac{a_\alpha x}{\alpha_0(1+a_\alpha)}\right] + a_\lambda \Phi\left[1, \frac{a_\alpha x}{\alpha_0(1+a_\alpha)}\right] \cos(\delta) \right) \right] \quad (8)$$

in which α_0 plays the role of the scale parameter. Observe that the distribution of maxima becomes a Gumbel distribution (also called "base AM distribution") when $a_\alpha = a_\lambda = 0$ in Eq. (2). Observe also that for Eq. (8) the mean and standard deviation of the distribution are not analytical.

3 Model application

Ignoring the possible effects of seasonality when adapting a distribution to a set of data is a rather common simplification, especially when few observations are available. In the following we explore the consequences of such a simplification, as they could affect the return period estimation both for the distribution of exceedances and the distribution of maxima.

To quantify the error associated to the selection of the wrong (non-seasonal) distribution, we consider the T-year event with the base (non-seasonal) distribution and compute the corresponding return period, T^*, with the correct seasonal distribution. The return period ratio

$$R_T = \frac{T}{T^*} \tag{9}$$

provides an indication of the magnitude of the underestimation (overestimation) in design values related to neglecting seasonality. In particular, if $R_T > 1$ the adoption of the base distribution corresponds to an underestimation of the real design value.

3.1 Peak over threshold

Consider precipitation exceedances to follow the distribution in Eq. (6), having mean μ_x as defined by Eq. (7). As anticipated, in the homogeneous case $a_\alpha = a_\lambda = 0$, and the distribution of exceedances becomes an exponential distribution, called base distribution. The design event $x_T^{(\mathrm{POT})}$ with the base distribution is determined as

$$x_T^{(\mathrm{POT})} = -\mu_x \log\left(\frac{1}{\lambda_0 T}\right). \tag{10}$$

Using the seasonal distribution Eq. (6) one finds that the same event $x_T^{(\mathrm{POT})}$ has a return period T^*, obtained by setting Eq. (10) for x in Eq. (6), and considering that $\lambda_0 T^* = 1/(1-F(x_T^{(\mathrm{POT})}))$ (e.g., Claps and Laio, 2003).

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Given the simplicity of the model the return period ratio is obtained in analytical form, and reads

$$R_T^{(\mathrm{POT})} = \lambda_0 T \left(\frac{1}{\lambda_0 T}\right)^{\frac{z(1+a_\alpha)}{a_\alpha}} \left(\Phi\left[0, z\log\left(\frac{1}{\lambda_0 T}\right)\right] - a_\lambda \Phi\left[1, z\log\left(\frac{1}{\lambda_0 T}\right)\right]\cos(\delta)\right) \qquad (11)$$

with $z = [a_\alpha - (1+a_\alpha)(\sqrt{(1+2a_\alpha)/(1+a_\alpha)^2} - 1)a_\lambda \cos(\delta)]/\sqrt{1+2a_\alpha}$. Note that Eq. (11) is independent of the mean rainfall intensity α_0 because α_0 is a scale parameter for both the base and the seasonal distribution.

The sensitivity of the return period ratio to the parameters of Eq. (6) is explored in Fig. 2 (gray-shaded contour areas). The results are obtained by varying two parameters at a time and by fixing the other parameters. In particular, in Fig. 2, λ_0 and a_α are assumed to vary. In each point of the plane the intensity of the grey-shade is proportional to the value of the return period ratio, with large values of the return period ratio $R_T^{(\mathrm{POT})}$ corresponding to precipitation events $R_T^{(\mathrm{POT})}$ times more frequent in the "seasonal reality" than if the exponential distribution was adopted. We find a strong sensitivity of the return period ratio to large values of a_α and λ_0 (with values that can be as large as 7). The sensitivity of $R_T^{(\mathrm{POT})}$ to the variation of the two other parameters T and δ is shown in Fig. 3: it emerges that $R_T^{(\mathrm{POT})}$ significantly increases both with the return period T and the phase shift δ. From Fig. 3 it can be deduced that $R_T^{(\mathrm{POT})}$ assumes slightly lower values when the regimes are in phase, as in the case illustrated in Sect. 4.

One could argue that the difference between the two distributions would be recognized if the data were closely scrutinized. This is verified by applying a goodness of fit test for exponentiality to the samples generated by the seasonal model. In details, 1000 samples of length N', generated from the distribution Eq. (6) under different parameterizations, are tested with the Anderson-Darling test (see Laio, 2004) against the hypothesis of exponential distribution. In Fig. 2, black labelled contour lines show the percentage of cases recognized as non-exponential by the Anderson-Darling test

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(at a 5% level) for the case of $N = 20$ yr (which entails a sample length of $N' = \lambda_0 N$) and for the same parameter set that was used before. It is observed that, for small a_α values, the percentages in the graph are very close to the significance level of the test, meaning that the distribution in this region is substantially undistinguishable from an exponential distribution. The percentage of cases recognized as non-exponential increases up to 50% for larger values of a_α(i.e. for very peaky seasonal regimes) and λ_0.

By comparing the two sets of curves (gray-shaded areas and countours) we observe that the pattern of the $R_T^{(\mathrm{POT})}$ values and of the test performances (as functions of λ_0 and a_α) are rather similar: low efficiencies of the Anderson-Darling test emerge in correspondence of low $R_T^{(\mathrm{POT})}$ values, whereas better performances are found where $R_T^{(\mathrm{POT})}$ is high. However, it is clear that the power of the test remains rather low even when the $R_T^{(\mathrm{POT})}$ values are large, with a 70% probability of not being able to recognize a 5-fold increase in return period, with possible serious detrimental consequences in design-value estimation.

3.2 Annual maxima

Suppose that a sample of annual maxima follows the distribution function in Eq. (8), which is described by the five parameters α_0, λ_0, a_α, a_λ and δ. The Gumbel distribution (which is characterized by two parameters, called $\alpha_0^{(\mathrm{AM})}$ and $\lambda_0^{(\mathrm{AM})}$ in the following) is the form that Eq. (8) would assume for $a_\alpha = a_\lambda = 0$, i.e. in the absence of seasonality. An analysis of the error that would derive from the adoption of a Gumbel distribution in the presence of seasonal maxima is provided in the following.

As in the previous case, the two parameters $\alpha_0^{(\mathrm{AM})}$ and $\lambda_0^{(\mathrm{AM})}$ of a Gumbel distribution are estimated with the method of moments

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$$\alpha_0^{(\mathrm{AM})} = \frac{\sqrt{6}\sigma_{\mathrm{AM}}}{\pi},$$
$$\lambda_0^{(\mathrm{AM})} = \exp\left[\frac{\mu_{\mathrm{AM}}}{\alpha_0} - \gamma_E\right], \tag{12}$$

where μ_{AM} is the mean and σ_{AM} the standard deviation of the samples taken from the distribution in (8) and γ_{E} is the Euler constant. We then define $x_T^{(\mathrm{AM})} = \alpha_0 \log[-1/\lambda_0 \log[1-1/T]]$ as the design T-year event with the Gumbel distribution. The same event would be characterized by a return period T^* with the seasonal distribution (8). The return period ratio $R_T^{(\mathrm{AM})}$ is defined by Eq. (9) as the ratio of the reference return period T to the return period T^* of the $x_T^{(\mathrm{AM})}$ event. In this case the relation between the return period ratio and the distribution parameters cannot be expressed in closed analytical form. The ratio is computed several times by varying the parameters of Eq. (8). The values assumed by $R_T^{(\mathrm{AM})}$, after considering the variation of λ_0 and a_α and by keeping the other parameters constant, are shown in Fig. 4 (shaded areas). It can be observed that $R_T^{(\mathrm{AM})}$ assumes values up to 1.45, for high values of a_α and low λ_0, whereas $R_T^{(\mathrm{AM})}$ is unaffected by the variation of the scale parameter α_0 (not shown). This implies that the assumption of a Gumbel model leads to less relevant overestimation of the return period of the design event than in the POT case.

To check if these errors would be detectable we apply the same testing procedure as for the POT case. The application demonstrates that the test is unable, for sample dimensions $N \leq 50$ yr, to discern between a sample generated from (8) and a Gumbel-distributed sample. In Fig. 4 the percentage of cases recognized as non-Gumbel by the Anderson-Darling test (at a 5% level) is shown by the black contour lines, for variable values of a_α and λ_0. It can be observed that the percentages in the graph are very close to the test significance level, i.e. the two distributions are statistically undistinguishable.

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4 Case study

In this Section 294 rainfall stations located in the North-West of Italy are examined, with the aim of demonstrating the importance of precipitation seasonality in the area of study. Daily totals of precipitation depths are available between year 1913 and 1986. The peak over threshold series are extracted by imposing a threshold value of 20 mm on the precipitation depths. The threshold corresponds to about $\lambda = 20$ events per year over the whole data set, which seems a reasonable approximation for the average number of significant rain events in a year. The site-specific regime curves are obtained by aggregating the exceedance amounts and the occurrence rates at the monthly timescale and by subsequently averaging the results for each station. A representation of the at-site regimes $h(t)$ and $k(t)$ (respectively, the amount and occurrence regimes divided by their mean, see Eq. (2)) is given in Fig. 5 for all the stations in the region (grey lines).

A marked seasonal behavior in both the amount and occurrence rate of the regimes emerges: in particular most of the curves are characterized by a double peak. The two curves also appear to have approximately the same oscillation period and to be in phase. The time shift ($\delta = 1 \div 12$ months) between the two curves is determined by varying the value of δ and evaluating, for each station, a distance measure (D) between the site-specific regimes. D is defined as the average of the absolute differences between the 12 monthly values of $h(t)$ and $k(t)$. The minimum D is found in correspondence of $\delta = 0$ for 291 (out of 294) stations. The global regimes (black curves in Fig. 5) of the mean precipitation intensity and rainfall rate, derived by averaging the station-specific curves, are then assumed to be in phase for the whole region of study.

The parameters α_0 and λ_0 are estimated in each station as the average (over-threshold) rainfall intensity and rainfall rate. a_α and a_λ are estimated as functions of the coefficients of variation (CV) of $\alpha(t)$ and $\lambda(t)$. The reasoning to find the relation between CV_λ and a_λ is described hereinafter. Similarly one can find the relation between CV_α and a_α.

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Under the hypothesis of poissonianity, the cumulative distribution of λ, $F_\Lambda(\lambda)$, can be obtained as a derived distribution of the times of occurrence, provided that the relation $\lambda(t)$ is monotonic. To meet this condition, we restrict the domain of $\lambda(t)$ to the interval $[\pi/2n;3\pi/2n]$ (which should be multiplied by $365/2\pi$ to be expressed in days). The distribution of λ is unaffected by this assumption, due to the periodicity of the $\lambda(t)$ function. On this domain, the cumulative distribution of times $F_T(t)$ is uniform, Equation (2) relates λ to t, and the cumulative distribution function of λ reads

$$F_\Lambda(\lambda) = \frac{1}{\pi}\arcsin\left(\frac{\lambda_0-\lambda}{a_\lambda\lambda_0}\right)-\frac{1}{2}, \tag{13}$$

which is valid in the domain $[\lambda_0(1-a_\lambda);\lambda_0(1+a_\lambda)]$. The mean and standard deviation are respectively $\mu_\lambda=\lambda_0$ and $\sigma_\lambda=a_\lambda\lambda_0/\sqrt{2}$ and the relation between CV_λ and a_λ is found accordingly as $a_\lambda=\sqrt{2}\cdot\mathrm{CV}_\lambda$. Analogously, one obtains $a_\alpha=\sqrt{2}\cdot\mathrm{CV}_\alpha$.

$R_T^{(\mathrm{POT})}$ values are then obtained from Eq. (11) for each station, as shown in Fig. 6. Rather large values are found, pointing out the risk to significantly underestimate the design values if seasonality is neglected. When the annual maxima are considered, $R_T^{(\mathrm{AM})}$ varies between 1 and 1.45 (not shown), demonstrating that in this case the errors induced by neglecting seasonality are not extremely relevant.

5 Discussion and conclusions

The effects of disregarding seasonality of extremes when evaluating the return period of a design event are investigated. Two very simple examples are illustrated: (1) the case of using an exponential distribution to describe POT values derived from a non-homogeneous Poisson process (Sect. 3.1); and (2) the case of using a Gumbel distribution for AM values extracted from a seasonal (i.e., with time dependent parameters) distribution (Sect. 3.2). The entity of the return period overestimation (i.e., apparently less frequent extreme events) when seasonality is not accounted for is quantified by a coefficient R_T. It is found that, when seasonality is ignored and threshold exceedances

(POT) are analyzed, the return period of an event can be overestimated up to 7 times. Neglecting seasonality therefore corresponds to significantly underestimating the design values; moreover, standard goodness-of-fit techniques are not very efficient at recognizing the unsuitability of the non-seasonal distribution.

Resorting to peak over threshold analysis is, nevertheless, an advantageous technique, especially in the presence of few years of observation, as acknowledged by numerous works (see e.g. Wilks, 1993; Lang et al., 1999). This allows, in fact, to capture more accurately the upper tail of the distribution, increasing at the same time the sample dimension. In this sense, the difficulties pointed out by this study in analyzing seasonal POT values should not discourage the adoption of an over-threshold modelling, but rather reassert the need for accurate preliminary analyses of the data based on visual inspection of the regimes or, possibly, on unsupervised testing techniques (e.g., "analysis of variance" or ANOVA techniques, see e.g., Salas et al., 1980; Kottegoda and Rosso, 1998).

Less relevant errors are associated to the use of the annual maximum distribution, even if seasonality still induces a systematic downward bias in design value estimators. In particular, the Gumbel distribution is found to represent an acceptable choice to describe the annual maxima generated by a Poisson exponential process with seasonally varying parameters. The reason behind this better performance probably lies in the greater flexibility of the Gumbel distribution (compared to the exponential).

However, while the Gumbel distribution generally performs well, it is of interest to note that, in the presence of seasonality, the estimated $\alpha_0^{(AM)}$ and $\lambda_0^{(AM)}$ values (see Eq. (12)) lose (some of) their physical significance. In fact, $\alpha_0^{(AM)}$ and $\lambda_0^{(AM)}$ do not correspond to the average rainfall intensity and average rainfall rate, α_0 and λ_0. In Fig. 7A the relative variations of the $\lambda_0^{(AM)}$ to λ_0 ratio are shown for varying a_α and a_λ. It is observed that the $\lambda_0^{(AM)}$ to λ_0 ratio takes values between 1 (in the homogeneous case) and 0.5, meaning that the estimated $\lambda_0^{(AM)}$ could be one half of the real λ_0. In panel B the behavior of the non-dimensional product $\lambda_0^{(AM)}\alpha_0^{(AM)}/\lambda_0\alpha_0$ is shown in the same

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parameter range. One can observe that the product values are fan-shaped around the unit value, with greater differences corresponding to higher a_α values. This entails that in the parameters estimation process the total amount of "over-threshold precipitation" $\lambda_0^{(AM)} \cdot \alpha_0^{(AM)}$ tends to remain constant, with lower values assumed by λ being compensated by an increase in the estimated α. This systematic underestimation of λ and overestimation of α should be accounted for when the parameters $\lambda_0^{(AM)}$ and $\alpha_0^{(AM)}$ are used to infer the statistical properties of the extreme-value process in the area.

In conclusion, the message we want to convey with this work is about the need of accounting for seasonality when dealing with hydro-climatic extreme values. We show that disregarding their non-homogeneity may have detrimental consequences in design value estimation. Moreover, the errors may be difficult to detect by means of standard goodness of fit testing techniques. Future work will focus on the formulation of ad-hoc metrics to identify and quantify the seasonality in hydrological extremes.

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Effects of seasonality on the distributions of hydrological extremes

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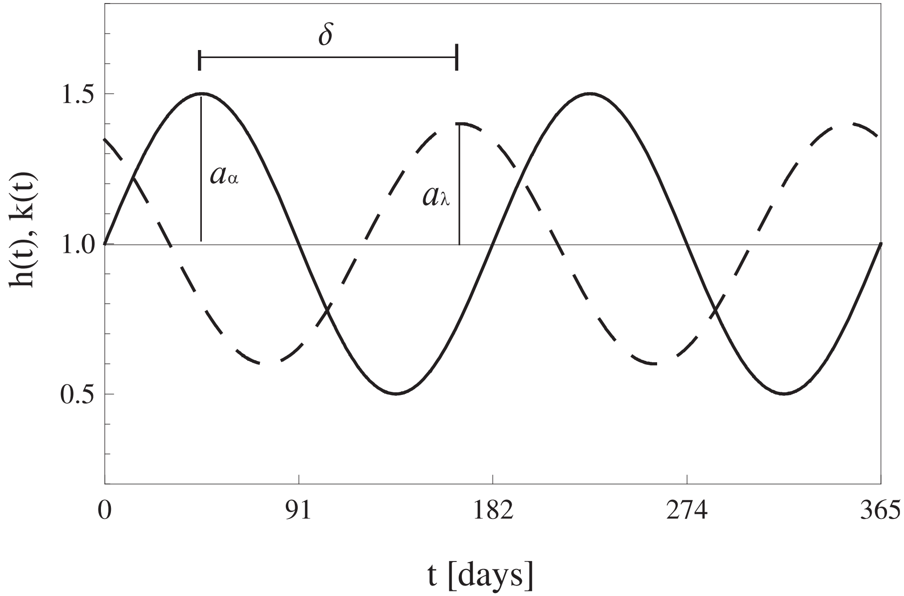

Fig. 1. Example of the $h(t)$ (solid) and $k(t)$ (dashed) curves, see Eq. (2). The temporal shift δ between the seasonal peaks is indicated, as well as their amplitudes, a_α and a_λ. In this case $n = 2$.

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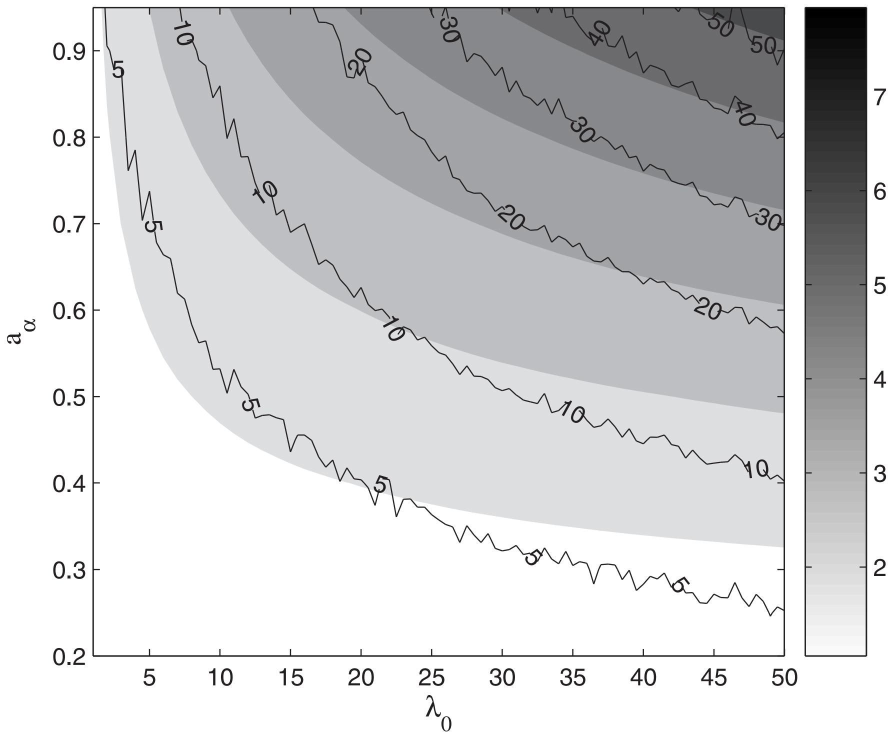

Fig. 2. Sensitivity of the return period $R_T^{(POT)}$ to variations of λ_0 and a_α, with $T = 100$ yr, $\delta = \pi$, $n = 1$ and $a_\lambda = 0.5$. In each point of the plane the intensity of the grey-shade is proportional to the value of the return period ratio R_T^{POT}. The percentage of cases recognized as non-exponential by the Anderson-Darling test (at a 5% level) is indicated by the black (labelled) contour lines. $N = 20$ yr is considered.

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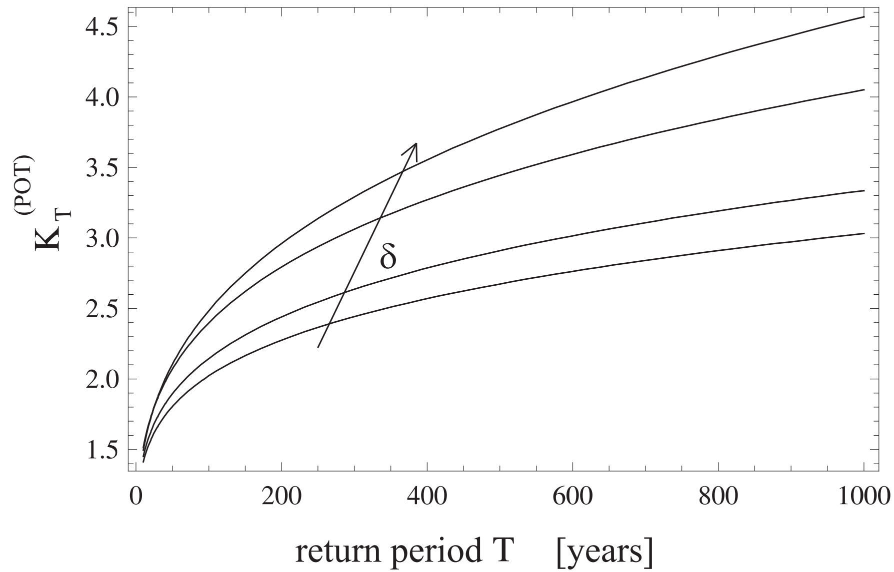

Fig. 3. Relation between the return period ratio $R_T^{(POT)}$ and T for increasing values of δ $(0, 365/8, 365/4, 365/2)$, with $a_\alpha = a_\lambda = 0.5$, $n = 1$ and $\lambda_0 = 20$.

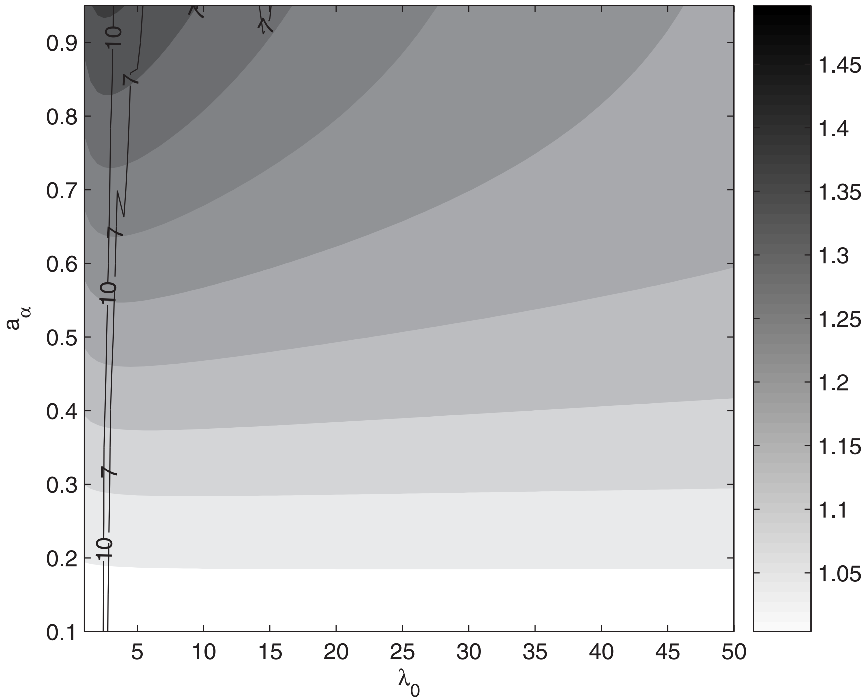

Fig. 4. Sensitivity of the return period ratio $R_T^{(AM)}$ to variations of λ_0 and a_α, with $T = 100$ yr, $n = 1$, $\delta = \pi$ and $a_\lambda = 0.5$. In each point of the plane the intensity of the grey-shade is proportional to the value of the return period ratio. The percentage of cases recognized as non-Gumbel by the Anderson-Darling test (at a 5% level) is indicated by the black (labelled) contour lines. $N = 50$ yr is considered.

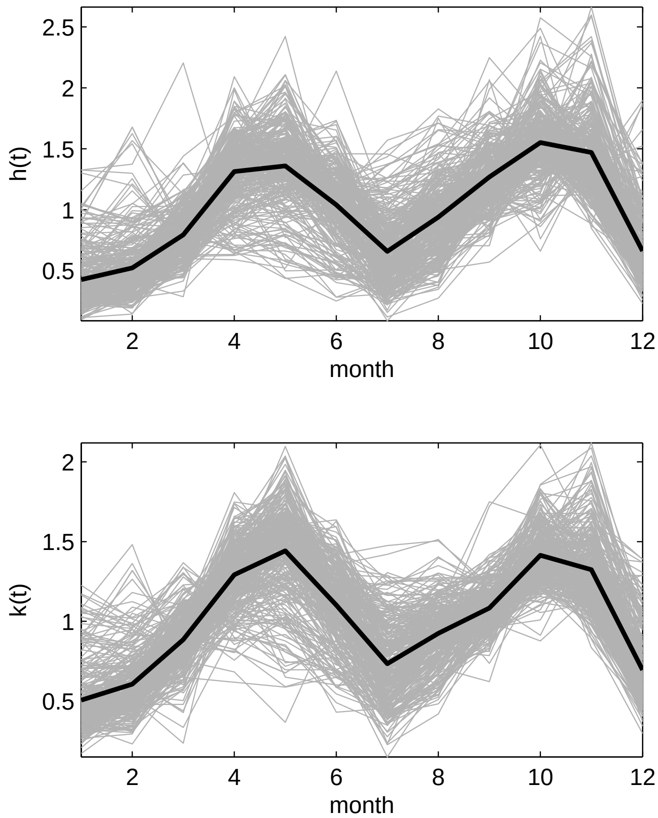

Fig. 5. Non-dimensional monthly average rainfall intensity $h(t)$ and rainfall rate $k(t)$ for 294 gauging stations in North-Western Italy (grey lines). The overall spatial mean values are reported in black.

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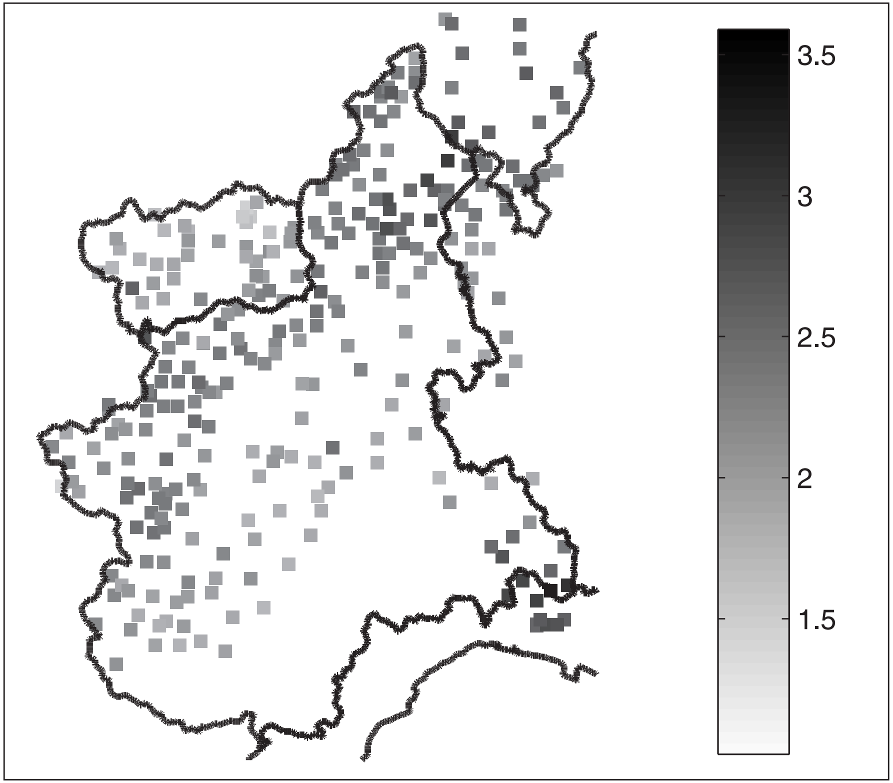

Fig. 6. Return period ratios ($R_T^{(POT)}$) evaluated for the 294 rain gauging stations in the North-West of Italy. The color scale refers to the resulting range of variation of $R_T^{(POT)}$.

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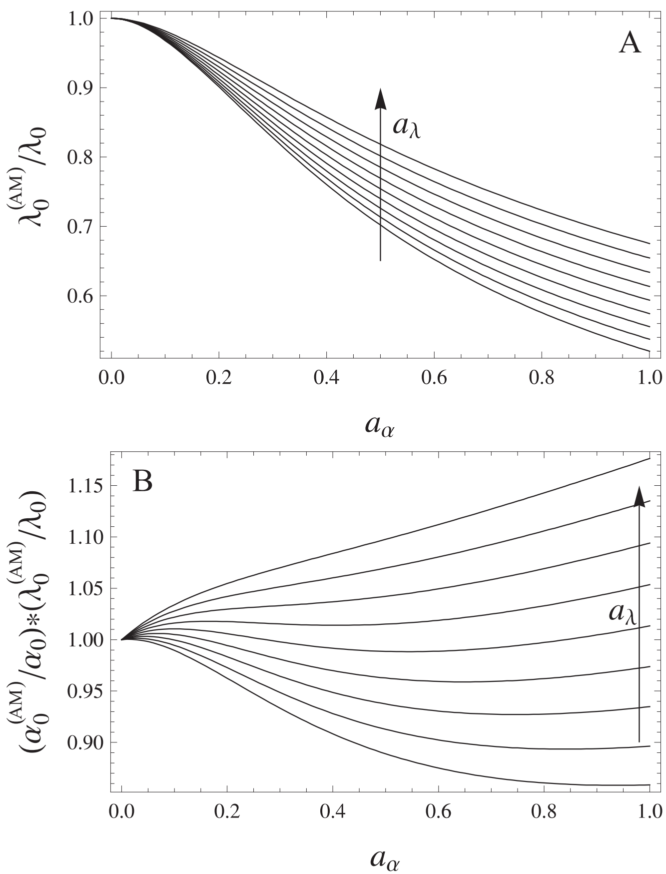

Fig. 7. Panel A: sensitivity of the $\lambda_0^{(AM)}$ to λ_0 ratio to varying a_α and a_λ values (with $T = 100$ yr, $\delta = 0$, $n = 1$ and $\lambda_0 = 20$ events/year). Panel B: Sensitivity of the non-dimensional product $\lambda_0^{(AM)}\alpha_0^{(AM)}/\lambda_0\alpha_0$ to the same parameter variations.

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