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Hydrology and Earth System Sciences Discussions

Hydrologic system complexity and nonlinear dynamic concepts for a catchment classification framework

B. Sivakumar[1,2] **and V. P. Singh**[3]

[1]School of Civil and Environmental Engineering, The University of New South Wales, Sydney, Australia
[2]Department of Land, Air and Water Resources, University of California, Davis, USA
[3]Department of Biological and Agricultural Engineering & Department of Civil & Environmental Engineering, Texas A & M University, College Station, USA

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Correspondence to: B. Sivakumar (s.bellie@unsw.edu.au)

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Abstract

The absence of a generic modeling framework in hydrology has long been recognized. With our current practice of developing more and more complex models for specific individual situations, there is an increasing emphasis and urgency on this issue. There have been some attempts to provide guidelines for a catchment classification framework, but research in this area is still in a state of infancy. To move forward on this classification framework, identification of an appropriate basis and development of a suitable methodology for its representation are vital. The present study argues that hydrologic system complexity is an appropriate basis for this classification framework and nonlinear dynamic concepts constitute a suitable methodology. Discussing the utility of hydrologic data in describing the complexity of the underlying system, the study also offers a three-step procedure for a classification framework: (1) detection of possible patterns and determination of complexity levels of hydrologic systems; (2) classification of hydrologic systems into groups and sub-groups based on patterns and complexity; and (3) verification of the classification framework through establishing relationships between the data patterns/complexity and the catchment/process properties. The framework is expected to lead to a much more effective and efficient procedure for identifying the appropriate structure and complexity of models for hydrologic systems and, thus, save significant time, data collection, and computational requirements.

1 Introduction

As in most other fields of science and engineering, growth in the field of hydrology during the past century has been unprecedented, largely driven by the invention of powerful computers, measurement devices, remote sensors, geographic information systems (GIS), digital elevation models (DEM), and networking facilities. This growth may be viewed in terms of: (1) the various sub-fields that have been created essentially to "break down" hydrology into specific components for more focused and detailed

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studies (e.g. surface hydrology, subsurface hydrology, groundwater hydrology, forest hydrology, mountain hydrology, urban hydrology, isotope hydrology, snow and glacier hydrology, ecohydrology); and (2) the numerous scientific theories and mathematical techniques that have been developed/applied for modeling and prediction of hydrologic systems and the associated processes (e.g. deterministic techniques, stochastic methods, scaling and fractal theories, artificial neural networks, chaos theory, wavelets, genetic algorithms).

Despite this growth, there remain many grand challenges in performing good hydrologic teaching, research, and practice. Among others, two major concerns are dominating discussions and debates on current hydrologic studies: (1) hydrologic models being developed are often more complex, having too many parameters and requiring too much data, than perhaps needed; and (2) models are often developed for specific situations, and their extensions and generalizations to other situations are rather difficult. In addition, our general lack of emphasis in studying the crucial connections between the (model) theories and the actual system properties (e.g. data), our increasing emphasis in applying specific (and often pre-selected) mathematical techniques independently as opposed to the integration of techniques for modeling hydrologic systems, and our focus mainly on local-scale hydrologic problems rather than global-scale hydrologic issues have also come under severe scrutiny (e.g. Beven, 2002; Kirchner, 2006; Sivakumar, 2008a, b). With general consensus on the occurrence of global climate change and its potential impacts on water resources and the environment (including more frequent and greater magnitudes of extreme events, such as floods and droughts), the limitations of the "confines of traditional hydrology" and the need to go beyond and perform cross-disciplinary research integrating hydrology with atmospheric sciences, geomorphology, geochemistry, ecology, and other areas have also been increasingly recognized (see, for example, Paola et al. (2006) and Wagener et al. (2010) for some details).

In view of these concerns, many studies during the past decade or so have emphasized the need for simplification in modeling wherever possible as well as a common

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framework in hydrology (e.g. Grayson and Blöschl, 2000; McDonnell and Woods, 2004; Sivapalan, 2005; Sivakumar, 2008a). Within this context, some attempts have also been made towards a catchment classification framework (e.g. Sivakumar et al., 2007; Wagener et al., 2007), with an aim to streamline catchments into different groups and sub-groups on the basis of their salient characteristics (e.g. data and process complexity) and to provide directions to model developers on the level of model complexity to invoke. Nevertheless, these attempts are only preliminary and research in this direction is still in a state of infancy. Indeed, there are even questions on the basic form of the classification framework and on the components to be included (e.g. Wagener et al., 2007). Therefore, identification of an appropriate basis for the classification framework and development of a suitable methodology are crucial for moving forward in hydrology.

The present study attempts to offer some workable guidelines for an appropriate basis and a suitable methodology towards a classification framework in hydrology. The study, in essence, argues that system complexity is an appropriate basis for the classification framework and nonlinear dynamic concepts constitute a suitable methodology for assessing system complexity. The relevance of complexity and nonlinearity in hydrologic systems is highlighted, and the utility of nonlinear dynamic tools for pattern recognition and complexity determination is demonstrated. Discussing the utility of "data" in assessing "system" complexity, an effective procedure to use system complexity and nonlinear dynamic concepts for formulation of a catchment classification framework is also proposed.

The rest of this paper is organized as follows. Section 2 presents a brief account of major attempts on classification in hydrology. Section 3 highlights the property of complexity and its role in hydrologic systems. Section 4 discusses the nonlinear and chaos properties of complex systems and their relevance in hydrology; two basic methods for their identification are also described. Section 5 illustrates the utility of these methods for hydrologic time series. Section 6 proposes a procedure for the formulation of a catchment classification framework. Conclusions and directions for further research are presented in Sect. 7.

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2 Classification in hydrology: a brief history and scope

The realization of the need for a classification framework in hydrology is not entirely new. It had indeed been discussed some time ago, and since then several studies have also attempted to advance the idea (e.g. Haines et al., 1988; Chapman, 1989; Nathan and McMahon, 1990; Burn and Boorman, 1992; McMahon and Finlayson, 1992; Olden and Poff, 2003; Merz and Blöschl, 2004; Snelder et al., 2004, 2005; Isik and Singh, 2008). These studies have investigated different ways for developing such a framework, including regionalization, river regimes, landscape and land use parameters, similarity indices, and ecohydrologic factors factors, among others. Although useful in their own ways, these studies and their different forms do not adequately account for some inherent properties of hydrologic systems and processes (e.g. nonlinearity and chaos) and, thus, are largely insufficient for a generic classification framework. At the least, a coherent effort to bring these disparate forms together for a workable classification framework is missing. The urgency to formulate a generic classification framework in hydrology is increasingly realized now, especially with our current practice of employing more and more sophisticated mathematical techniques and developing more and more complex models for each and every individual hydrologic system/situation, rather than the emphasis needed on addressing broader-scale hydrologic issues (e.g. Sivakumar, 2008a).

The fundamental idea behind a classification framework in hydrology is to streamline hydrologic systems into groups and sub-groups to recognize salient characteristics that are emblematic and to develop suitable methods/models. This classification and subsequent modeling approach also serve as a middle-ground to the two extreme approaches: (1) treatment of all hydrologic systems in the same way, regardless of the differences among them; and (2) treatment of each and every individual hydrologic system in its own way, regardless of the similarities among them. Either of these approaches has enormous implications for modeling, including complexity of the models, data and computer requirement, accuracy of results, and overall understanding of the systems. The classification framework, therefore, is aimed at providing an optimum way of studying hydrologic systems, taking into account both minimization of costs and maximization of benefits. In the end, it should help modelers identify suitable catchments to apply their models to and also users to identify suitable models for their catchments.

For its usefulness to be realized both at the global and at the regional/local levels, the classification framework should be able to accommodate important general as well as specific characteristics of hydrologic systems/processes. The framework must also be simple enough and commonly agreeable to provide a "universal" language for communication and discussion in hydrology and water resources. The crucial questions now are: (1) What form should the classification framework assume? (2) What components need to be included? and (3) What is the appropriate methodology for its formulation?

Wagener et al. (2007) made a preliminary attempt to address these questions and relevant issues. After reviewing the existing approaches to define hydrologic similarity, which has often been invoked for classification purposes, they offered some general guidelines for catchment classification that include the use of catchment structure, hydroclimatic region, and catchment functional response, among others. They also identified the following requirements for a classification framework: (1) mapping catchment form/hydroclimatic conditions on catchment function across spatial and temporal scales; (2) including partition, storage, and release of water in catchment functions; (3) consideration of uncertainty in the metrics/variables used; and (4) basing on functions characterized by streamflow to start with and subsequently expanding to other more complex functions.

Using the Shannon entropy, Krasovskaia (1995, 1997) developed a quantitative methodology for river flow regime classification. The entropy-based methodology involves: (1) classification of mean monthly flows into different types; (2) identification of discriminating periods for different classes; (3) specification of instability index; (4) computation of instability index value for each regime type; and (5) computation of instability index for all flow series. Another method for grouping river regimes, developed

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by Krasovskaia (1997), employs minimization of an entropy-based objective function. This function uses a concept of information loss resulting from flow aggregation and determining the difference between the series aggregated into one group.

Sivakumar et al. (2007) explored the utility of a simple nonlinear data reconstruction approach, called phase space reconstruction, for assessing the complexity of hydrologic systems and, thus, for their classification. They used the "region of attraction of trajectories" in the phase space to identify data as exhibiting "simple" or "intermediate" or "complex" behavior and, correpondingly, classify the system as potentially low-, medium-, or high-dimensional. The utility of this reconstruction concept was first demonstrated on two artificial time series possessing significantly different characteristics and levels of complexity (purely random and low-dimensional deterministic), and then tested on a host of river-related data representing different geographic regions, climatic conditions, basin sizes, processes, and scales. The ability of the phase space to reflect the river basin characteristics and the associated mechanisms, such as basin size, smoothing, and scaling, was also observed. The "dimensionality" and "complexity" ideas used by Sivakumar et al. (2007) were along the lines of the dominant processes concept (DPC), which was originally introduced in the context of hydrologic model simplification (Grayson and Blöschl, 2000) and subsequently suggested as a potential means for formulation of a classification framework (e.g. Woods, 2002; Sivapalan et al., 2003; Sivakumar, 2004a).

Following up on the ideas by Sivakumar et al. (2007), we attempt here to offer a generic framework for catchment classification. In doing so, we resort to an inverse approach, wherein the complexity of a given hydrologic system is assessed by studying the outputs (i.e. data) from that system. The extent of "complexity" of the system is considered to be reflected by the "variability" of observed data, which, in turn, is assessed by its "dimensionality." Nonlinear dynamic tools are used for studying data dimensionality and system complexity.

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3 Complexity and hydrologic systems

3.1 Complexity: what do we mean by it?

Although words “complex” and “complexity” are widely used both in scientific theory and in common practice, there is no general consensus on the definition. The difficulty in arriving at a consensus definition comes from the fact that it is oftentimes subjective; what is “complex” for one person may not be complex at all for another person, or even when viewed by the same person from a different perspective or at a different time. Nevertheless, one workable definition may be this: “consisting of interconnected or interwoven parts.” With this definition, however, it is also important to clarify why the nature of a complex system is inherently related to its parts, since simple systems are also formed out of parts. Therefore, to explain the difference between “simple” and “complex” systems, the terms “interconnected” or “interwoven” are somehow essential.

Qualitatively, to understand the behavior of a complex system, we must understand not only the behavior of the parts but also how they act together to form the behavior of the whole. This is because: (1) we cannot describe the whole without describing each part; and (2) each part must be described also in relation to other parts. As a result, complex systems are difficult to understand. This is relevant to another definition of “complex”: “not easy to understand or analyze.” These qualitative ideas about what a complex system is can be made more quantitative. Articulating them in a clear way is both essential and fruitful in pointing out the way toward progress in understanding the universal properties of these systems.

For a quantitative description, the central issue again is defining quantitatively what “complexity” means. In the context of classification of systems, such as the one addressed in this study, it may perhaps be even more useful to ask: (1) What do we mean when we say that one system is more complex than another? and (2) Is there a way to identify the complexity of one system and to compare it with the complexity of another system? To develop a quantitative understanding of complexity, a variety of tools can be used. These may include: statistical (e.g. coefficient of variation), nonlinear dynamic

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(e.g. dimension), information theoretic (e.g. entropy) or some other measure. In this study, we discuss the nonlinear dynamic tools, which allow identification of complexity of different systems and interpretations and distinctions on "more complex" and "less complex" systems. In particular, we attempt to relate the complexity of the system (i.e. variability of the data) to the number of variables dominantly governing the system; in other words, the amount of information necessary to describe the system, so to speak (see Sect. 4 for more details).

During the past few decades, numerous attempts have been made to define, qualify, and quantify "complexity" and to apply complexity-based theories for studying natural and physical systems, including their classification. Extensive details can be found in Ferdinand (1974), Cornaccchio (1977), Nicolis and Prigogine (1989), Waldrop (1992), Cilliers (1998), Buchanan (2000), Barabási (2002), McMillan (2004), Johnson (2007), and Érdi (2008), among others.

3.2 Complexity in hydrologic systems

Hydrologic phenomena arise as a result of interactions between climate inputs and landscape characteristics that occur over a wide range of space and time scales. Due to the tremendous heterogeneities in climatic inputs and landscape properties, such phenomena may be highly variable and complex at all scales (although simplicity is also possible). Consequently, they are not fully understood. In the absence of perfect knowledge, a simplified way to represent them may be through the concept of "system." There are many different definitions of a system, but perhaps the simplest may be: "a system is a set of connected parts that form a whole." Chow (1964) defined a system as an aggregate or assemblage of parts, being either objects or concepts, united by some form of regular interaction or inter-dependence. Dooge (1967a), however, defined a system as: "any structure, device, scheme, or procedure, real or abstract, that inter-relates in a given time reference, an input, cause, or stimulus, of matter, energy, or information and an output, effect, or response of information, energy, or matter." This definition by Dooge is much more comprehensive and instructive and it brings out,

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among others, the following important characteristics of the system: (1) a system can consist of more than one component; (2) these components are separate, and they may be inter-dependent; (3) these components are put together following some sort of a scheme, i.e., a system is an ordered arrangement; (4) a system inter-relates input and output, cause and effect, or stimulus and response; (5) a system does not require that input and output be alike or have the same nature; and (6) a system can be composed of a number of sub-systems, each of which can have a distinct input-output linkage.

With this system concept, the entire hydrologic cycle may be regarded as a hydrologic system, whose components might include precipitation, interception, evaporation, transpiration, infiltration, detention storage or retention storage, surface runoff, interflow, and ground water flow, and perhaps other phases of the hydrologic cycle. Each component may be treated as a sub-system of the overall cycle, if it satisfies the characteristics of a system set out in its definition. Thus, the various components of the hydrologic system can be regarded as hydrologic sub-systems. To analyze the total system, the simpler sub-systems can be treated separately and the results combined according to the interactions between the sub-systems (especially with the assumption of linearity). Whether a particular component is to be treated as a system or sub-system depends on the "objective of the inquiry" (Singh, 1988).

In this "objective of the inquiry" context, Sivakumar (2008a) suggests that hydrologic systems may be viewed from three different, but related, angles: process, scale, and purpose of interest. Depending upon the angle at which they are viewed, hydrologic systems may be either simple or complex; for example, the rainfall occurrence in a desert may be treated as an extremely simple process since there may be no rainfall at all, while the runoff process in a large river basin may be highly complex due to the basin complexities and heterogeneities, in addition to rainfall variability. Consequently, hydrologic modeling must also be viewed from these three angles; in other words, the appropriate model to represent a given hydrologic system may also be either simple or complex. The obvious question, however, is: how simple or how complex the models should be? This issue is addressed in this study, since the basic purpose behind

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formulation of a catchment classification framework is the identification of the most appropriate model (type and complexity) for a given catchment. Therefore, the present study is, in a way, an inverse approach to study the classification framework.

Since complexity is a fundamental and central characteristic of hydrologic systems, and is also a representation of their generality and specificity, it should form the basis for a classification framework. The studies by Sivakumar (2004a) and Sivakumar et al. (2007) offer clues as to the use of complexity (defined in terms of extent of data variability) as a viable means for a classification framework.

4 Nonlinear dynamic concepts and relevance to hydrology

4.1 Nonlinearity in hydrologic systems

Much of the research in hydrologic systems, at least until recently, has been based on the assumption of "linearity;" i.e., the relation between cause (e.g. input) and effect (e.g. output) is linear or proportional. One of the important factors that contributed to, or necessitated, this linear approach was the lack of computational power to develop the (perhaps more complex) nonlinear mathematical methods. However, the "nonlinear" behavior of hydrologic systems had been known for a long time (e.g. Minshall, 1960; Izzard, 1966; Amorocho, 1967; Dooge, 1967b; Amorocho and Brandstetter, 1971).

The nonlinear behavior of hydrologic systems is evident in various ways and at almost all spatial and temporal scales. The hydrologic cycle itself is an example of a system exhibiting nonlinear behavior, with almost all of the individual components themselves exhibiting nonlinear behavior as well. The climatic inputs and landscape characteristics are changing in a highly nonlinear fashion, and so are the outputs, often in unknown ways. The rainfall-runoff process is nonlinear, almost regardless of the basin area, land uses, rainfall intensity, and other influencing factors. In fact, the effects of nonlinearity can be tremendous, especially when the system is sensitively dependent on initial conditions. This means, even small changes in the inputs may

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result in large changes in the outputs (and large changes in the inputs may turn out to cause only small changes in the outputs), a situation popularly termed as "chaos" in the nonlinear science literature (e.g. Lorenz, 1963; see also Gleick (1987) for details).

With significant developments in computational power during the past three decades or so, and also with major advances in measurement technology and mathematical concepts, studies on the nonlinearity and related properties of hydrologic systems have started to gain attention. Nonlinear stochastic methods (e.g. Kavvas, 2003), artificial neural networks (e.g. Govindaraju, 2000), data-based mechanistic models (e.g. Young and Beven, 1994), and nonlinear dynamics and chaos (e.g. Sivakumar, 2000) are some of the popular nonlinear techniques that have found extensive applications in hydrology. This study discusses the utility of nonlinear dynamic techniques as a suitable methodology for studying the complexity of hydrologic systems and, thus, for formulation of the catchment classification framework.

It must be pointed out, at this point, that there is still some confusion on the definition of "nonlinearity" in hydrology, and perhaps in many other fields as well. This is highlighted, for example, by Sivapalan et al. (2002), who discuss two definitions of nonlinearity that appear in the hydrologic literature. One is with respect to the dynamic property, such as the rainfall-runoff response of a catchment, where nonlinearity refers to a nonlinear dependence of the storm response on the magnitude of the rainfall inputs (e.g. Wang et al., 1981). The other definition is with respect to the dependence of a catchment statistical property, such as the mean annual flood, on the area of the catchment (e.g. Goodrich et al., 1997). The present study does not make any attempt to discuss the confusion behind the use of the term "nonlinearity" in hydrology. However, the ideas presented herein are mainly concerned with the dynamic property of hydrologic processes.

4.2 Nonlinear dynamics and chaos: some basics

During the past three decades or so, significant advances have been made in the field of nonlinear sciences to study complex systems. Numerous methods have been

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developed and applied in various fields, including physics, chemistry, biology, earth sciences, ecology, economics, engineering, medicine, and psychology. Popular books on nonlinear dynamic and chaos theories and their applications are those by Tsonis (1992), Strogatz (1994), Abarbanel (1996), and Kantz and Schreiber (1997), among others. For a more general and non-mathematical description of nonlinear dynamic and chaos concepts, the reader is referred to Gleick (1987) and, to some extent, Goerner (1994). Reviews of applications of nonlinear dynamics and chaos concepts in hydrology and geophysics at large are found in Sivakumar (2000, 2004b, 2009).

Popular among the methods developed within the context of nonlinear dynamic and chaos theories are phase space reconstruction, correlation dimension, Lyapunov exponent, false nearest neighbors, nonlinear prediction, surrogate data, close returns plot, and redundancy methods. All of these methods involve data embedding and nearest neighbor search, identifying different yet related properties of the underlying system dynamics. In the following, we briefly present the phase space reconstruction and correlation dimension methods for pattern recognition and complexity determination of time series. In Sect. 5, we illustrate the utility of these methods for hydrologic time series and also discuss their superior performance over widely used linear tools.

4.2.1 Phase space reconstruction

Phase space is a useful tool to represent the evolution of a system in time (or in space). It is essentially a graph or a coordinate diagram, whose coordinates represent the variables necessary to describe the state of the system completely at any moment; in other words, the variables that enter the mathematical formulation of the system (e.g. Packard et al., 1980). The trajectories of the phase space diagram describe the evolution of the system from some initial state, which is assumed to be known, and hence represent the history of the system. The “region of attraction” of these trajectories in the phase space provides important qualitative information on the “extent of complexity” of the system, which can be verified quantitatively using other methods, such as those based on the concept of dimensionality (see below).

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For a dynamic system with known partial differential equations (PDEs), the system can be studied by discretizing the PDEs, and the set of variables at all grid points constitutes a phase space. One difficulty in constructing the phase space for such a system is that the (initial) values of many of the variables may not be known. However, a time series of a single variable of the system may be available, which may allow the attractor (a geometric object that characterizes the long-term behavior of a system in the phase space) to be reconstructed. The idea behind such a reconstruction is that a (nonlinear) system, such as a hydrologic system, is characterized by self-interaction, so that a time series of a single variable can carry the information about the dynamics of the entire multi-variable system.

It must be pointed out that the assumption of only a single-variable time series for phase space reconstruction was made in due consideration to: (1) the general availability of such a time series for a system (e.g. streamflow in a catchment); and (2) the difficulties in observing data of multiple variables, for various reasons. There is no restriction, however, on the use of multi-variable time series for such a reconstruction, if available (see, for instance, Cao et al. (1998) for details on multi-variable data reconstruction); in this case, the reconstruction is normally termed as state space reconstruction. Therefore, in a way, phase space is one, and a simplified, version of state space.

Many methods are available for phase space reconstruction from an available time series. Among these, the method of delays (e.g. Takens, 1981) is the most widely used one. According to this method, given a single-variable series, X_i, where $i = 1, 2, ..., N$, a multi-dimensional phase space can be reconstructed as:

$$\boldsymbol{Y}_j = (X_j, X_{j+\tau}, X_{j+2\tau}, \ldots, X_{j+(m-1)\tau}) \quad (1)$$

where $j = 1, 2, ..., N-(m-1)\tau$; m is the dimension of the vector $\boldsymbol{Y}_j$, called embedding dimension; and τ is an appropriate delay time (an integer multiple of sampling time). A correct phase space reconstruction in a dimension m generally allows interpretation of the system dynamics (if the variable chosen to represent the system is appropriate) in

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the form of an m-dimensional map f_T, given by:

$$\boldsymbol{Y}_{j+T} = f_T(\boldsymbol{Y}_j) \quad (2)$$

where $\boldsymbol{Y}_j$ and $\boldsymbol{Y}_{j+T}$ are vectors of dimension m, describing the state of the system at times j (current state) and $j+T$ (future state), respectively.

4.2.2 Correlation dimension method

One of the purposes behind determining the dimension of a time series is that dimensionality furnishes information on the number of variables dominantly governing the evolution of the corresponding dynamic system. Dimension analysis also reveals the extent to which the variations in the time series are concentrated on a subset of the space of all possible variations. In the context of identification of the nature of governing mechanisms and, thus, the potential complexity level of the system, the central idea behind the application of the dimension approach is that systems whose dynamics are governed by stochastic dynamic processes are thought to have an infinite (or at least very high) value for the dimension, whereas a finite and small value of the dimension is considered to be an indication of the presence of nonlinear deterministic chaotic dynamic processes.

Correlation dimension is a measure of the extent to which the presence of a data point affects the position of the other points lying on the attractor in (a multi-dimensional) phase space or coordinate system. The correlation dimension method uses the correlation integral (or function) for determining the dimension of the attractor in the phase space and, hence, for distinguishing between low-dimensional and high-dimensional systems. The concept of the correlation integral is that a time series arising from deterministic dynamics will have a limited number of degrees of freedom equal to the smallest number of first-order differential equations that capture the most important features of the dynamics. Thus, when one constructs phase spaces of increasing dimension, a point will be reached where the dimension equals the number

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of degrees of freedom, beyond which increasing the phase space dimension will not have any significant effect on correlation dimension.

Many algorithms have been formulated for the estimation of the correlation dimension of a time series. Among these, the Grassberger-Procaccia algorithm (Grassberger and Procaccia, 1983) has been and continues to be the most widely used one, especially in hydrologic studies. The algorithm uses the concept of phase space reconstruction for representing the dynamics of the system from an available single-variable time series, as presented in Eq. (1). For an m-dimensional phase space, the correlation function $C(r)$ is given by

$$C(r) = \lim_{N\to\infty} \frac{2}{N(N-1)} \sum_{\substack{i,j \\ (1\le i<j\le N)}} H\left(r - \left|\boldsymbol{Y}_i - \boldsymbol{Y}_j\right|\right) \tag{3}$$

where H is the Heaviside step function, with $H(u) = 1$ for $u > 0$, and $H(u) = 0$ for $u \le 0$, where $u = r - |\boldsymbol{Y}_i - \boldsymbol{Y}_j|$, r is the vector norm (radius of sphere) centered on $\boldsymbol{Y}_i$ or $\boldsymbol{Y}_j$. If the time series is characterized by an attractor, then $C(r)$ and r are related according to:

$$\underset{\substack{r\to 0 \\ N\to\infty}}{C(r)} \approx \alpha r^{\nu} \tag{4}$$

where α is a constant and ν is the correlation exponent or the slope of the Log $C(r)$ versus Log r plot. The slope is generally estimated by a least square fit of a straight line over a certain range of r (i.e. scaling regime) or through estimation of local slopes between r values.

The distinction between low-dimensional and high-dimensional systems can be made using the ν versus m plot. If ν saturates after a certain m and the saturation value is low, then the system is generally considered to exhibit low-dimensional and chaotic behavior. The saturation value of ν is defined as the correlation dimension (d) of the attractor, and the nearest integer above this value is generally an indication of the number of variables dominantly governing the system dynamics. On the other

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hand, if v increases without bound with increase in m, the system under investigation is generally considered to exhibit high-dimensional and stochastic behavior.

5 Identification of complexity of hydrologic time series

In the context of identification of system properties and behaviors, it is customary to use some basic linear tools, such as the autocorrelation function and power spectrum. The autocorrelation function is a normalized measure of the linear correlation among successive values in a time series. The power spectrum is particularly useful for characterizing the regularities/irregularities in observed signals.

While the autocorrelation function and power spectrum yield reliable results in identification of system properties in certain situations, they are inadequate in certain others. For instance, while they provide convincing distinctions between random and periodic (or quasi-periodic) systems, they are not reliable for distinguishing between random and chaotic signals. A clear demonstration of this limitation of linear tools and the superiority of nonlinear tools has been presented in Sivakumar et al. (2007). For the sake of brevity, we highlight here only some important results, and the reader is directed to the original article for further details.

Sivakumar et al. (2007) analyzed two synthetic time series that are "similar" in appearance and "complex and random" in nature but possess totally different characteristics: one generated using a pseudo random number function and the other obtained from a deterministic nonlinear two-dimensional map. They observed that the autocorrelation function and power spectrum failed to distinguish the properties of the two systems, and the failure was not just in "visual" or "qualitative" terms but also in quantitative terms: for instance, for both series, the time lag at which the autocorrelation function first crossed the zero line was equal to one (especially no exponential dacay for the chaotic series) and the power spectral exponent was equal to zero (indicating randomness in the underlying dynamics of both). However, the phase space reconstruction and correlation dimension methods were able to distinguish between the two

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time series. In the phase space reconstruction, for the stochastic series, the points (of trajectories) were scattered all over the phase space (i.e. absence of attractor), a clear indication of a "complex" and "random" nature of the underlying dynamics and potentially of a high-dimensional system; the projection for the chaotic series yielded a very clear attractor (in a well-defined region), indicating a "simple" and "deterministic" (yet non-repeating) nature of the underlying dynamics and potentially of a low-dimensional system. In the correlation dimension method, there was no saturation of the correlation exponent for the stochastic series, suggesting a high-dimensional system; for the chaotic series, correlation exponent saturated at a value of 1.22 (correlation dimension = 1.22), indicating a two-dimensional system.

Sivakumar et al. (2007) also analyzed several river-related time series to test the utility of phase space reconstruction concept and its superiority over linear tools. For illustration here, we present the results obtained for two such series, i.e., daily river flow datasets from the USA: (1) the Mississippi River flow at St. Louis, Missouri; and (2) the Kentucky River flow near Winchester, Kentucky. Table 1 presents some important statistics of these time series.

Figure 1 shows the time series (top) and autocorrelation functions (bottom) for these datasets. Both flow series look "complex" and "random," with peaks and dips. The autocorrelation functions are also very similar, albeit positively showing persistence and seasonality properties in both. These results do not provide any clues as to whether the underlying dynamic properties are low-dimensional or high-dimensional (or deterministic or stochastic), and, thus, do not offer any help in any type of classification for modeling purposes. As a result, selection of the appropriate type and structure of models for these two series, as well as data collection, are difficult.

On the other hand, the phase space diagrams (Fig. 2, top) and correlation dimension results (Fig. 2, bottom) reveal similarities as well as differences between the two series. They reveal certain order and determinism in the underlying dynamics of both time series, i.e. clear attractor structures in well-defined regions in the phase space and low correlation dimensions; they also provide crucial information on the extent of complexity

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of each, i.e. the first series has more order and determinism with just three dominant governing variables in the underlying dynamics (dimension = 2.32) when compared to five in the other (dimension = 4.22). These results are particularly useful for classifying these time series in terms of their nature (deterministic or stochastic) as well their extent of complexity (low-dimensional or high-dimensional). In view of this, these results are also helpful in the selection of the appropriate type and structure of models and, consequently, in determining data requirements.

These observations and numerous others on nonlinear dynamic analysis of hydrologic time series (see Sivakumar (2000, 2004b, 2009) for reviews) provide a reliable means for classification of hydrologic systems, and, thus, are used as an important basis for the classification framework proposed below.

6 Catchment classification framework: a proposal

With hydrologic system complexity as a basis and nonlinear dynamic tools as a methodology, a three-step procedure is proposed here for a classification framework:

1. Detection of possible patterns in data and determination of complexity levels of hydrologic systems;

2. Classification of hydrologic systems into groups and sub-groups based on data patterns and system complexity; and

3. Verification of the classification framework through: (a) establishing relationships between the data patterns/complexity and the actual catchment/process properties; and (b) studying the outputs simulated from existing hydrologic models and varying their complexities.

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An outline of these steps is shown in Fig. 3, and their details are presented next.

6.1 Detection of data patterns and determination of level of complexity

The possible patterns and the level of complexity of hydrologic data (systems) can be studied by applying a variety of methods. These methods can include both linear and nonlinear ones, so as to capture as many of the salient features of the system as possible. The linear methods may include: autocorrelation function, power spectrum, statistical moment scaling method, to detect seasonality, cyclicity, and scale properties, among others. The nonlinear methods may include: phase space reconstruction, correlation dimension, close returns plot, and local prediction, to detect system evolution, dimensionality, complexity, and predictability, among others. Furthermore, although each of these methods provides its own information, the results can also be easily verified through those from the others. For instance, the complexity of system evolution assessed from the phase space diagram (attractor) can be easily verified using the dimensionality determined using the correlation dimension method.

6.2 Classification framework based on patterns and complexity

The results from the above analysis (Step 1) offer important information on many of the system (data) properties and underlying dynamics. These include patterns, order, dimensionality, number of dominant governing variables, type and optimum complexity of model, type and amount of data required, and prediction accuracy and horizon. All this information need to be analyzed, cross-verified, and interpreted to streamline the hydrologic systems into different groups and sub-groups, as appropriate, possessing emblematic features of their own. Both qualitative and quantitative analyses can be performed for this purpose, including visual inspection of patterns, statistical analysis of dimensions, and assessment of model complexity and data requirements.

It is premature, at this stage, to provide definitive guidelines on the exact structure of the classification framework and on the specific number of groups and sub-groups.

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As Wagener et al. (2007) pointed out, there may be several possibilities and different components, including hydroclimate, catchment heterogeneity, process function, and scale. However, any of the properties identified in Step (1) (e.g. dimensionality), and their combinations, can serve as an important component to formulate this framework. For example, considering dimensionality, the systems may potentially be treated broadly as high-dimensional, medium-dimensional, and low-dimensional, and then further grouped as medium-high and low-medium, thus clearly emphasizing the complexity. Since the structure of this framework and its effectiveness to classify hydrologic systems can be verified via catchment/process properties as well as through simulation of outputs from existing hydrologic models by varying their complexities (Step 3; see Sect. 6.3), appropriate checks and balances are possible. Therefore, if necessary, the framework can be modified for improvement.

6.3 Verification of classification framework

This step evaluates the effectiveness of the above classification framework in two ways, as discussed next:

(a) Establishing relationships with catchment/process properties: This can be done with due consideration of numerous properties of catchments and processes. However, oftentimes only a few of these catchment/process properties are dominant, as has been illustrated through many case studies in Grayson and Blöschl (2000). Based on our knowledge and experience with catchments and processes, the following properties may be considered important. As for catchments, general climatic conditions (e.g. humid, semi-arid, arid), catchment area, channel length, soil type, land use, and time of concentration can be studied. As for processes, rainfall (e.g. type, amount, intensity) and streamflow (e.g. magnitude, runoff coefficient) properties can be the main focus. The established relationships can also offer crucial information on the properties that may not be dominant in specific catchments. This information may help to modify or further simplify the classification framework, through elimination of these properties in the original framework.

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(b) Studying the outputs simulated from hydrologic models and varying their complexities: Since identification of the optimum complexity of model for catchments is one of the main goals of the classification framework, simulating the hydrologic outputs for different complexity levels of a selected hydrologic model(s) for the catchment(s) of interest and studying their properties is a reliable means to verify/confirm the effectiveness of the classification framework. The results obtained for the observed data and, hence, the classification groups and sub-groups may also offer some guidelines. To this end, varying complexity levels of some selected hydrologic models (physics-based or other) that are commonly used, such as the HEC-HMS and MIKE-SHE, can be used to simulate hydrologic outputs (e.g. streamflow). Important properties of these outputs can be studied and compared with those of the observed data. This will help identify the appropriate model complexities for the catchments and also how they match against the classification groups and sub-groups. Analysis can also be carried out to assess the effects of structural and data uncertainties. These will particularly be useful for modelers to identify suitable catchments to apply the models to and for users to identify models that would be appropriate for catchments.

7 Conclusions and further research

Hydrologic models play a crucial role in the assessment of water resources availability and decisions on water planning and management. Consequently, hydrologic modeling has become an important research endeavor, particularly facilitated by recent technological and methodological advances. Although numerous hydrologic models have been developed (often with increasing structural complexity and mathematical sophistication), identifying which model is appropriate for which catchment remains a fundamental problem. To this end, the need for a classification framework that streamlines catchments into different groups and sub-groups for a more effective and efficient model selection is increasingly realized. However, an appropriate basis and a suitable methodology for such a framework are still elusive.

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This study offers one possible way to view the classification problem in hydrology through an inverse approach; i.e., going backward from system outputs. It argues that hydrologic system complexity forms an appropriate basis for the classification framework and nonlinear dynamic concepts constitute a suitable methodology for assessing system complexity. Discussing the relevance of complexity and nonlinearity in hydrologic systems and also the utility of nonlinear dynamic tools for pattern recognition and complexity determination, the study presents a three-step procedure for formulation of a catchment classification framework.

The proposed classification framework will lead to a far more effective and efficient procedure for identifying the appropriate structure and complexity of model for a given hydrologic system and to a far better practice in hydrologic modeling and prediction. It will help modelers identify appropriate catchments to apply the models to and users to identify appropriate models for catchments. It will help towards a more reliable assessment of the type, quantity, and quality of data requirements. This information will potentially yield significant savings in time, data, and computational requirements for hydrologic modeling. The framework will also provide a common language for communication and discussion among water researchers and managers and will significantly improve teaching, research, and practice in the field of water.

Finally, to assess its usefulness and effectiveness, the proposed framework needs to be tested on a wide variety of catchments and hydrologic data representing different climatic conditions, catchment characteristics, land use properties, and types of data, among others. As a starting point, efforts are underway to study a large network of catchments in the United States (western US), Australia, and India. Details of such studies will be reported elsewhere.

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Table 1. Statistics of daily river flow time series.

Statistic	Mississippi River	Kentucky River
Number of data	10226	10958
Mean ($m^3 s^{-1}$)	5309.9	151.9
Maximum ($m^3 s^{-1}$)	24100	2806
Minimum ($m^3 s^{-1}$)	980	329
Number of zeros	0	0

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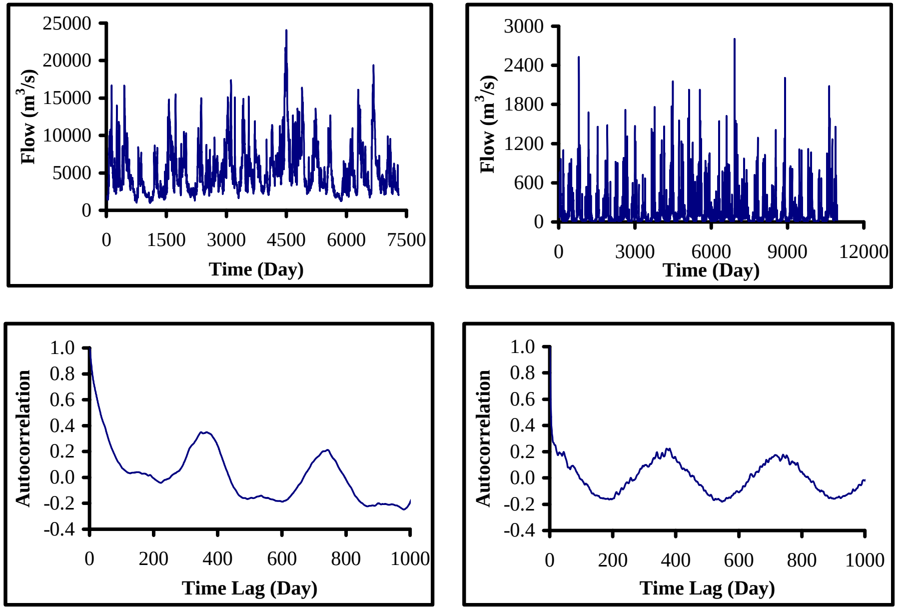

Fig. 1. Time series (top) and autocorrelation function (bottom) of daily river flow from the Mississippi River, Missouri, USA (left) and the Kentucky River, Kentucky, USA (right).

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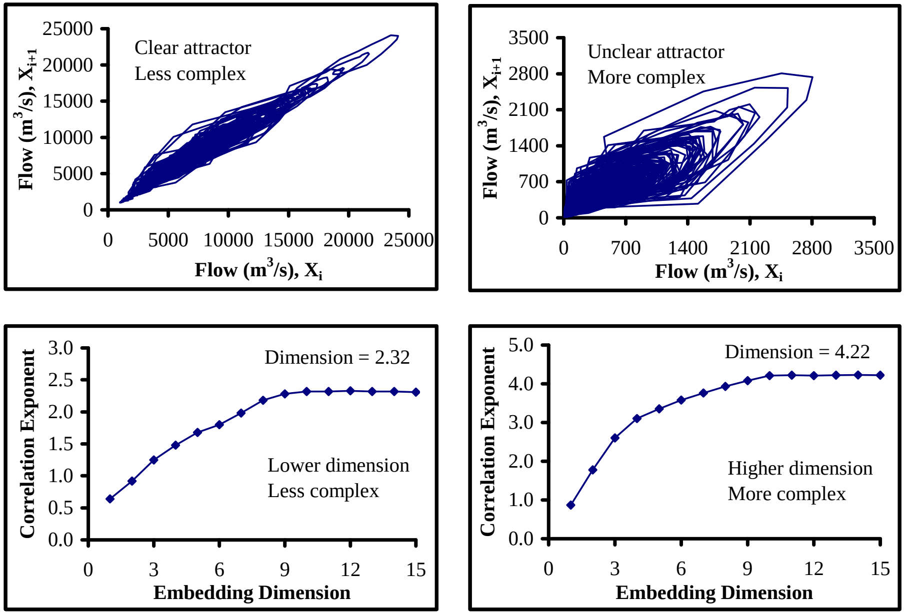

Fig. 2. Phase space (top) and correlation dimension (bottom) of daily river flow from the Mississippi River, Missouri, USA (left) and the Kentucky River, Kentucky, USA (right).

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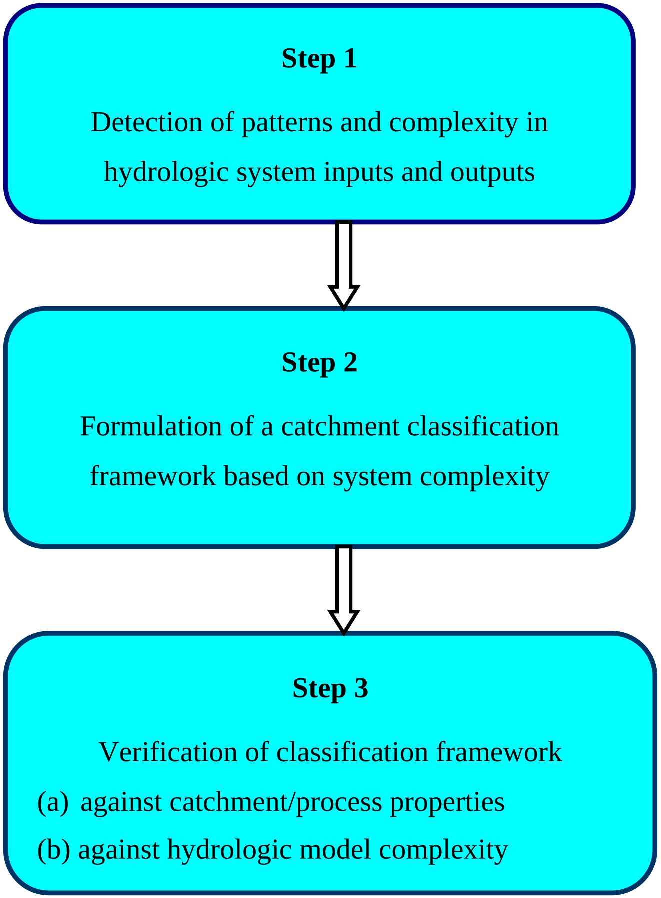

Fig. 3. Outline of steps for formulation of a catchment classification framework.

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