Hydrol. Earth Syst. Sci. Discuss., 8, 3961–3992, 2011 www.hydrol-earth-syst-sci-discuss.net/8/3961/2011/ doi:10.5194/hessd-8-3961-2011 © Author(s) 2011. CC Attribution 3.0 License.



This discussion paper is/has been under review for the journal Hydrology and Earth System Sciences (HESS). Please refer to the corresponding final paper in HESS if available.

Towards reconstruction of the flow duration curve: development of a conceptual framework with a physical basis

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 Received: 28 March 2011 – Accepted: 29 March 2011 – Published: 20 April 2011

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Published by Copernicus Publications on behalf of the European Geosciences Union.





Abstract

In this paper we investigate the climatic and landscape controls on the flow duration curve (FDC) with the use of a physically-based rainfall-runoff model. The FDC is a stochastic representation of within-year variability of runoff, which arises from the transformation, by the catchment, of within-year variability of precipitation that can itself be 5 characterized by a corresponding duration curve for precipitation (PFDC). Numerical simulations are carried out with the rainfall-runoff model under a variety of combinations of climatic inputs (i.e., precipitation, potential evaporation, including their withinyear variability) and landscape properties (i.e., soil type and depth). The simulations indicated that the FDC can be disaggregated into two components, with sharply dif-10 fering characteristics and origins: the FDC for surface (fast) runoff (SFDC) and the FDC for subsurface (slow) runoff (SSFDC). SFDC closely tracked PFDC and can be approximated with the use of a simple, nonlinear (threshold) filter model. On the other hand, SSFDC tracked the FDC that is constructed from the regime curve (ensemble mean within-year variation of streamflow), which can be closely approximated by a lin-15

- ear filter model. Sensitivity analyses were carried out to understand the climate and landscape controls on each component, gaining useful physical insights into their respective shapes. In particular the results suggested that evaporation from dynamic saturated areas, especially in the dry season, can contribute to a sharp dip at the
- ²⁰ lower tail of the FDCs. Based on these results, we develop a conceptual framework for the reconstruction of FDCs in ungauged basins. This framework partitions the FDC into: (1) a fast flow component, governed by a filtered version of PFDC, (2) a slow flow component governed by the regime curve, and (3) a correction to SSFDC to capture the effects of high evapotranspiration at low flows.





1 Introduction

The flow duration curve (FDC) is a classical signature of temporal (within-year) streamflow variability in a river basin (see Vogel and Fennessey, 1994, 1995; Smakhtin, 2001). It is effectively an alternative representation of the cumulative distribution function of

- daily (sometimes hourly) streamflow. Hydrologists have traditionally analyzed the FDC using purely graphical representations (Ward and Robinson, 1990), or using stochastic models that focus on fitting appropriate statistical distributions and estimating associated parameters (see most recent work by Castellarin et al., 2004a; Iacobellis, 2008). Many of the past efforts have focused on relating the characteristics of the FDCs (e.g.,
- shape measures or parameters of the statistical distributions, as the case may be), to the catchment's climatic and physiographic characteristics, to assist in regionalization of the FDCs and as a precursor to estimation in ungauged catchments.

Work over the past few decades has contributed to the accumulation of considerable empirical knowledge on the effects of a single or several characteristics of watersheds

- ¹⁵ upon the shape of FDCs. Musiake et al. (1975) investigated the effects of geology and climate type on the shape of FDCs in Japanese mountainous watersheds. Ward and Robinson (1990) provide a summary of the effects of dominant soil types on FDCs in UK catchments. Burt and Swank (1992) investigated the effects of vegetation type on the FDCs. Sefton and Howarth (1998) explored the effects of morphometric, soil,
- ²⁰ land use, and climate properties of watersheds on the FDCs in the UK. Castellarin et al. (2004b) presented a regional statistical model to construct FDCs based on watershed morphology and climate characteristic in Italian catchments. Despite such empirical results we have not yet succeeded in developing a comprehensive understanding of the relative contributions of climatic and watershed characteristics on the
- shape of the FDCs, especially towards establishing globally applicable relationships. In other words, if one were to think of the several empirical studied reviewed above as pieces of a jigsaw puzzle, we have not yet acquired the understanding and the methodology to complete that puzzle. The work presented in this paper is a small step in developing a general process-based characterization of the FDC, fully reflecting the





complex interactions between climate (i.e., precipitation and radiation) and catchment physiographic characteristics that contribute to the generation of runoff by many different mechanisms.

- In recent times there have been promising efforts that approach FDCs from a process perspective. Botter et al. (2007) presented the mathematical formalisms for the derivation of the probability distribution (which is equivalent to FDCs), associated with withinyear variation of the baseflow component of daily streamflow. Their stochastic-dynamic model captured the interaction of within-year sequences of precipitation events with a simple lumped model of subsurface drainage that is governed by a field capacity threshold and a characteristic catchment residence time. The model was success-
- fully tested in a number of catchments across the United States. Yilmaz et al. (2008) approached the same question with use of the Sacramento Soil Moisture Accounting Model (SAC-SMA) and explored, through sensitivity analyses, the effect of the upper layer tension water capacity (a model parameter) on the shape of FDCs. Muneepeer-
- ¹⁵ akul et al. (2010) extended the stochastic-dynamic model of Botter et al. (2007) to include fast runoff processes, and in this way developed a stochastic framework to mimic the within-year variability of the fast and slow flow components of the FDCs in a number of US catchments.

The work presented in this paper can be viewed as a further extension of the work of ²⁰ Botter et al. (2007) and Muneepeerakul et al. (2010) but different from their work in several ways: (i) this numerical modeling study will use a more advanced physically-based, continuous water balance model that includes runoff generation by several mechanisms; (ii) being a numerical model, the simulations will be able to capture the effects of not only the randomness of precipitation events but also, explicitly, the effects of sea-

²⁵ sonality of both precipitation and potential evaporation, which together govern the variability of antecedent soil moisture conditions and their effects on runoff generation; and (iii) the model used is a quasi-2-D model, and does include lateral flow processes (e.g., saturation excess overland flow and subsurface stormflow), and can therefore be used to assess their relative effects of on the shape of the FDCs.





The water balance model used here is taken from Yokoo et al. (2008). This model was developed on the basis of governing equations for mass and momentum balance derived at the scale of a representative elementary watershed (REW), which have been the basis of numerous distributed modeling efforts (e.g., Zhang and Savenije, 2005; Zehe et al., 2006; Lee et al., 2007; Li et al., 2011; Li and Sivapalan, 2011). Previous applications of the model used here have involved the exploration of (1) mean annual water balance within the Budyko (1974) framework (Reggiani et al., 2000), and (2) the ensemble mean within-year variations of runoff (i.e., regime curve) (Yokoo et al., 2008), with both studies including the partitioning into surface and subsurface runoff as well. The current paper thus represente a further application of the model to develop

- 10 well. The current paper thus represents a further application of the model to develop insights into the FDC. The main aims of the paper are: (1) to generate insights into the shape of the FDCs, and to determine the relative controls of climate and landscape properties on the FDCs, including its various components; (2) to develop a conceptual framework that can be utilized, in combination with the insights gained into the climate and landscape controls on the FDCs, to help reconstruct FDCs in ungauged basins.
- The approach adopted in this paper to achieve these aims includes the use the adopted water balance model under various combinations of climatic and catchment physiographic parameters to explore the sensitivity of the shapes of the FDCs to these parameter combinations. We examine the causes for these sensitivities through a detailed examination of how the variability in the climatic inputs propagates through
- detailed examination of how the variability in the climatic inputs propagates through the catchment system and the various transformations that take place within various subsystems. On the basis of these results we draw up broad conclusions about the controls of climatic and physiographic characteristics on the shapes of the FDCs. Note that, for the present, this is a purely theoretical study and therefore has the disadvan-
- tage that it is not based on observed data. However, our intention is that the framework we will develop here, through generating plausible hypotheses, would be a necessary precursor to systematic empirical analysis that could help validate these hypotheses through the use of empirical observations.





2 Methodology

The key components of the methodology involve the use of (i) a stochastic rainfall model to generate synthetic rainfall event sequences, under different assumed climates, and (ii) the lumped, physically based rainfall-runoff model. The two models are

⁵ used in sequence for a variety of combinations of soils and climate to generate runoff time series (including surface and subsurface runoff components) from which the FDCs are derived, including its two components. Next we present the brief summaries of the rainfall and rainfall-runoff models.

2.1 Stochastic rainfall model

- ¹⁰ We employ an event-based stochastic model of precipitation time series developed by Robinson and Sivapalan (1997) to generate multiple random realizations of synthetic precipitation inputs. This model is capable of reproducing multi-scale temporal variability of rainfall intensities, including random within-storm and between-storm variability, the parameters of which can, if needed, vary seasonally in a deterministic manner.
- ¹⁵ Both storm durations and inter-storm periods are assumed to follow the exponential distribution, the parameters of which also vary sinusoidally over the year. Mean rainfall intensities during storms are assumed to follow a conditional gamma distribution, subject to the chosen storm duration, the parameters of which could also vary sinusoidally over the year. The mean storm intensity is further disaggregated to hourly intensity pat-
- terns (within-storm patterns) using stochastically generated mass curves (Huff, 1967; Chow et al., 1988). This disaggregation is carried out with the use of the random cascade model (Koutsoyiannis and Foufoula-Georgiou, 1993): the random weights chosen to sequentially disaggregate the rainfall depth at finer time steps are assumed to follow the beta distribution.
- ²⁵ Details of the synthetic rainfall model can be found in the original paper by Robinson and Sivapalan (1997). For convenience, the model parameters used for the simulations reported in this paper are the same as those in Table 2 of Robinson and





Sivapalan (1997), derived for the raingauge at Salmon Creek in Western Australia (which is used mainly for convenience). In this study we generated hourly rainfall intensities for a period of 13 yr, which were then rescaled so that the mean annual rainfall over the 13 yr period becomes approximately 1000 mm, consistent with the notion that this is a completely theoretical study.

The attraction of using such a model for generating synthetic precipitation inputs is that it allows us to perform diagnostic analyses whereby we can switch on and off different components of the natural variability, and investigate their effects on the shape of the FDCs. This is a significant advantage over the use of historical data.

10 2.2 Water balance model

5

For the rainfall-runoff simulations in this paper, we employ the simple, lumped and physically based water balance model previously used by Yokoo et al. (2008), which was an extension of the model originally developed by Reggiani et al. (2000). The description presented is taken from Yokoo et al. (2008) for completeness. Figure 1 presents a schematic description of the model. To keep the model simple, the model structure has been simplified to include just two zones, an unsaturated zone and a saturated zone below it. Within this simple structure, and the associated governing equations, the model monitors as state variables the saturated and unsaturated area fractions (as geometric functions of the saturated area fractions (as geometric functions of the saturated area fractions). The model previously used by Yokoo et al. (2008)

- 20 geometric functions of the saturated area thickness). The model predicts key physical processes such as runoff generation by infiltration excess, saturation excess and subsurface stormflow, as well recharge, capillary rise and evaporation and transpiration through root water uptake. Further details about the model can be found in the original paper by Reggiani et al. (2000) and also in the subsequent paper by Yokoo et
- ²⁵ al. (2008). In this paper we only give a brief outline of the model and the parameter sets used in the simulations reported here.





2.2.1 Governing equations

5

The water balance model by Reggiani et al. (2000) consists of three coupled governing equations: mass balance in the unsaturated zone, momentum balance in the unsaturated zone, and mass balance in the saturated zone, as shown in Eqs. (1–3), respectively.



The function $\delta[0, t_r]$ (resp. $\delta[t_r, t_m]$) in Eq. (1) is equal to 1 if time *t* in a meteorological period, which consists of a storm duration and the subsequent inter-storm period, falls between 0 (resp. t_r) and t_r (resp. t_m), and is zero otherwise. The variables s_u , y_u , ω_u , Λ_u , ψ_u , v_u , *K*, *P*, and PET are, respectively, saturation degree in the unsaturated zone,





average thickness of the unsaturated zone, unsaturated surface area fraction, a characteristic length scale for infiltration, pressure head in the unsaturated zone, upward velocity in the unsaturated zone, unsaturated hydraulic conductivity, precipitation intensity, and a seasonally-varying potential evaporation. The variables y_s , ω_s , ω_o , γ_o , and

- Λ_s are, respectively, the average thickness of the saturated zone, area fraction of the saturated zone, saturated surface fraction, slope angle of the overland flow plane with respect to horizontal, and a typical length scale for seepage outflow. The definitions of the other variables used in these equations are summarized in Table 1 as well as in the appendix of Reggiani et al. (2000). These equations contain 7 unknowns, and
 therefore 4 additional closure relations are required. Most of those are simple geomet-
- therefore 4 additional closure relations are required. Most of those are simple geometric relations, but the parameterization of the seepage area fraction ω_o is a non-trivial one requiring further assumptions, as outlined below:

$$\omega_{\rm o} = \frac{y_{\rm s} - Z_{\rm r} + Z_{\rm s}}{Z - Z_{\rm r} + Z_{\rm s}}$$

 $\dot{\omega}_{\circ} = -\dot{\omega}_{u}$

In Eq. (4), Z is the average thickness of the subsurface zone, z_r is channel bed elevation with respect to a datum, z_s is the average elevation of the bottom surface of the REW with respect to the datum, y_s is the average thickness of the subsurface zone along the vertical, and ω_u is the area fraction of the unsaturated zone.

To solve the governing equations, we need constitutive relationships regarding the hydraulic properties of the soil. We use the VK model for hydraulic conductivity (Kosugi, 1994) for the water retention curve, which has the advantage that it does not have a discontinuous point near saturation, contains only three physical parameters, and thus permits easy calibration to measured water retention data. Equations (6) or (7) are the functional forms of the VK model,

$$\psi_{u} = \begin{cases} \psi_{c} - (\psi_{c} - \psi_{0}) \cdot \left\{ \frac{(s_{u})^{-m^{-1}} - 1.0}{m} \right\}^{1.0-m} (s_{u} < 1.0) \\ \psi_{c} (s_{u} = 1.0) \\ 3969 \end{cases}$$



(4)

(5)

(6)



$$s_{u} = \begin{cases} 1 / \left\{ 1 + m \left(\frac{\psi_{c} - \psi_{u}}{\psi_{c} - \psi_{0}} \right)^{1/(1-m)} \right\}^{m} & (\psi_{u} < \psi_{c}) \\ 1.0 & (\psi_{u} \ge \psi_{c}) \end{cases}$$

where ψ_c , ψ_0 , and m are bubbling pressure, capillary pressure at the inflection point on the $s_u - \psi_u$ curve, and dimensionless parameter, respectively. For the unsaturated hydraulic conductivity *K*, we use Eq. (8) from Reggiani et al. (2000), which was in turn taken from (Brutsaert, 1966),

 $K = K_{\rm s} \cdot (S_{\rm u})^{\lambda}$

 $R = \frac{\text{PET}_{a}}{P_{a}}$

(8)

(9)

(7)

where λ is pore-disconnectedness index.

2.2.2 Numerical solution of governing equations and water balance calculations

- ¹⁰ In solving the governing equations, we need to provide initial conditions for the saturation degree in the unsaturated zone s_u and water table thickness y_s , in addition to parameter settings for soil properties, climatic inputs, and the two geometric parameters of γ_o and Λ_s in Table 1. Arbitrary initial values for soil moisture and water table depth are appropriate so long they are not very different from physically acceptable values. We set the initial values of 0.5 for s_u and $z_r - z_s$ m for y_s , in common to all the numerical experiments. The soil hydraulic properties are taken from the literature (Bras, 1990). As in Reggiani et al. (2000) the two geometric parameters, γ_o and Λ_s , are taken to be 0.0 and 10 m, respectively. For the climatic inputs, we assumed the annual rainfall to be 1000 mm and annual potential evaporation was then chosen on the basis of the climatic dryness index *R*, the ratio of annual potential evaporation over
- annual precipitation, given by:

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where PET_a and P_a are annual evaporation (m) and annual precipitation (m), respectively. For reference, Table 1 presents the list of all the parameters and the ranges of values used in this paper.

In this paper we utilize a fourth-order Runge-Kutta integration method for solving the coupled governing equations simultaneously. Firstly, we gave initial condition for s_u and y_s to solve Eq. (2) as mentioned above. Secondly, we solved Eqs. (1) and (3) to obtain s_u and y_s in the next step of the Runge-Kutta integration method. The simulations were carried out for a period of 13 yr with a time step of 5 min. Only the last 3 yr of the runoff time series produced by the model were used to estimate the FDCs; in this way, the effect of initial conditions on the resulting FDCs can be neglected.

2.3 Setup for the numerical experiments

The main analysis we perform in this paper is a series of numerical experiments with the numerical model of water balance. These experiments take the form of sensitivity analyses with the rainfall-runoff model to investigate the controls of climate, soil, and topography on the shapes of the FDCs, including the SFDCs (surface runoff) and SSFDCs (subsurface stormflow). Average annual precipitation P_a was set to 1000 mm in all of the simulations. The synthetically generated precipitation time series P(t)based on the stochastic model of Robinson and Sivapalan (1997) are used in all the simulations. Dryness index *R* was varied from 0.5 to 1.5 by changing the annual potential evapotranspiration PET(*t*), with both P(t) and PET(*t*) including seasonal variabilities that are perfectly in-phase or perfectly out-of-phase. We also consider three different soil types: silty loam, sandy loam, and sand; assumed soil depths ranged from 6 m to 8 m.





3 Results

3.1 FDC separation into constituent elements

Our goal in this paper is to use carefully defined rainfall-runoff simulations to elucidate the physical meaning of the shape of the FDC in terms of its underlying process con-

- ⁵ trols. Figure 2 shows the result of a test run for a hypothetical watershed (with default parameters slope gradient of 0.006, soil type: silty loam, soil depth is 8 m), in a humid climate with the seasonality of *P* and PET that are in phase. In this case the FDC (thick black curve) is presented along with the surface flow duration curve (SFDC, thin blue curve) and the subsurface flow duration curve (SSFDC, thin black curve). We can
- clearly see that the upper tail of the FDC is quite close to that of the SFDC, whereas the middle section and the lower tail of the FDC track well the SSFDC. In addition, we can see that SFDC is a slightly modified (filtered) version of the PFDC, and shows a level of intermittency that is similar to that of precipitation. Likewise we can see that the SSFDC closely tracks (is slightly below) the FDC of the mean within-year variation of streamflows (i.e., the regime curve). This is suggestive of the potential of constructing
- the middle part and lower tail of the FDC from the regime curve.

These results point to the decomposition of the FDC into two component building blocks: (i) the first component is a slightly filtered version of precipitation, which preserves the intermittence of the original time series, and (ii) a second component is a highly smoothed one, representing the result of a competition between subsurface

²⁰ a highly smoothed one, representing the result of a competition between subsurface drainage and evapotranspiration.

In the sections below we will present results of simulations for several combinations of climate and soils to confirm that the breakdown suggested above remains valid in all or most cases. If these features persist for all combinations of climate and landscape

²⁵ properties, this would then present an elegant and physically meaningful way to perform the separation of the FDC into its two component building blocks. In addition, we will explore the climatic and landscape (i.e., soils) controls on the two building blocks.





3.2 Sensitivity of the FDC to climate factors

The initial set of simulations involves different combinations of climate variability, which are the principal drivers of runoff variability. As shown in Fig. 3 four different cases are considered. Two different values of climatic dryness (defined as PET/P) are assumed,

namely 0.5 (humid) and 1.5 (semi-arid). In each case, two different types of seasonality are assumed, i.e., in-phase and out-of-phase seasonality of *P* and PET. Apart from these default values of soil type (silty loam), a soil depth of 8 m, and the topographic gradient assumed of 0.006 are assumed. Figure 3a, c and e is for the case for in-phase seasonality, whereas Fig. 3b, d and f is for out-of-phase seasonality. Figure 3c and d
 presents the FDCs for surface runoff, whereas Fig. 3e and f presents the FDCs for subsurface runoff.

Figure 3a and b presents the FDCs for R = 0.5 (humid) and 1.5 (arid), and the corresponding FDCs for the subsurface flow component (SSFDC). The results show that, in both cases, the middle section and the lower tail of the FDCs very well track the

- SSFDC. The FDCs deviate from the SSFDCs towards the upper tail, which is suggestive of surface runoff component. This leads to the FDCs of the surface runoff component (SFDC), which are presented in Fig. 3c and d, along with the FDCs of the precipitation inputs (PFDC). The results reflect the presence of an infiltration loss, with a larger loss term in the arid case, and a smaller loss in the humid case, with seasonality not having a significant impact. The transformation between the PFDC and SFDC
- are suggestive of a nonlinear (threshold) filter.

Figure 3e and f presents the model generated FDC for subsurface runoff (SSFDC) for both R = 0.5 and R = 1.5. Along with these, we also present the corresponding FDCs associated with the predicted regime curve. The results indicate that the FDCs

derived from the regime curve approximate the SSFDCs in a humid climate, regardless of climatic seasonality. However, in the arid climate where ET becomes the dominant process, the SSFDCs deviate from the FDCs derived from the regime curves, especially for low flows, resulting in a sharp dip in the FDCs towards the lower tail. Indeed,





in arid climate with out-of-phase seasonality, there is a slight dip in the FDC of the regime curve as well towards the lower tail.

3.3 Sensitivity of the FDCs to soil type

Figure 4 shows the results of sensitivity analyses with respect to soil type and climatic
seasonality (in-phase and out-of-phase). Otherwise, these simulations use default values of climate dryness of *R* = 0.5 (humid), a soil depth of 8 m and a topographic gradient of 0.006. As shown in Fig. 4 four different cases are considered: sand vs. silty loam, and in-phase vs. out-of-phase seasonality. Figure 4a, c and e is for the case for in-phase seasonality, whereas Fig. 4b, d and f is for out-of-phase seasonality. Figure 4c
and d presents the FDCs for surface runoff, whereas Fig. 4e and f presents the FDCs for subsurface runoff.

Figure 4a and b presents the FDCs for two different types of soil (sand and silt), and the corresponding FDCs for the subsurface flow component (SSFDC). The results show that, in both cases, the middle section and the lower tail of the FDCs almost perfectly track the SSFDC. The FDC for sand deviates from that for silt in the lower tail in the case of in-phase seasonality; the deviations are much more in the case of outof-phase seasonality. In either case, the net result is that the FDC for sand is steeper than those for silty loam. These results suggest that a combination of out-of-phase seasonality and well drained soils push the response towards ephemeral systems.

- The FDCs deviate from the SSFDCs towards the upper tail for silt (suggesting that the deviation is due to surface runoff due to infiltration excess runoff). Interestingly, in the case of sand, there is very little deviation between the FDC and SSFDC for the entire range of flows, suggesting that subsurface flow makes a contribution to high flows as well. This leads us to look at the FDCs of the surface runoff component (SFDC), which
- are presented in Fig. 4c and d, along with the FDCs of the precipitation inputs (PFDC). The difference reflects infiltration loss, with larger loss in sand.

Figure 4e and f presents the model generated FDCs for subsurface runoff (SSFDC) along with the corresponding FDCs associated with the regime curve. The results





indicate that the FDCs derived from the regime curve nicely track the SSFDCs in a humid climate, regardless of climatic seasonality. However, in the case of out-of-phase seasonality, the FDCs for sand deviate from those for silty loam.

The above results can be compared to those reported in Ward and Robinson (1990).
⁵ Ward and Robinson showed the FDCs for catchments in clay soils and chalk. Clay has low hydraulic conductivity and high porosity, and chalk has the opposite properties; the empirical results in Ward and Robinson (1990) indicate that soils with low hydraulic conductivity and high porosity generate steeper FDCs, and vice versa. This contradicts our results shown in Fig. 4, where we predicted that FDC for sand would be steeper than for silty loam. It is possible that this contradiction may be explained by the presence of macropores or other kind of biotically influenced preferential path-

ways. Hence our results for the effects of soil type on the FDCs have to remain as a hypothesis to be eventually tested against observed data in the future.

3.4 Sensitivity of the FDCs to soil depth

- Figure 5 shows the results of sensitivity analyses with respect to soil depth and climatic seasonality (in-phase and out-of-phase), with default values of climate dryness of R =0.5 (humid), silty loam, and a topographic gradient of 0.006. As can be seen in Fig. 5, four different cases are considered: two different soil depths (6 m and 8 m), and inphase and out-of-phase seasonality. Figure 5a, c and e is for in-phase seasonality, and Fig. 5b, d and f is for out-of-phase seasonality. Figure 5c and d present the FDCs
- for surface runoff, whereas Fig. 5e and f present the FDCs for subsurface runoff.

Figure 5a and b indicates that SSFDCs closely approximate the FDCs in the middle region and lower tail, in all four cases. However, the FDCs for shallow soil are steeper than for deep soil, with the steepness being bigger in the case of out-of-phase sea-

²⁵ sonality. This is suggestive of the role of ET, which tends to be larger in the shallow soil, because the moisture is more accessible to the influence of atmospheric demand. There is however a deviation between the FDCs and SSFDCs at the upper tail. As before, this being a silty loam, the discrepancy is due to surface runoff contribution. This





is also reflected in Fig. 5b and d; as before, the FDCs for surface runoff track PFDC of precipitation. However, since the surface runoff (especially by infiltration excess) is a surface phenomenon, it is not affected by the depth of soil.

- Figure 5e and f presents a comparison between the SSFDCs and the FDCs derived
 from the regime curve. The results indicate that the SSFDCs generally track the FDCs generated from the regime curve, especially when the soil is deep. However, there is a deviation towards the lower tail of the FDCs, and the deviation is larger in the case of out-of-phase seasonality. One can also see that the FDC generated from the regime curves also deviate in shallow soils from that for deep soils during low
 flows, which becomes even more significant when *P* and PET are out of phase. This is because shallow soil has smaller water holding capacity and hence streamflow is sensitive to precipitation and evapotranspiration. If precipitation stops, then ET would become more dominant during such dry periods. These observations lead us to the idea that ET may be playing a dominant role under dry conditions, as also highlighted
- in Figs. 3 and 4.

saturated surface.

3.5 Possible reason lower tail of the FDCs

Through Figs. 3 to 5, we have shown that the SSFDCs deviated from the FDCs generated from the regime curve whenever ET is a dominant flux. This suggests that there is a higher tendency to generate lower subsurface flows, including zero flows under ²⁰ dry conditions in particular years. On the other hand, due to the smoothing involved in obtaining average flows between years, there is less chance for the flow to go to zero. In order to understand the causes of these differences, we constructed the duration curves for outflows OF from the saturated zone along with the corresponding SSFDCs. In the model, subsurface flow Q_{ss} is calculated as outflow OF minus ET from saturated surface (which happens at the potential rate PET: indeed, when ET is higher than OF, then Q_{ss} is put to zero Therefore, any difference between the duration curves for OF and Q_{ss} , and the dip in the SSFDCs at low flows must be due to this ET from the





The results are presented in Fig. 6. Panels (a) and (b) show the results for different climatic dryness and seasonality. Panels (a) and (c) are for in-phase seasonality, and panels (b) and (d) are for out-of-phase seasonality. Default value of soil type is silty loam, soil depth is 8 m, and the topographic gradient is 0.006. Through these results we assess the controls on the deviations by exploring the effects of climate dryness in panels (a) and (b), and the effects of soil depth in panels (c) and (d). The results confirm that even when OF is non-zero during dry periods, the FDC for subsurface flows deviate downwards at low flow conditions, reaching zero for some 25% of the time in semi-arid catchments. As shown here, this effect is due to evaporation from saturated areas, and cannot be modeled using lumped formulations; only a 2-D or guasi-2-D model can capture this effect.

4 Discussions and conclusions

25

The flow duration curve represents the distillation of intra-annual variability of runoff, and presented in the frequency (probability) domain. It can be seen as a manifestation

- of the filtering by the catchment of within-year variability of precipitation. Precipitation variability comprises variability at a range of scales, including random within-storm and between-storm variability as well as more systematic (e.g., seasonal) variability. In this paper we investigated the effects of climate, soils, and topography on the shape of the FDC using a simple, physically based water balance model and synthetic rainfall data.
- ²⁰ The study focused on the fundamental questions: what does the shape of the FDC represent, and what are its process controls?

Despite the fact that the exploration of these questions was based on model simulations in hypothetical watersheds, they did produce important results and revealed new insights that are consistent with our physical intuition as embedded in the rainfallrunoff model. We found that the slope of the FDCs is strongly affected by the nature of seasonality of the climate drivers: precipitation and potential evaporation. The FDC is steeper when the seasonality of P and PET are out of phase, in comparison to





permeable soils and in shallow soils. On the other hand, the effect of climate is such that with increasing aridity the flow becomes more ephemeral, with the result that the FDC tends to get cut off at low flows.

- The simulation results presented in this paper indicated that filtering of the precipitation variability occurs in four different ways: (i) at the ground surface in the transformation of precipitation to surface runoff – this takes place at short time scales, and the filtered product retains characteristics of small-scale variability, including intermittency, (ii) in the unsaturated zone in terms of the conversion between rates at which precipitation infiltrates into the soil surface and the recharge to the saturated zone at
- the bottom of the unsaturated zone this involves a competition between forces that drive downward percolation, capillary forces that contribute to the storage of water in the unsaturated zone and upward movement due to the drying power of the atmosphere, (iii) filtering of the rates of percolation in the saturated zone, which results in subsurface stormflow – this transformation is governed by the soil permeability and the
- topographic gradient. Due to the longer flow pathways and small flow velocities in the subsurface, the mean residence time is generally long. This means that small, event scale variability of percolation is filtered out and the filtering applies to the longer-term, between event and seasonal variability.

The model simulations also indicated that, in circumstances where the effect of ET is strong (i.e., arid climate, shallow soils, out-of-phase seasonality) we need to explicitly consider a correction to the lower tail to account for the reduction of subsurface drainage due to the effects of ET losses over the near-stream saturation area.

Figure 7 presents a schematic that describes the conceptual understanding that we have gained from these model simulations, as a hypothesis that needs to be rigorously tested with the use of observed data. Figure 7 highlights three components to the shape of the FDC: a surface runoff component SFDC, a subsurface component SSFDC, and a correction to account for the effects of ET from near-stream saturation areas. The model simulations also showed that the SFDC component closely tracks the FDC of within-year precipitation variability. The transformation from PFDC to SFDC





reflects the effects of infiltration losses, a climate dominated, surface phenomenon that can be captured using a nonlinear (threshold) filter. The factors that control it are rainfall intensity patterns and soil (infiltration) characteristics, with very little influence of climate seasonality, soil depth or surface topography.

The SSFDC component, in most cases, is a much more smoothed component, with much of the event structure smoothed out. Instead, the shape of the SSFDC reflects the competition between seasonal variability of precipitation and potential evapotranspiration in the vadose zone, which governs recharge to the water table, and then the filtering of the groundwater recharge flux by the dynamic aquifer through subsurface drainage. Previous work has explored the process controls on the recharge process (Struthers et al., 2006; McGrath et al., 2007; Harman et al., 2011), and on the process controls on shallow subsurface flow in hillslopes (Harman et al., 2009).

Finally the dip of the flow duration curve at the lower tail arises due to the relatively higher evapotranspiration from saturated surfaces, which happens in watersheds in arid climates or with lower water holding capacity of soil. This is a feature that can only captured by a 2-D model that pays explicit attention to the water table profile

- and its intersection with the land surface. The main process controls are, therefore, topography, landscape organization, depth to bedrock, and lateral saturated hydraulic conductivity.
- On the basis of these considerations we are now in a position to formulate a conceptual framework for the reconstruction of the flow duration curve in ungauged basins. This is presented in Fig. 8. The conceptual framework comprises three components: (1) a simple nonlinear (threshold) filter model that captures surface infiltration losses, and in this way provide the transformation from PFDC to SFDC; (2) a simple
- two component model of the vadose zone coupled to a shallow subsurface flow model, as a way to simulate realistic patterns of recharge to the water table and then its filtering in the shallow aquifer below; and (3) finally, we need a 2-D model in order to simulate the dynamics of the near-stream saturated area so we can estimate that ET correction during low flow periods.





The insights gained into the shapes of FDCs, including their process controls, give us confidence that there is considerable potential for estimating the shapes of FDCs in ungauged basins from daily precipitation data, monthly flow data, climatic dryness and water holding capacity. Work being undertaken by the authors is aimed at imple-

- menting the conceptual framework developed here in over 200 catchments around the continental United States, and using the data from these catchments to explore the spatial (regional) patterns of variations of the FDCs across the country, explain these patterns on the basis of available evidence on climate, soils and topography, and evaluate the power of the conceptual framework developed here to reconstruct FDCs in ungauged basins. The results of this study will be presented in a series of forthcoming
- publications.

Acknowledgements. The first author is grateful for the financial support provided by the JSPS Research Fellowship for Young Scientists and the Grant-in-Aid for JSPS fellows that enabled the long-term collaboration that culminated in the publication of this paper. He also thanks the

- Is Grant-in-Aid for Young Scientists (B, 18760381, 21760381), Grant-in-Aid for Scientific Research (B, 22360192, PI: So Kazama at Tohoku University, Japan), research grant from Maeda Engineering Foundation, Environment Research and Technology Development Fund (S-8-1(4)) of the Ministry of the Environment, Japan, and the Science and Technology Research Partnership for Sustainable Development, JST-JICA, Japan, for providing additional support. The present
- study was also supported by "Wisdom of Water" (Suntory) Corporate Sponsored Research Program at the University of Tokyo, provided by the Suntory Corporation, Japan. The authors are also grateful to Taikan Oki of the University of Tokyo for his constructive comments on earlier drafts of this paper.





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Table 1. Meaning, values and the ranges of model parameters used in the numerical experiments. The calibration involved manually adjusting the parameters in Kosugi's (Kosugi, 1994) water retention curve (VK model) with those from the Brooks-Corey model (Brooks et al., 1966) and the parameters in Bras (1990) as much as possible. The "ini." indicates initial condition.

Group	Name	Description (unit)		Value and the range
Climate	Pa	Annual precipitation (mm)		1000 (mean)
	Ŕ	Dryness index		0.5 (0.0-2.0)
	PET _a	Potential evapotranspiration (mm)		Pa·R
Geographic	Ζ	Depth of soil layer (m)		5–20
	Zr	Average elevation of channel bed from datum (m)		3.0-7.0
	Zs	Average elevation of the bottom end of REW from datum (m)		0
	y _s	Average thickness of saturated zone (m)		Z – y _u @ _u , 0.5Z (ini.)
	V.,	Average thickness of unsaturated zone (m)		$(Z - \gamma_c)/\omega_{ij}$
	<i>w</i> .,	Unsaturated surface area fraction of unsaturated zone		$(Z - V_c)/V_u$
	Ø.	Saturated surface area fraction of unsaturated zone		1-0
	Ø.	Horizontal area fraction of saturated zone		1
	s	Saturation degree of unsaturated zone		0.5(ini.)
	Λ	Typical length scale for infiltration		S. V.
	۸	Typical length scale for seepage outflow (m)		10
	v.	Slope gradient of the overland flow plane, which is assumed to be nearly flat.		0.0
	Ĺ,	Representative hillslope length of a REW in Fig. 1 (m)		500
	G	Slope gradient of a REW		0.002-0.010
Soil	К	Hydraulic conductivity (m s ⁻¹)		-
	ĸ	Saturated hydraulic conductivity (m s ⁻¹)*	Silty loam	3.4×10^{-6}
	's		Sandy loam	3.1×10^{-5}
			Sandy Ioan	0.4 × 10
	1	Dere disconnectedness indevt	Sanu	0.0 × 10
	л	Pore-disconnectedness index	Silly Idam	4.7
			Sand	3.0
	0	Porocity*	Silty loom	0.25
	2	Folosity	Silly Idam	0.35
			Sand	0.25
	m	Dimonsionloss parameter related to the	Silty loom	0.20
		width of the pere radius distribution**	Sandy loom	0.44
		wider of the pore radius distribution	Sandy Ioan	0.70
		Pubbling procedure (m)**	Silty loom	0.77
	ψ_c	Bubbling pressure (III)	Sandy loom	-0.20
			Sand	-0.10
		Capillary pressure at the inflection point	Silty loam	-0.30
	Ψ_0	on the A warra (m)**	Sinty Iodin Sandy Joam	-0.30
		on the $\theta - \psi$ curve (m).	Sandy loam	-0.25
	117	Sano		-0.10
	Ψu V.,	Velocity in the unsaturated zone (m s^{-1}), positive when directed upward.		_
Others	-	Water density $(ka m^{-3})$	•	1000
	μ	water density (kg m)		0.00
	g	Gravitational acceleration (ms ⁻)		9.80
	t	Ime		-

* indicates parameters are taken from Bras (1990), and ** indicates the parameters are obtained by calibration.







Fig. 1. Conceptual drawing of Reggiani et al.'s (2000) REW-scale water balance model: *P*: precipitation, ET: evapotranspiration, PET: potential evapotranspiration, Q_S : surface runoff, Q_{SS} : subsurface runoff, I_u : infiltration from the ground surface, I_s : infiltration to the saturated zone, *C*: capillary rise, F_o : outflow from saturated zone, *Z*: average elevation of ground surface from datum, z_r : average elevation of channel bed with respect to datum, z_s : average elevation of the BEW with respect to datum, y_s : average thickness of saturated zone, L_h : averaged horizontal length of one side of REW.







Fig. 2. An example of the decomposition of the flow duration curve normalized by mean annual daily flow $Q_{\rm m}$. "nQ", " $nQ_{\rm s}$ ", " $nP_{\rm ss}$ ", "nP" are normalized duration curves of daily flow, daily surface flow, daily subsurface flow, and daily precipitation; "nMQ" is flow duration curve associated with the regime curve – ensemble averaged mean within-year daily variation normalized by mean annual daily flow $Q_{\rm m}$). Dryness index is 0.5, and seasonal peaks of precipitation and potential evapotranspiration are in phase making a humid summer climate. Soil type is set as silty loam. Soil depth is 8 m. Surface gradient is 0.006.







Fig. 3. Effect of dryness index *R* on the FDC for different types of climatic seasonality: (a), (b): total flow FDC, (c), (d): surface flow FDC, (e), (f): subsurface flow FDC. (a), (c), (e): seasonal peaks of *P* and PET are in phase, (b), (d), (f): seasonal peaks of *P* and PET are of opposite phase. The numbers after *R* are dryness indices. Q_m is mean annual daily flow (mm d⁻¹). "nMQ" refers to FDC associated with the regime curve – ensemble averaged mean within-year daily variation normalized by mean annual daily flow Q_m . Default value of soil type is silty loam, soil depth is 8 m, and the topographic gradient is 0.006.







Fig. 4. Effect of soil type for different climatic seasonality: (a), (b): total flow FDC, (c), (d): surface flow FDC, (e), (f): subsurface flow FDC. (a), (c), (e): seasonal peaks of *P* and PET are in phase, (b), (d), (f): seasonal peaks of *P* and PET are of opposite phase. "SAL" and "SAN" indicate silty loam and sand. Q_m is mean annual daily flow (mm d⁻¹). "nMQ" refers to FDC associated with the regime curve – ensemble averaged mean within-year daily variation normalized by mean annual daily flow Q_m . Default value of climate dryness is R = 0.5 (humid), soil depth is 8 m and a topographic gradient is 0.006.







Fig. 5. Effect of soil depth for different climatic seasonality: (a), (b): total flow FDC, (c), (d): surface flow FDC, (e), (f): subsurface flow FDC. (a), (c), (e): seasonal peaks of *P* and PET are in phase, (b), (d), (f): seasonal peaks of *P* and PET are of opposite phase. The "6 m" and "8 m" indicate soil depth for each experimental case. Q_m is mean annual daily flow (mm d⁻¹). "nMQ" refers to FDC associated with the regime curve – ensemble averaged mean within-year daily variation normalized by mean annual daily flow Q_m . Default value of climate dryness is R = 0.5 (humid), soil type is silty loam, and topographic gradient is 0.006.







Fig. 6. Relationships between FDCs of the outflow OF and subsurface flow Q_{ss} normalized by mean annual daily flow Q_m : (a) seasonal peaks of *P* and PET are both in phase but dryness indices are 0.5 and 1.5; (b) seasonal peaks of *P* and PET are both of opposite phase and dryness indices are 0.5 and 1.5; (c) seasonal peaks of *P* and PET are both in phase but soil depths are 6 m and 8 m; and (d) seasonal peaks of *P* and PET are both of opposite phase but soil depth is 6 m and 8 m. The difference between Q_m and OF is equal to the evapotranspiration from the saturated fraction of the ground surface. Default value of soil type is silty loam, soil depth is 8 m, and the topographic gradient is 0.006.















Fig. 8. A conceptual model for reconstruction of the flow duration curves in ungauged basins, consisting of models of (i) partitioning of precipitation into fast (surface) runoff and wetting, which involves nonlinear (threshold filtering), (ii) the partitioning of the wetting into slow (subsurface) flow and evapotranspiration, which involves mainly linear filtering, and (iii) a correction to the FDC in flow situations due to evaporation from saturated areas.

