Hydrology and Earth System Sciences Discussions

Copula-based statistical refinement of precipitation in RCM simulations over complex terrain

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Abstract

This paper presents a new Copula-based method for further downscaling regional climate simulations. It is developed, applied and evaluated for selected stations in the alpine region of Germany. Apart from the common way to use Copulas to model the extreme values, a strategy is proposed which allows to model continous time series. In this paper, we focus on the positive pairs of observed and modelled (RCM) precipitation. As the concept of Copulas requires *independent and identically distributed* (*iid*) random variables, meteorological fields are transformed using an ARMA-GARCH time series model. The dependence structures between modelled and observed precipitation are conditioned on the prevailing large-scale weather situation. The impact of the altitude of the stations and their distance to the surrounding modelled grid cells is analyzed. Based on the derived theoretical Copula models, stochastic rainfall simulations are performed, finally allowing for bias corrected and locally refined RCM simulations.

1 Introduction

GCMs and RCMs are a central prerequisite for the conduction of climate change impact studies that require time series of climatic variables. The projections of future climate usually follow the so-called delta-change approach, considering the differences between present and future climate. If time series of RCMs are used directly as input for external impact models such as hydrological or agricultural models with nonlinear responses to the climate signal, the delta-change approach may fail (e.g., Graham et al., 2007; Sennikovs and Bethers, 2009). One reason is the spatial resolution which does not allow for local scale climate differences, particularly in complex terrain. Usually there exist tremendous biases between modelled and observed climate statistics (e.g. Schmidli et al., 2007; Smiatek et al., 2009). Therefore, further statistical refinement and bias correction methods are required to obtain reliable meteorological information at local scale (Wilby and Wigley, 1997). Precipitation is one of the most important

variables for climate change impact studies (Schmidli et al., 2006). At the same time it is the most difficult to model.

Statistical downscaling and bias correction methods can be divided into (i) direct point-wise techniques which relate specific (mostly adjacent RCM gridd cells) to a station of interest such as mean value adaptation (e.g. Kunstmann et al., 2004; Jung and Kunstmann, 2007), quantile mapping/histogram equalization methods (e.g. Leung et al., 1999; Wood et al., 2002; Themeßl et al., 2010; Senatore et al., 2011) or local intensity scaling (e.g. Schmidli et al., 2006; Yang et al., 2010), and (ii) indirect methods which relate meteorological fields to the station such as the analogue method (e.g. Bliefernicht and Bárdossy, 2007), weather or circulation pattern classification techniques (e.g. Bárdossy et al., 2002), or empirical orthogonal functions (e.g. von Storch and Zwiers, 1999).

The dependence structure of hydrometeorological data such as between modelled and observed rainfall is usually very complex, both in time and space. Using simple correlation of the multivariate normal is often not appropriate (Bárdossy and Pegram, 2009). It depends on the rainfall generating process, i.e. stratiform or convective events, and thus, on the season. For the mid-latitudes, large-scale stratiform events can be represented well by climate models resulting in a relatively good agreement between modelled (grid cell) and measured rainfall amounts (point scale). The models generally perform worse for convective events, which are highly variable in time and space. As a result, the discrepancies between modelled and observed rainfalls can be very large especially during the summer season with prevailing convective rainfall processes (e.g. Schmidli et al., 2007). Potential reasons for this are e.g. (i) the variability of observed rainfall within one corresponding modelled grid cell could be very high; rain gauges in the near surrounding can measure large differences in rainfall amounts; (ii) difficulties to capture the location of convective rainfall events by the climate models; this is mostly due to coarse resolution of the land surface model (LSM) and the partly chaotic nature of convection, and (iii) wrong or inadequate model parametrisations for convection.

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The dependence structures of multivariate distributions can be modelled using classical distributions such as a multivariate normal. Alternatively, a Copula approach can be used to describe the dependence structure independently from the marginal distributions (e.g. Genest and Favre, 2007; Dupois, 2007), and thus, to use different
5 marginal distributions at the same time without any transformations.

There is an increase in applications of Copulas in hydrometeorology over the past years. Copula-based models have been introduced for multivariate frequency analysis, risk assessment, geostatistical interpolation and multivariate extreme value analyses (e.g. De Michele and Salvadori, 2003; Bárdossy, 2006; Genest and Favre, 2007; Re-
10 nard and Lang, 2007; Schölzel and Friederichs, 2008; Bárdossy and Li, 2008; Zhang and Singh, 2008). For rainfall modelling, De Michele and Salvadori (2003) used Copulas to model intensity-duration of rainfall events. Favre et al. (2004) utilized Copulas for multivariate hydrological frequency analysis. Zhang and Singh (2008) carried out a bivariate rainfall frequency analysis using Archimedean Copulas. Renard and Lang
15 (2007) investigated the usefulness of the Gaussian Copula in extreme value analysis. Kuhn et al. (2007) employed Copulas to describe spatial and temporal dependence of weekly precipitation extremes. Serinaldi (2008) studied the dependence of rain gauge data using the non parametric Kendall's rank correlation and the upper tail dependence coefficient (TDC). Based on the properties of the Kendall correlation and TDC,
20 a Copula-based mixed model for modelling the dependence structure and marginals is suggested. Recently, Copula-based models for estimating error fields of radar information are developed (Villarini et al., 2008; AghaKouchak et al., 2010a,b)

The intermittend nature of daily and sub-daily rainfall time series (zero-inflated data) can be modelled by mixed distributions, which are distributions with a continuous part
25 describing data larger than zero and a discrete part accounting for probabilities to observe zero values (Serinaldi, 2009). While the univariate form of such distributions was studies by many authors, the bi- or multivariate case received less attention (Serinaldi, 2009).

Conventional rainfall models operate under the assumption of either constant variance or season-dependent variances using an *AutoRegressive Moving Average* (ARMA) model. However, it could be shown that daily rainfall data are affected by non-linear characteristics of the variance often referred to as variance clustering or volatility, in which large changes tend to follow large changes, and small changes tend to follow small changes. This nonlinear phenomenon of the variance behaviour can be found e.g. in monthly and daily streamflow data (Wang et al., 2005), but also in meteorological time series such as temperature (Romilly, 2005). It is analyzed in this paper if volatility in daily precipitation series can be modelled using *Generalized AutoRegressive Conditional Heteroskedasticity* (GARCH) models.

This paper addresses the questions of (i) how to model the temporal characteristics, i.e. serial dependence and time varying variance (volatility) of daily rainfall series, and (ii) how to describe the complex joint dependence structure of measured daily rainfall series and corresponding simulated rainfall series obtained from a RCM model. The method of choice in this paper is the bivariate Copula. The dependence structure is investigated for each observation station separately.

The innovation of this paper mainly is:

– Application of an ARMA-GARCH algorithm to analyze daily precipitation time series for seasonal variation and volatility, and to generate *independent and identically distributed* (hereinafter *iid*) residuals for the Copula approach.

– Description and modelling of the joint dependence structure between RCM modelled and observed precipitation, accounting for the prevailing flow situations caused by large-scale circulation patterns.

2 Regional climate model simulations and observed data

Regional climate simulations used in this study are based on the Penn State/NCAR Mesoscale Model (MM5) and ECMWF/ERA15 reanalysis data for 1979–1993 at

19.2 km spatial resolution. The climate simulations have been carried out within the framework of the QUIRCS-DEKLIM project (Kotlarski et al., 2005). A comparison of the MM5 simulations with gridded observation data for Germany obtained from the German weather service (DWD) reveal that rainfall is overestimated by MM5 for the eastern part of Germany, and strongly underestimated for the Rhine valley and the alpine region of Germany (Fig. 1, bottom). The underestimation in the alpine region is possibly due to the complex terrain with strong gradients of altitude (Fig. 1, top).

In order to statistically refine and correct precipitation obtained by RCM (MM5) climate simulations, daily rainfall data of 132 observation stations within the alpine region of Germany are retrieved from the webwerdis data portal of the DWD. Our study focusses on the alpine subregion round Garmisch-Partenkirchen. For the analysis of the dependence structure between modelled and observed precipitation, a subset of 14 observation stations with large altitudinal differences is selected (see Fig. 2 and Table 1). These stations correspond to three different grid cells of the RCM output where the model bias is comparatively large.

3   Modelling the dependence structure between modelled and observed rainfall

The procedure followed in this paper to model the dependence structure between RCM modelled and observed rainfall, and to finally generate random samples of locally refined and bias corrected pseudo-observations, requires multiple steps (Fig. 4) that can be comprised as follows:

1. A suitable ARMA-GARCH model is fitted to the modelled and observed rainfall series (positive values only) to capture the seasonal variation of variance (see Sect. 3.1.1) of both RCM simulated and station observed precipitation.

2. The marginals are fitted to semi-parametric Generalized Pareto Distributions (GPD) for an improved representation of the tails (see Sect. 3.1.2).

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3. The bivariate empirical Copula (bivariate probability density plot), which is independend from their corresponding marginal distributions, is derived from the residuals of the ARMA-GARCH model.

4. Using the marginal distributions of the original data (positive pairs only) and the *iid* residuals obtained from the ARMA-GARCH model, a theoretical Copula model is estimated (see Sect. 3.2.2).

5. Stochastic simulations are performed using the conditional CDF of the theoretical Copula (see Sect. 3.2.3).

6. Stochastic simulations are performed using conditional CDFs of the theoretical Copula of different large-scale weather patterns (see Sect. 3.3).

3.1 Modelling the marginals

Modelling the single marginal distributions requires the observations to be *iid*. Most climatological time series, however exhibit some degree of autocorrelation and heteroskedasticity. In the sequel the ARMA-GARCH composite model to generate *iid* variables is introduced, followed by the description of how to fit a GPD to the marginals, and to derive a joint distribution function (Copula) to model the dependence between modelled and observed rainfall time series.

3.1.1 ARMA-GARCH filter

This section describes briefly the theory of the ARMA/GARCH composite model and how it is used to simulate the univariate time series in the presence of conditional mean as well as conditional time-varying variance, i.e. heteroskedasticity or volatility, to produce *iid* residuals. An ARMA model is used to compensate for autocorrelation, and a GARCH model to compensate for the heteroskedasticity.

The term *conditional* in GARCH – Generalized Autoregressive Conditional Heteroskedasticity – implies explicitly the dependence on a past sequence of

observations, and *autoregressive* describes a feedback mechanism that incorporates past observations into the present. GARCH is a time series modelling technique that includes past variances for predicting present or future variances.

A univariate model of an observed time series y_t can be written as

$$y_t = f(t - 1, X) + \varepsilon_t \tag{1}$$

In this equation, the term $f(t - 1, X)$ represents the deterministic component of the current value as a function of the information known at time $t - 1$, including past innovations $\{\varepsilon_{t-1}, \varepsilon_{t-2}, ...\}$, past observations $\{y_{t-1}, y_{t-2}, ...\}$ and other relevant time series data X. Bollerlslev (1986) developed GARCH as a generalization of the ARCH volatility modelling technique (Engle, 1982). The distribution of the residuals, conditional on the time t, is given by

$$\text{Var}_{t-1}(y_t) = E_{t-1}\left(\varepsilon_t^2\right) = \sigma_t^2 \tag{2}$$

where

$$\sigma_t^2 = \kappa + \sum_{i=1}^{P} G_i \, \sigma_{t-i}^2 + \sum_{j=1}^{Q} A_j \, \varepsilon_{t-j}^2 \tag{3}$$

One can see that σ_t^2 is the prediction of the variance, given the past sequence of variance predictions, σ_{t-i}^2, and past realizations of the variance itself, ε_{t-j}^2. When $P = 0$, the GARCH(0,Q) model becomes the original ARCH(Q) model introduced by Engle (1982). This equation mimics the variance clustering of the variable (i.e. precipitation and temperature). The lag lengths P and Q and the coefficients G_i and A_j determine the degree of persistence.

A common assumption is that the innovations are serially independent, however, GARCH(P,Q) innovations, $\{\varepsilon_t\}$, are modelled as

$$\varepsilon_t = \sigma_t \, z_t. \tag{4}$$

σ_t is the conditional standard deviation given by the square root of Eq. (3), and z_t is the standardized *iid* random draw from some specified probability distribution. Usually, a Gaussian distribution is assumed such that $\varepsilon \sim N(0,\ \sigma_t^2)$. Reflecting this, Eq. (5) illustrates that a GARCH innovations process $\{\varepsilon_t\}$ simply rescales an *iid* process $\{z_t\}$ such that the conditional standard deviation incorporates the serial dependence of Eq. (3).

3.1.2 Generalized pareto distribution

This subsection describes the fitting of semi-parametric cumulative distribution functions (CDFs). First, the empirical CDF of each parameter is estimated using a Gaussian kernel function (using a kernel width of 50 points) to eliminate the staircase pattern. This provides a reasonably good fit to the interior of the distribution of the residuals. This procedure, however, tends to perform poorly when applied to upper and lower tails.

The upper and lower tails therefore are fitted separately from the interior of the distribution. For this reason, the *peaks over threshold* (POT) method is applied: A threshold value of 0.1 is chosen, i.e. the upper and lower 10% of the residuals are reserved for each tail. The extreme residuals (beyond the threshold) are fitted to a parametric GPD, which can be described as

$$y = f(x|k,\ \sigma,\ \theta) = \left(\frac{1}{\sigma}\right)\left(1 + k\left(\frac{(x-\theta)}{\sigma}\right)\right)^{-1-\frac{1}{k}} \tag{5}$$

using a maximum likelihood approach. Given the exceedances in each tail, the negative log-likelihood function is optimized to estimate the tail index/shape parameter k and the scale parameter σ of the GPD. The composite GPD function allows for interpolation in the interior of the CDF but also for extrapolation in the lower and upper tails.

3.2 Copula based joint distribution functions of modelled and observed rainfall

Copulas are functions that link multivariate distribution $F(x_1, \ldots x_n)$ to their univariate marginals $F_{X_i}(x_i)$. Sklar (1959) proved that every multivariate distribution $F(x_1, \ldots x_n)$ can be expressed in terms of a Copula C and its marginals $F_{X_i}(x_i)$:

$$F(x_1, \ldots x_n) = C\left(F_{X_1}(x_1), \ldots, F_{X_n}(x_n)\right) \tag{6}$$

$$C : [0, 1]^n \to [0, 1]. \tag{7}$$

Copulas allow to merge the dependence structure and the marginal distributions to form a joint multivariate distribution. The Copula function is unique when the marginals are steady functions. As the Copula is a reflection of the dependency structure itself, its construction is reduced to the study of the relationship of the correlated *iid* variables, giving freedom for the choice of the univariate marginal distributions. Further information about Copulas can be found e.g. in Joe (1997); Frees and Valdez (1998); Nelson (1999); Salvadori et al. (2007).

The Copula approach allows to account for the fact that the dependence structure between regional and local meteorological fields and between simulated and observed fields is more complex than it can be modelled by the multivariate normal distribution or ordinary dependency measures such as e.g. the Pearson correlation coefficient. The complex multivariate dependence structure is analyzed between RCM modelled precipitation (MM5 output) and station observed precipitation.

As there exist no unique characterization of the Copula for dry days, our work focusses on the positive pairs (RCM precipitation > 0, observed precipitation > 0). For dry days, only the conditional marginals can be identified (Yang, 2008).

3.2.1 Empirical Copula

The dependence structure of daily measured precipitation and simulated precipitation is studied. Since the underlying (theoretical) Copula is not known in advance, it is

necessary to analyze the empirical Copula, which is purely based on the data (Deheuvels, 1979). The ranks of the residuals of modelled and observed rainfall from day 1 to day n, obtained from the original data as well as the ARMA-GARCH time series model, are $\{r_1(1), ..., r_1(n)\}$ and $\{r_2(1), ..., r_2(n)\}$, respectively. The empirical Copula is defined as:

$$C_n(u,v) = 1/n \sum_{t=1}^{n} 1\left(\frac{r_1(t)}{n} \leqslant u, \frac{r_2(t)}{n} \leqslant v\right) \tag{8}$$

where u indicates the percentile of the modelled rainfall residuals, v indicates the percentile of the measured rainfall residuals and 1(...) is denoting the indicator function.

3.2.2 Estimation of the theoretical Copula

A goodness-of-fit test for Copulas is applied comparing the empirical Copula C_n (Eq. 8) with the parametric estimate of a theoretical Copula model C_θ derived under the null hypothesis. The test is based on the Cramér-von Mises statistic (Genest and Favre, 2007):

$$S_n = n \sum_{t=1}^{n} \{C_\theta(u_t, v_t) - C_n(u_t, v_t)\}^2. \tag{9}$$

As the definition of S_n involves the theoretical Copula function, the distribution of this statistic depends on the unknown value of θ under the null hypothesis that C is from the class C_θ (Grégoire et al., 2008). Therefore, the approximate p-values for the test statistic are obtained using a parametric bootstrap (Genest and Remillard, 2008; Genest et al., 2009) as well as a fast multiplier approach (Kojadinovic and Yan, 2011,?).

3.2.3 Copula-based rainfall simulations

After the estimation of the Copula-based joint distribution – that is $F_X(x)$, $F_Y(y)$ and $C_\theta(u,v)$ are obtained – conditional random samples from this distribution are generated

through Monte Carlo simulations. We follow the procedure of Salvadori for the conditional simulation using Copulas Salvadori et al. (2007). The simulation is based on conditional probabilities of the form:

$$P(V \leq v|U = u) = \frac{\partial}{\partial u} C(u, v); \tag{10}$$

$$P(U \leq u|V = v) = \frac{\partial}{\partial v} C(u, v). \tag{11}$$

For the Gumbel-Hougaard Copula e.g. it is:

$$\frac{\partial}{\partial u} C(u,v) = u^{-1} e^{-(\log(u)^\theta) + (\log(v)^\theta)^{1/\theta}} (-\log(u))^{-1+\theta}((-\log(u)^\theta) + (-\log(v))^\theta)^{-1+1/\theta}. \tag{12}$$

The concept pseudo-observation simulation from model data is as follows: a pair of variates (u, v) with Copula $C(u, v)$ needs to be generated which finally can be transformed into (x, y), using the probability integral transformation

$$U = F_X(x) \Leftrightarrow X = F_X^{-1}(U) \tag{13}$$

$$V = F_Y(y) \Leftrightarrow Y = F_Y^{-1}(V). \tag{14}$$

The complete algorithm is divided into three steps:

1. Computation $u = F_X(x)$, where x denotes one value of the modelled rainfall and $F_X(x)$ is the marginal distribution of the variate X.

2. Generation of random samples for the variate v^* from the conditional CDF $C_{V|U}(v|u) = c_u(v)$ and calculation of $v = c_u^{-1}(v^*)$, where c_u^{-1} denotes the generalized inverse of c_u (Nelsen, 1999).

3. Calculation of the corresponding y-values using the probability integral transformation $F_Y^{-1}(v) = y$.

The final result for y is a sample of pseudo-observations which lies in the original data space and can be compared with the observed data series.

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3.3 Usability of weather patterns for conditional simulations

Especially for complex terrain, it is assumed that the direction of advection is of crucial importance for the observed precipitation amounts. The combinations of terrain exposition and advection direction leads to luv and lee side effects, i.e. the stations can lie in the rainshadow or can be exposed to intense rainfall. As independent from the RCM simulations, large-scale weather patterns are used to further improve the results of the bias correction. Besides the advection direction, large scale information about cyclonality and tropospheric humidity is evaluated. The objective weather pattern classification method of the German Weather Service is used (Bissolli and Dittmann, 2001). The classification domain is Germany, and the meteorological criteria for the classification are (i) the direction of advection of air masses, (ii) the cyclonality, and (iii) the humidity of the troposphere. This leads to numerical indices from which the weather types are derived (Bissolli and Dittmann, 2001). There exist 40 predefined types, which can be used. Due to the limited occurrence frequencies of single weather types, their usability for conditional simulations is restricted. However, the usage of the numerical indices provides the possibility to group the types to different classes.

For this study, the following grouping strategies are evaluated:

1. Grouping types due to the *direction of the advection of air masses at 700 hPa*: the weather types (WTs) are grouped into northeasterly, southeasterly, southwesterly, and northwesterly flow.

2. Grouping types due to the *cyclonality at 950 hPa and 500 hPa*: this leads to four classes, namely anticyclonal – anticyclonal (AA), anticyclonal – cyclonal (AC), cyclonal – anticyclonal (CA), and cyclonal – cyclonal (CC).

3. Grouping types due to the *humidity of the troposphere*: this leads to the discrimination of dry (D) and wet (W). Therefore, a humidity index is calculated as the weighted areal mean of the precipitable water integrated over the 950, 850, 700, 500, and 300 hPa levels.

For each group of weather types, a theoretical Copula model is estimated separately. For sake of simplicity, the Gumbel-Hougaard Copula model is used.

4   Simulation results

In this section simulation results of both, the obtained RCM and corresponding observed precipitation time series are exemplarily presented. Based on *iid* residuals obtained by ARMA-GARCH models the empirical and theoretical Copulas, and the marginal distributions are estimated and analyzed and locally refined and bias corrected pseudo-observations are generated.

4.1   Analysis of ARMA-GARCH time series models

The autocorrelation function and Ljung-Box Q-test is applied to the original time series, the squared original time series as well as the resulting standardized residuals and standardized squared residuals of the ARMA-GARCH model. According to the autocorrelation function plots the original time series show serial dependence and heteroskedasticity, i.e. non-*iid* behaviour (Fig. 3). After application of the ARMA-GARCH model, the time series of the residuals can be seen as serially independent (Fig. 5). The Ljung-Box Q-test confirms the results of the autocorrelation function (not shown here).

The K-plots indicate that there remains a positive dependence in the upper tails of the residuals (Fig. 6). Further information about how to calculate and to interpret the K-plots can be obtained e.g. by Genest and Favre (2007). Therefore, a sensitivity analysis is conducted accounting for the order of ARMA-GARCH models (using orders for AR, MA, P, and Q between 1 and 3, respectively), the threshold value for a wet day (0.01 mm, 0.1 mm, and 1 mm), and the peak-over-threshold (POT) value for lower and upper tail (10 % and 20 %). It is found that the larger the wet day threshold, the higher is

the distortion of the upper tail, which can be partly explained by the fat tail behaviour. The POT and the order of the ARMA-GARCH are less sensitive.

Figure 7 (top) shows the empirical and fitted exceedance probability for the upper tail of the observed rainfall residuals at station Garmisch-Partenkirchen. Both, for observed and modelled rainfall, the *Generalized Pareto Distribution* seems to be a good choice to fit the upper tails of the data. Figure 7 (bottom) illustrates the composite of the three piecewise CDFs for modelled and observed rainfall residuals.

4.2   Analysis of empirical and theoretical Copula models

Figure 8 (top) shows the empirical Copula density between modelled and measured rainfall for station Garmisch-Partenkirchen. Only the positive pairs of modelled and measured rainfall are shown, using a threshold of 0.01 mm to define a wet day. It can be seen from the figure that the distribution is strongly asymmetrical for the minor diagonal, and that the density in the upper corner is highest. This implies that modelled and observed rainfall are strongly concordant in the higher ranks of the distribution, whereas the concordance is weaker in the lower ranks. This empirical density structure may be remarkably different compared to the ARMA-GARCH transformed residuals (Fig. 8, bottom).

Table 2 shows the results for the goodness-of-fit (GOF) test statistics using the parametric bootstrap procedure. In order to chose between the three different Copula families, namely Normal, Gumbel-Hougaard, and Clayton Copula, the parametric bootstrap algorithm of Genest and Remillard (2008) is applied to observed and modelled precipitation. 1000 bootstrap values of the Cramér-von-Mises test statistic are produced, and the proportion of those values that are larger than S_n (p-values) is estimated. From the p-values obtained the usability of the Gumbel-Hougaard Copula is concluded. The Copula parameters which are used for Copula-based stochastic simulations are also given in Table 2.

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4.3 Dependence on altitude and distance

The dependence of the altitude is shown in Table 1. The variability of the simulated results (shown as empirical CDFs) are obviously not related to differences in the Copula parameters, just weak differences of θ can be found (see Table 2). No clear functional dependence between the altitude of the stations and the Copula parameter θ exists. Additionally, neither shape parameter nor scale parameter of the marginal distributions are systematically differing with the station altitude. At least one could suspect that the scale parameter for the observed precipitation is increasing with height, but there are just three grid cells available for this inspection which is clearly not significant. The dependence of the distance between observed rainfall and modelled rainfall of the corresponding and surrounding RCM grid cells is analyzed using the Kendall's τ (Fig. 17). In general, higher coefficients are obtained for nearest grid cells, and the correlations decrease anisotrophically around the stations. Moreover, the pattern shown in this figure mimics well the regional topography given in the RCM model. In some cases, the corresponding grid cell does not show the highest correlation.

There is a functional relationship between the classical dependence parameters such as Kendall's τ and Spearman's ρ namely

$$\rho = 12 \iint_{[0,\,1]^2} u\,v\,dC_\theta\,(u,\,v) - 3 = 12 \iint_{[0,\,1]^2} C_\theta\,(u,\,v)\,du\,dv - 3 \tag{15}$$

and

$$\tau = 4 \iint_{[0,\,1]^2} C_\theta\,(u,\,v)\,dC_\theta\,(u,\,v) - 1 \tag{16}$$

or for Archimeadean Copulas with generator φ

$$\tau = 1 + 4 \int_{[0,\,1]} \frac{\varphi(t)}{\varphi'(t)}. \tag{17}$$

For the Gumbel-Hougaard Copula with its generator $\varphi(t) = (-\ln(t))^\theta$ it is found that $\theta = \frac{1}{1-\tau}$, so θ is a increasing function of τ. Thus the "Copula map" shown in Fig. 17

may be interpreted in the light of dependence. Higher Copula parameters θ reveal a stronger dependence.

4.4 Dependence of large-scale weather situation

The dependence structure between modelled and observed rainfall, given the large-scale weather situation, is analysed. The method used for classifying large-scale weather types is described in Sect. 3.3. The empirical Copulas are calculated using different grouping strategies for the WTs. Based on the empirical Copulas, as well as the conditional CDFs, the usability for conditional simulations is investigated.

Using four different weather types and one indefinite type for advection (Fig. 10) can have additional value, and thus be used for conditional stochastic simulations. Figure 11 illustrates the empirical Copula density for modelled and observed precipitation for Garmisch-Partenkirchen grouping the weather types due to the cyclonality in 950 hPa and 500 hPa into four classes. One can observe that for the four classes significant differences within the dependence structure between modelled and observed rainfall exist. The classification due to the humidity of the troposphere (Fig. 12) does not lead to a clear discrimination between the empirical Copulas. Both, the wet and the dry Copula density is similar to the unconditional Copula densities (compare with Fig. 8). The empirical CDFs of observed precipitation in Garmisch-Partenkirchen based on a given WT and certain groups of WTs are illustrated in Fig. 13.

4.5 Conditional stochastic simulations of pseudo-observations

Figure 9 shows the results of Copula-based stochastic simulations (100 realizations) of pseudo-observations asssuming that the modelled RCM precipitation is given. A split-sampling approach is used to subdivide the data into calibration and validation period. It can be seen from the figure that the observations are usually underpredicted by the model, and that the Copula-based technique can partly correct for that effect. For the very high RCM rainfall amounts, the Copula-based approach tends to overestimate

the observations. After this first graphical comparison the improvements attained using the Copula approach are analyzed further with selected performance measures. A first hint is given by the correlations between the observations, RCM and the pseudo-observations i.e. the bias corrected prediction (see Table 3). The Pearson correlation
5 coefficient between observations and RCM is calculated as 0.3 for the *iid* transformed data of Garmisch-Partenkirchen, and between observations and the mean value of the random sample, generated through the Copula approach, it is 0.36. This corroborates the usability of the Copula based bias correction of precipitation. Including large-scale information about advection, cyclonality, and humidity is increasing the correlation co-
10 efficient between observations and pseudo-observations. The correlation between the RCM and the pseudo-observations is higher. However, the Pearson correlation is just a general measure, operating on the complete time series, and does not mirror the quality of the new method for specific subsets of the rank space. In turn, the probability plot (Fig. 15) provides a performance measure for the quantiles of the distribution.
15 It can be seen from Fig. 15 that the RCM underestimates the observations over the whole range of the distribution. Taking this as a reference, the Copula-based stochastic simulations of the pseudo-observations lead to significant improvements. However, the simulations still suffer from fat tail characteristics. Including large-scale conditional information contributes moderately to a reduction of the bias. Including information
20 about humidity of the troposphere, the performance skill is comparable to the Copula-based stochastic simulations without any large-scale information.

5 Discussion

It is shown that ARMA-GARCH models are able to model serial dependence and volatility in the precipitation time series. Consequently, they are generally useful to generate
25 *iid* random variables. However, even high order models are not able to fully capture the fat tail behaviour. Filtering the time series before fitting to a theoretical Copula model

is reducing the correlation between RCM (here: MM5) and observed precipitation and thus the estimated Copula parameter θ, which is directly related to Kendall's τ.

From the theoretical Copula models analyzed, the Gumbel-Hougaard Copula is found to be a suited choice to model the joint distribution of modelled (gridded) and observed precipitation. While the Copula parameter is relatively stable for the joint distribution functions between different locations within the same grid cell, the shape and scale parameters of the fitted marginal distributions of the observation stations can differ significantly. It is also found that the correlation between station and grid cell is not necessarily highest for the corresponding grid cell. Assessing the dependence of the distance between observed rainfall and rainfall of the surrounding RCM grid cells potentially allows for spatial interpolation in ungauged regions. However, further investigations will be necessary.

The empirical Copula density plot is used to analyze the dependence structure between modelled and observed precipitation. As computational inexpensive they are suitable to (i) find a theoretical Copula model, and (ii) screen variables such as e.g. large-scale weather types which could additionally improve the performance of the bias correction.

The objective weather pattern classification method of the German Weather Service (Bissolli and Dittmann, 2001) shows only moderate potential to further constraint the model. Including information about the humidity of the troposphere can slightly increase the skill for bias correction compared to the Copula-based stochastic simulations without using large-scale information. This can be seen from the conditional CDFs and the corresponding probability plots for the different groups, which are not very discriminative (compare Sects. 3.3 and 4.4). Using the single 40 weather types (without grouping) could potentially increase the discriminative power (see e.g. Fig. 16), but decreases the sample size for certain WTs far too much to reliably estimate the Copula parameter(s) and the marginal distributions. Another limitation of the approach shown in this paper is that the same theoretical Copula model, i.e. the Gumbel-Hougaard Copula, is fitted to each WT class. It is obvious from the empirical Copula density plots,

that this does not necessarily provide an adequate fit for all groups of weather types. It is also well-known from other studies that the domain size (here: whole Central Europe domain) can strongly impact the classification results (e.g. Laux, 2009), and thus the subsequent conditional modelling.

In this study a stationary approach for the Copula parameter θ is chosen. Further improvements are expected by accounting for the temporal variability of θ. Figure 17 illustrates the temporal variability of τ, which is empirically linked with the Copula parameter. As seen in Sects. 4.2 and 4.4, all the empirical Copulas derived in this study show a strong asymmetry with respect to the minor axis $u = 1 - v$ of $[0,\ 1]^2$. This asymmetry can not be depicted by the common Copula families such as the Clayton, Normal or Gumbel-Hougaard Copulas, acting as basic set of possible candidates for the performed GOF tests. Nonmonotonic transformation to construct asymmetric multivariate Copulas from the Gaussian (Bárdossy, 2006) could reflect the asymmetries in the empirical Copulas and thus also improve the bias correction.

6   Conclusions

The presented Copula-based approach is potentially useful for statistical downscaling, bias correction, and local refinement of RCMs. The performance will be evaluated and compared to established methods for bias correction.

Asymmetries are found in the empirical Copula densities which cannot be reproduced by the theoretical Copulas used in this study. Therefore, it is generally difficult to find a theoretical Copula model which is not rejected by the applied GOF test.

Fitting the marginal distributions is of crucial importance as it strongly impacts the simulation results (more than the Copula parameter θ).

Large-scale weather patterns could be used to further constrain the model, and thus, increase the performance of the simulation results.

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Table 1. Station informations corresponding to 3 chosen MM5 grid cells, shape (tail index), and scale parameters of the fitted *Generalized Pareto Distribution* (GPD) for positive pairs of modelled and observed precipitation respectively.

Station	Altitude [m a.s.l.]	Location lat	lon	Precipitation (MM5 output) Shape	Scale	Precipitation (observed) Shape	Scale
Garmisch-Partenkirchen	719	47.48	11.06	0.42	0.34	0.094	8.29
Grainau	760	47.47	11.02	0.29	0.40	0.13	6.757
Grainau (Eibsee)	1010	47.46	11.00	0.28	0.42	0.06	8.34
Bad Reichenhall	470	47.72	12.88	0.07	0.78	0.26	6.98
Schneizlreuth-Unterjettenberg	507	47.68	12.83	0.07	0.82	0.10	8.29
Schneizlreuth-Ristfeucht	523	47.67	12.77	0.08	0.80	0.17	8.06
Schneizlreuth-Weissbach	630	47.72	12.77	0.08	0.80	0.14	8.67
Anger-Oberhögl	690	47.8	12.9	0.06	0.81	0.28	5.50
Bischofswiesen-Winkl	690	47.69	12.94	0.07	0.82	0.13	8.20
Inzell	690	47.76	12.76	0.05	0.84	0.06	10.22
Anger-Stoissberg	830	47.8	12.82	0.08	0.77	0.32	7.00
Rottach-Egern	747	47.68	11.77	0.05	0.76	0.15	7.93
Kreuth	895	47.61	11.65	0.04	0.77	0.16	9.65
Schwarzkopfhütte	1336	47.66	11.91	0.06	0.74	0.09	10.09

Table 2. Goodness-of-fit (GOF) test using the Cramér-von-Mises test statistics and parametric bootstrap procedure (see Sect. 3.2.2). p-values exceeding 0.01 are highlighted in bold.

Station	Normal Copula			Gumbel-H. Copula			Clayton Copula		
	S_n	p-value	θ_N	S_n	p-value	θ_GH	S_n	p-value	θ_C
Garmisch-Partenkirchen	0.09	0.00	0.08	0.12	0.00	1.09	0.17	0.00	0.01
Grainau	0.10	0.00	0.09	0.15	0.00	1.10	0.16	0.00	0.01
Grainau (Eibsee)	0.07	0.00	0.12	0.08	0.00	1.11	0.12	0.00	0.06
Bad Reichenhall	0.10	0.00	0.19	0.04	**0.02**	1.15	0.37	0.00	0.13
Schneizlreuth-Unterjettenberg	0.14	0.00	0.15	0.10	0.00	1.13	0.29	0.00	0.08
Schneizlreuth-Ristfeucht	0.11	0.00	0.17	0.05	0.00	1.14	0.40	0.00	0.09
Schneizlreuth-Weissbach	0.08	0.00	0.13	0.06	0.00	1.11	0.21	0.00	0.07
Anger-Oberhögl	0.10	0.00	0.16	0.05	**0.01**	1.14	0.34	0.00	0.11
Bischofswiesen-Winkl	0.07	0.00	0.16	0.04	**0.07**	1.14	0.23	0.00	0.11
Inzell	0.08	0.00	0.16	0.05	0.01	1.13	0.24	0.00	0.13
Anger-Stoissberg	0.12	0.00	0.14	0.06	0.00	1.12	0.40	0.00	0.07
Rottach-Egern	0.05	0.00	0.17	0.03	**0.09**	1.14	0.21	0.00	0.13
Kreuth	0.06	0.00	0.12	0.06	0.00	1.10	0.17	0.00	0.06
Schwarzkopfhütte	0.05	**0.01**	0.15	0.03	**0.07**	1.12	0.22	0.00	0.12

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Table 3. Pearson correlation coefficients (all significant at $\alpha = 0.01$ level) between positive pairs of pseudo-observations (mean value) produced by Copula-based stochastic simulations without using large-scale information (uncond), including advection (advec), cyclonality (cyclo), and humidity (humi) of the troposphere, and the observed precipitation at station Garmisch-Partenkirchen and the corresponding grid cell precipitation of RCM.

	uncond	advec	cyclo	humi
observed	0.36	0.43	0.45	0.37
RCM	0.98	0.93	0.98	0.65
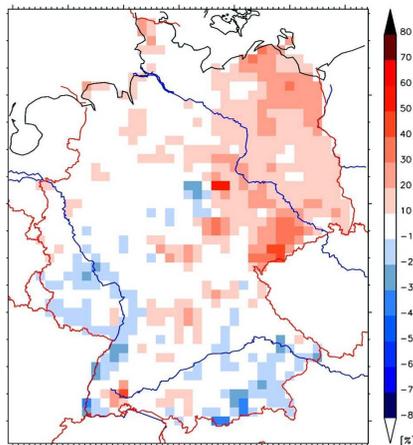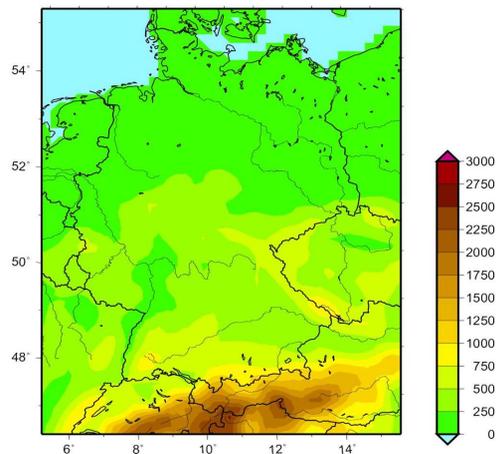
Fig. 1. Bias of mean annual total precipitation for the RCM (MM5) with respect to the DWD reference data set [%] (Kotlarski et al., 2005).
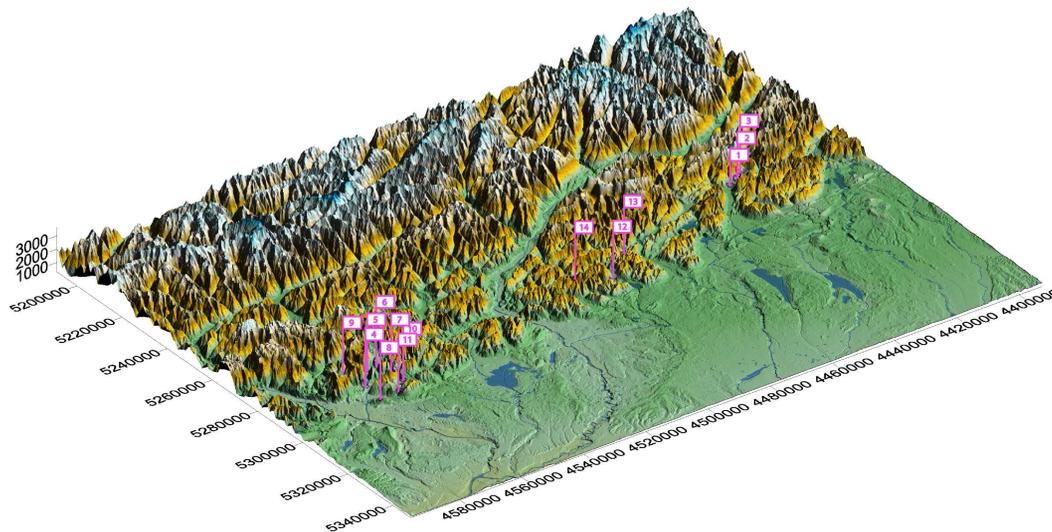
Fig. 2. Alpine region showing the location of observation stations used in this paper: 1 Garmisch-Partenkirchen, 2 Grainau, 3 Grainau (Eibsee), 4 Bad Reichenhall, 5 Schneizlreuth-Unterjettenberg, 6 Schneizlreuth-Ristfeucht, 7 Schneizlreuth-Weissbach, 8 Anger-Oberhögl, 9 Bischofswiesen-Winkl, 10 Inzell, 11 Anger-Stoissberg, 12 Rottach-Egern, 13 Kreuth, and 14 Schwarzkopfhütte.

Fig. 3. Autocorrelation function for precipitation (1979–1993) of Garmisch-Partenkirchen, Germany (top), and its corresponding squared time series (bottom).
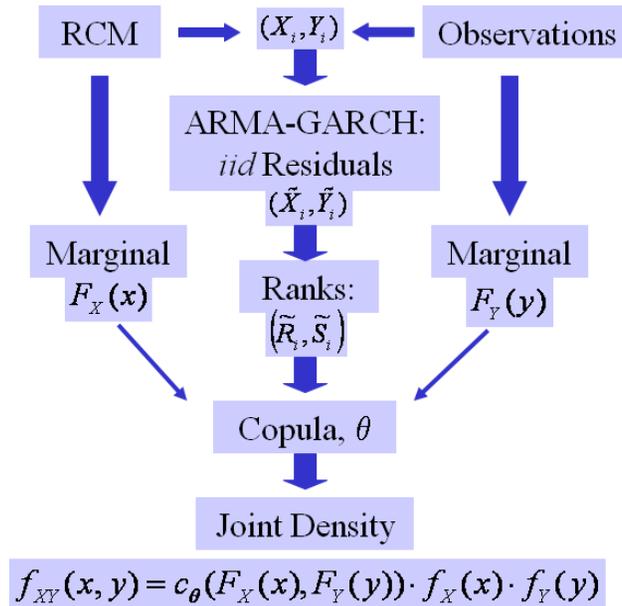
Fig. 4. Concept of the bias correction followed in this paper.
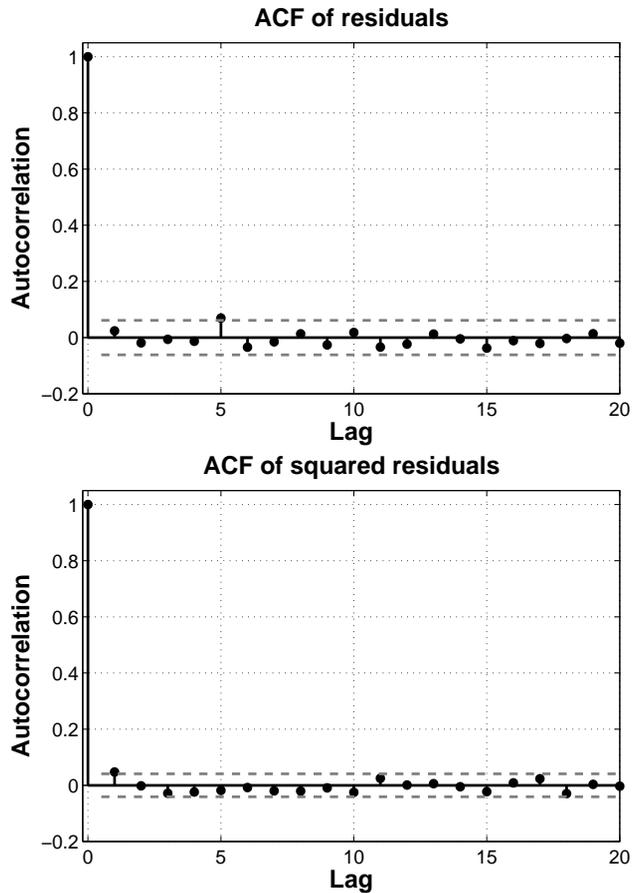
Fig. 5. Autocorrelation function for ARMA-GARCH residuals for precipitation (1979–1993) of Garmisch-Partenkirchen (top), and its corresponding squared residuals (bottom).

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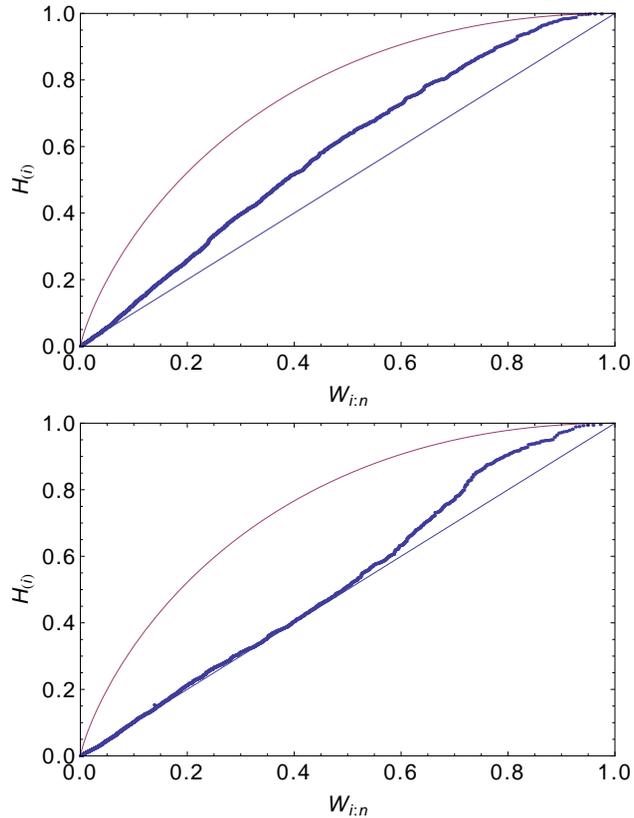
Fig. 6. K-plot of the observed rainfall time series at station Garmisch-Partenkirchen (Germany) before ARMA-GARCH transformation (top), and after ARMA-GARCH transformation (bottom). Superimposed on the graph are a straight line (blue) corresponding to the case of independence and a curve corresponding to perfect positive dependence (red).
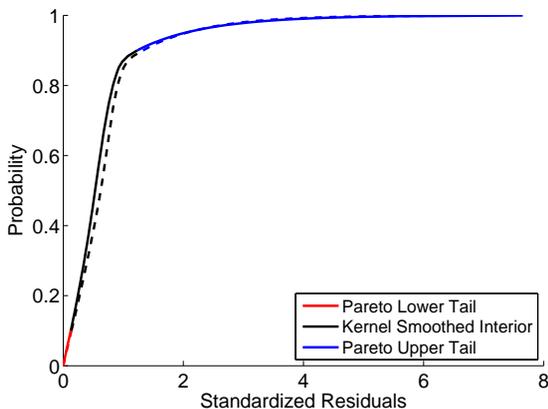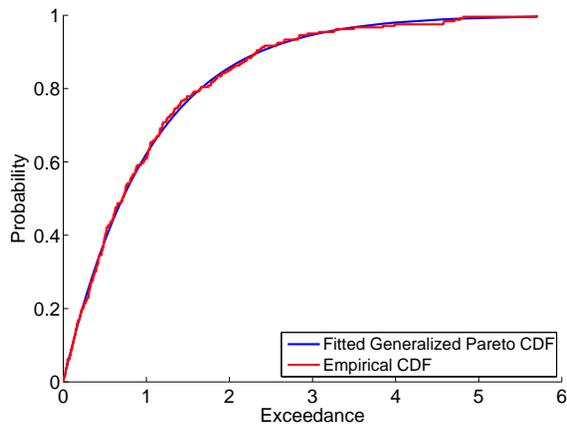
Fig. 7. Empirical and fitted exceedance probability of the upper tail of observed rainfall residuals (top), composite of the piecewise CDF of the modelled (solid lines) and observed (dashed lines) rainfall residuals at Garmisch-Partenkirchen (bottom).
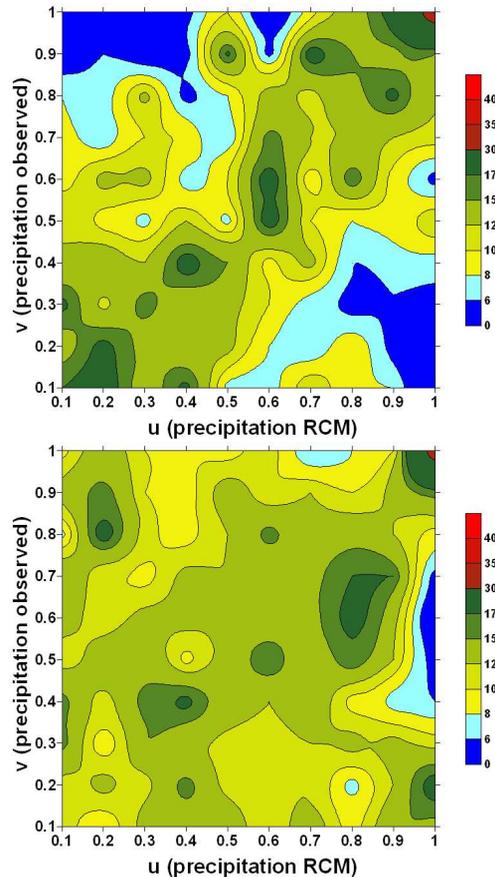
Fig. 8. Empirical Copula density for modelled (*u*) and observed precipitation (*v*) for station Garmisch-Partenkirchen, using the positive pairs of original data (top), and the positive pairs of the ARMA-GARCH transformed *iid* residuals (bottom).

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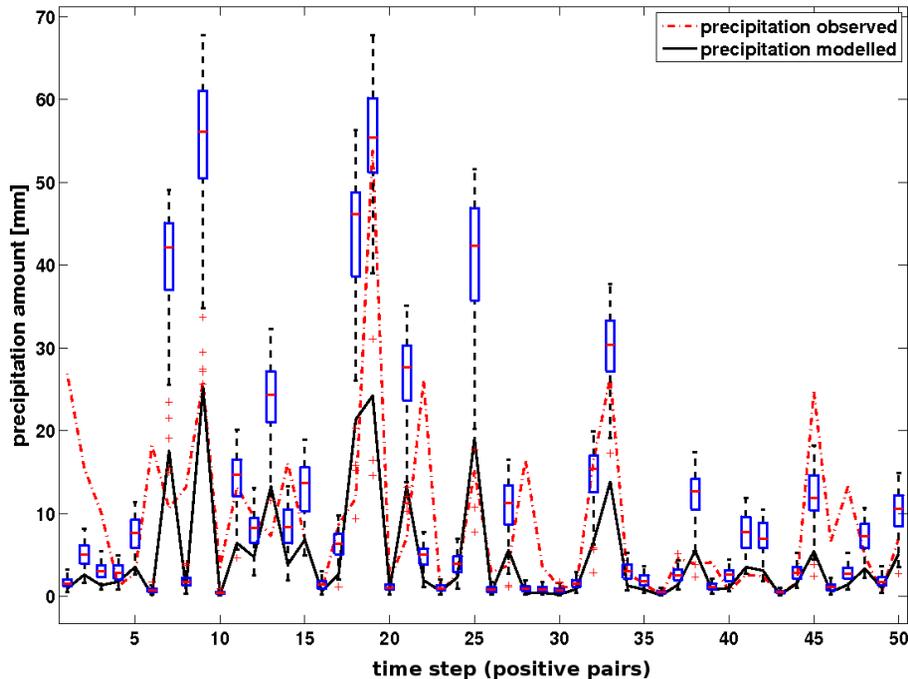
Fig. 9. Copula-based stochastic simulations of 50 consecutive positive pairs (precipitation > 0.01 mm) performing 100 realizations (illustrated as box-whiskers) of V (observed rainfall at gauge, illustrated as red line) asssuming that U (corresponding coarse scale MM5 precipitation, illustrated as black line) is known. The boxes have lines at the lower $Q1$ and upper quartile $Q3$ and the median values $Q2$ (middle horizontal lines). The whiskers (vertical lines) are lines extending from each end of the boxes to show the extent of the rest of the data. The maximum length of the whiskers is determined by 1.5 ($Q3$–$Q1$). Outliers (crosses) are data with values beyond the ends of the whiskers.

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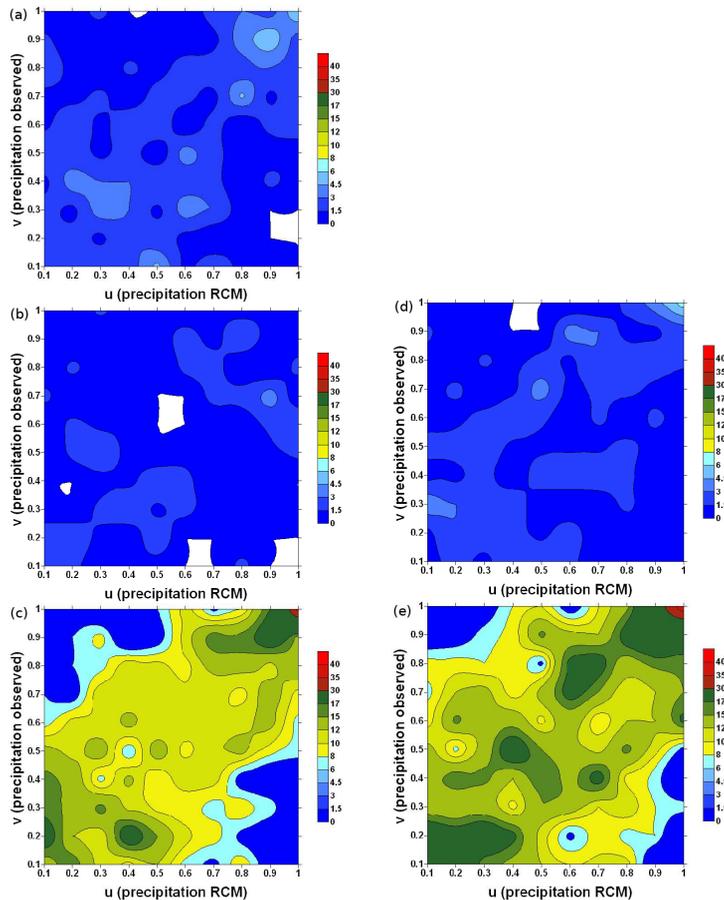
Fig. 10. Empirical Copula density for modelled (u) and observed precipitation (v) for Garmisch-Partenkirchen, using the weather type classification following the advection of air masses. The advection types correpond to: **(a)** Northeast, **(b)** Southeast, **(c)** Southwest, **(d)** Northwest, and **(e)** no prevailing direction. The white areas originate from interpolation effects using an ordinary kriging algorithm.

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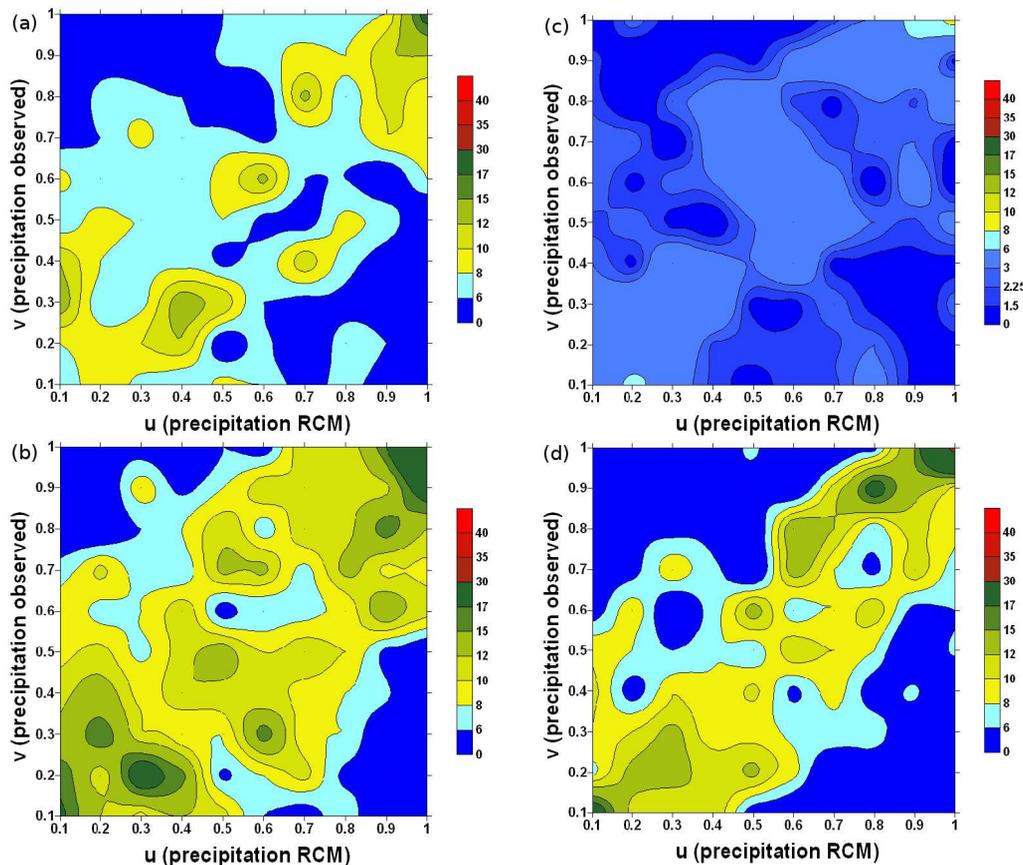
Fig. 11. Empirical Copula density for modelled (u) and observed precipitation (v) for Garmisch-Partenkirchen, using the weather type classification following the cyclonality in 950 hPa and 500 hPa respectively: **(a)** AA, **(b)** AC, **(c)** CA, and **(d)** CC (A – anticyclonic, C – cyclonic).
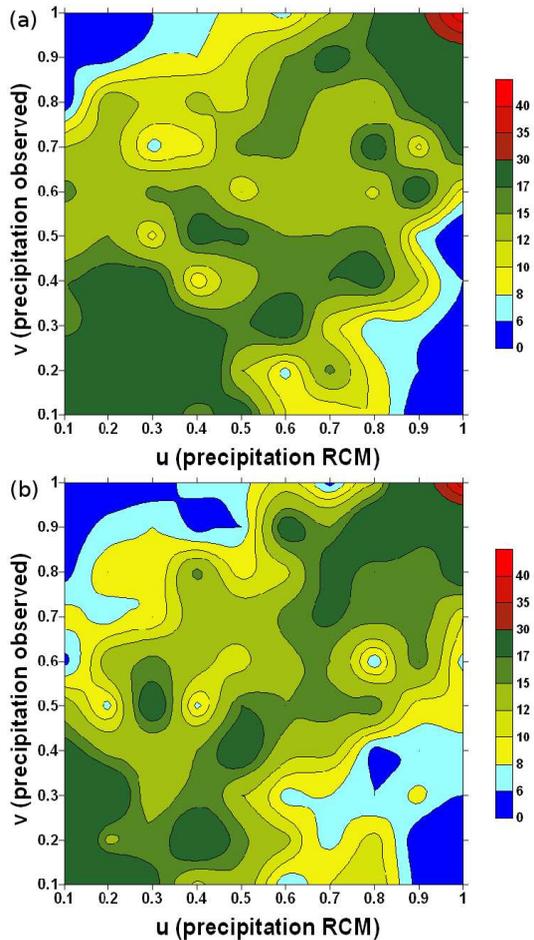
Fig. 12. Empirical Copula density for modelled (*u*) and observed precipitation (*v*) for Garmisch-Partenkirchen, using the weather type classification following the humidity of the troposphere: **(a)** D, and **(b)** W (D – dry, W –wet).
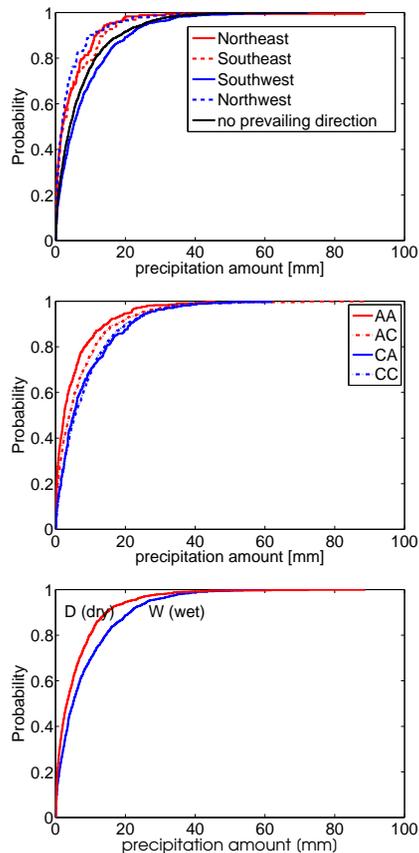
Fig. 13. Empirical CDF of observed precipitation in Garmisch-Partenkirchen, conditioned on the occurrence of (i) four different advection types plus one unspecified type, (ii) the following cyclonality types in 950 hPa and 500 hPa respectively: AA, AC, CA, and CC (A – anticyclonic, C – cyclonic), and (iii) the following humidity types of the troposphere: D, and W (D – dry, W – wet). Dry (wet) types are colored red (blue).
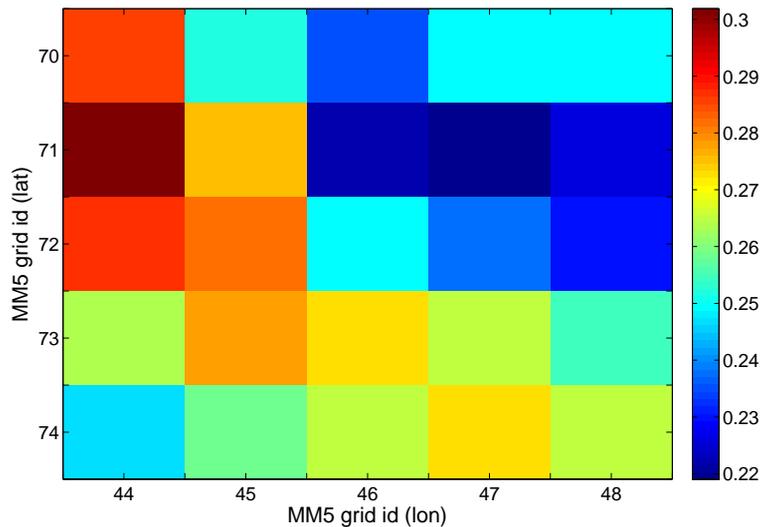
Fig. 14. Kendall's τ between observed precipitation at station Garmisch-Partenkirchen and its the surrounding RCM grid cells ($\Delta x = 19.2$ km). Garmisch-Partenkirchen is located in the middle grid cell (ID:46/72).

Fig. 15. Probability plots of observed and modelled precipitation time series at station Garmisch-Partenkirchen. If the quantiles of the two distributions agree, the plotted points fall on exactly on the thin dotted line. The red quadrates illustrates the agreement between observed and RCM rainfall. The black quadrates correspond to the Copula-based stochastic simulations without additional large-scale information. The Copula-based simulations including advection, cyclonality, and humidity of the troposphere are illustrated as blue, green, and orange quadrates respectively.

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Interactive Discussion
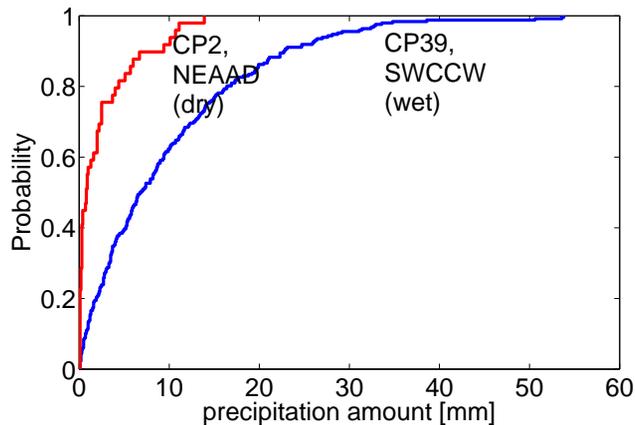
Fig. 16. Empirical CDF of observed precipitation in Garmisch-Partenkirchen, conditioned on the occurrence of the Northeast advection type WT 2 (NEAAD), and the Southwest advection type WT 39 (SWCCW).
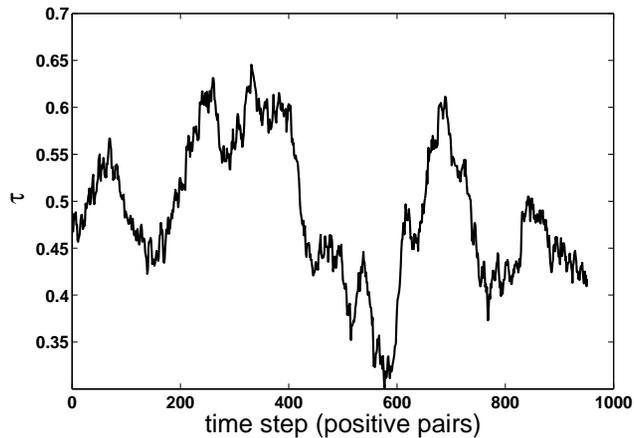
Fig. 17. Kendall's τ of 101 consecutive positive pairs for modelled and observed precipitation at station Garmisch-Partenkirchen.