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Parameter uncertainty and sensitivity analysis in sediment flux calculation

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Abstract

This paper examines uncertainties in the calculation of annual sediment budgets at the outlet of rivers. Emphasis is put on the sensitivity of power-law rating curves to degradations of the available discharge-concentration data. The main purpose is to determine how predictions arising from usual or modified power laws resist to the infrequence of concentration data and to relative uncertainties affecting source data. This study identifies cases in which the error on the estimated sediment fluxes remains of the same order of magnitude or even inferior to these in source data, provided the number of concentration data is high enough. The exposed mathematical framework allows considering all limitations at once in further detailed investigations. It is applied here to bound the error on sediment budgets for the major French rivers to the sea.

1 Introduction

Sediment exports in fluvial systems reflect earth surface denudation processes and have a controlling effect on the fluxes of nutrients, organic pollutants and heavy metals. The assessment of sediment fluxes therefore helps characterizing the impact of particulate transfers on water quality throughout river systems. Meanwhile, the prediction of realistic sediment budgets requires long-term discharge-concentration data (Walling and Webb, 1985; Ludwig and Probst, 1998; Delmas et al., 2009) which often suffer poor availability and reliability (Meybeck et al., 2003; Walling and Fang, 2003). In particular, uncertainties on both the sampling and calculation methods affect suspended sediment concentration data (Rode and Surh, 2007).

Monitoring uncertainties possibly ensue from incorrectly-gauged instruments or lack of precision in laboratory analyses. Moreover, significant drifts in the measured quantities may arise from the location of the sampling in the river section, as suspended sediment concentration varies within cross-sections of the rivers, pleading for series of depth and width-integrated measurements (Horowitz, 1997). In most cases though,

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only one sample is available, at a single location. In addition, the high temporal variability in measured concentrations inclines to relate decreasing uncertainties to increased sampling frequencies and duration of data collection (Coynel et al., 2004; Moatar and Meybeck, 2005; Rode and Surh, 2007).

Unfortunately, available sediment concentration data often result from programs of water quality monitoring that involve infrequent (monthly) samplings which should not be directly used to calculate annual sediment loads. A relevant alternative is to reconstruct continuous (daily) fluctuations in concentration from daily discharge data by resorting to $c(Q)$ rating curves linking concentration (c) to discharge (Q) values. The $c(Q)=aQ^b$ is often used for power laws allow showing deviations of at least two orders of magnitude from the nominal Q or c levels, not without a few empirical and theoretical objections when dealing with stretched Q or c distributions (Clauset et al., 2009). In the context of sediment transport, the occasional strong non-linearity in the $c(Q)$ relation and the presence of only a few extreme events refers to well-known problematic cases in fitting power laws (Laherrere, 1996; Goldstein et al., 2004) which suggests the use of truncated intervals of Q values, the adjunction of correcting terms (Laherrere and Sornette, 1998) or a systematic analysis of the error term in a more cautious $c(Q)=aQ^b+\varepsilon$ formulation.

Ferguson (1986, 1987) chose the latter option in his inaugural papers tackling the advantages and limitations of estimating sediment fluxes from power-law rating curves. As in other research domains, a pending question is whether to attribute a deterministic physical meaning to the a prefactor and the b exponent (Peters-Kümmerly, 1973; Morgan, 1995; Asselmann, 2000) or to consider them conceptually imperfect, although demonstrably statistically relevant, which opens the way for improved rating curves and variants advocated by Phillips et al. (1999), Asselmann (2000), Horowitz (2003) and Delmas et al. (2011a). Until then, attempts to cope with uncertain or infrequent raw data in deriving $c(Q)$ laws have lead to somewhat efficient strategies of data subdivision (Walling and Webb, 1981; Smart et al., 1999; Quilbé et al., 2006).

Complementary to the indirect procedures listed above, the present paper proposes a short mathematical development to link uncertainties in the predicted sediment fluxes to variations in the sampling frequency and the duration of data collection, as well as to uncertainties affecting the raw discharge and concentration data. Four rating curves are benchmarked; their propensity to transmit or attenuate the imperfections in source data is seen as their sensitivity to variations in the number and precision of discharge and concentration data.

2 Materials and methods

The present study is supported by daily discharge-concentration data collected over several years at 21 USGS stations among the wider set available at http://waterdata.usgs.gov/. The selected stations (Table 1) cover a wide range of basin sizes, discharge values and suspended sediment concentrations, disregarding intermittent or ephemeral streams that require specific descriptions. Various basin typologies and temperate climates are accounted for as the stations find themselves in California (CA), Illinois (IL), Iowa (IA), Missouri (MI), North Carolina (NC), Ohio (OH) and Virginia (VA).

In addition to the reference $c = aQ^b$ law ($f = 1$ fitting) different expressions and strategies have been tested and described into details by Delmas et al. (2011a). Table 2 provides a brief overview of their features and performances in their present states of development. The $f = 2$ fitting introduces correcting terms related to the instantaneous relative change in discharge ($\delta Q/Q$) along the rising or falling limbs of the hydrograph and to the quickness of discharge variations ($\Delta Q/\Delta t$) during events. In the $f = 1$ and $f = 2$ fittings all parameters are fitted "at once", i.e. the magnitude of the correcting terms is determined at the same time as that of the main aQ^b term. On the contrary, the $f = 3$ and $f = 4$ fittings involve pre-fitted (a, b) values then free adjustment of the remaining parameters, in a feedback procedure intended to allow more adaptable results. The expression of the $f = 3$ fitting is formally the same as that of its $f = 2$ counterpart and the differences between them only arise from the treatment of the (a, b)

parameters. The $f = 4$ fitting includes a correcting capacitor-like term defined as the variation in sediment stock between successive Q records, with values in $[0,1]$ and specific trends associated with increases or decreases of Q. All fittings were automated in a complex multi-stage procedure centered on the PEST parameter estimation software (Doherty, 2004) as indicated in Fig. 1. The purpose here is to examine how fittings and extrapolations react to changes in the available source data figuring uncertainty ranges, which necessitates a few theoretical developments.

The real total exported sediment mass in the $[0,T]$ time period is the unknown quantity:

$$M = \int_0^T Q(t)c(t)dt \quad (1)$$

where Q is discharge ($m^3\,s^{-1}$), t is time (s) and c ($kg\,m^{-3}$) is sediment concentration.

The straightforward approximation for direct calculation is:

$$M_N = \sum_{i=1}^{N} Q_i c_i \delta t_i \quad (2)$$

where Q_i and c_i are discrete time-averaged values over the δt_i intervals in the (δt_1, ..., δt_N) series covering the entire $[0,T]$ period without overlap. Expressions (1) and (2) are equivalent only if $\delta t_i \to 0$ and the discrepancy between M_N and M plausibly increases with the use of progressively bigger δt_i intervals. Nevertheless, certain combinations of δt_i values may lead to $M_N = M$ thus the drift of M_N away from M is neither expected to be linear nor even monotonous. By contrast, the use of δt_i intervals inferior to the characteristic time period of the fluctuations in Q and c values of the real system guarantees reliability of the approximation. In the following, M_N values arising from δt_i intervals all equal to 1 day are the best estimates and will be considered as "exact" solutions.

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In Eq. (2) the replacement of lacking concentration data by fitted values leads to:

$$M_{N,f} = \sum_{i=1}^{N} Q_i c_i^{(f)} \delta t_i \qquad (3)$$

where $M_{N,f}$ indicates that the estimation M depends both on the number of records N and on the series of fitted concentration values $c_i^{(f)}$. A close look identifies them as implicit functions of the entire sets of available discharge and concentration data, also depending on their collection in time:

$$c_i^{(f)} = c_i^{(f)}(Q_1,\ldots,Q_N,c_1,\ldots,c_n,\delta t_1,\ldots,\delta t_n,\ldots,\delta t_N) \qquad (4)$$

hypothesizing no discharge datum is lacking whereas $n<N$ concentration data are available, associated with as many time intervals.

For a fixed sampling period δt_s the total time period writes $T = N\delta t_s$ and the sub-period covered by the available concentration records is $T_s = n\delta t_s$. Five quantities (n, N, T, T_s, δt_s) are involved but only the two above relations exist between them, leaving three degrees of freedom. The independent triplet (n, N, T) facilitates graphical representations and was chosen among others, which yields:

$$c_i^{(f)} = c_i^{(f)}(Q_1,\ldots,Q_N,c_1,\ldots,c_n,n,N,T) \qquad (5)$$

A conceptual implication of the (n, N, T) choice is to emphasize the roles played by the ratio n/N over the total duration T of the experiment as an alternative to the classical studies in terms of sampling frequencies that rather debate influence of the $1/\delta t_s$ quantity.

To determine how infrequent concentration data may be used with satisfying relevancy to estimate the sediment flux M, a simple strategy is to study the convergence of the fitted estimation $M_{N,f}$ towards the closest approximation M_N of M, as defined in Eqs. (1)–(3). The quality of the estimation depends on the (n, N, T) triplet where n is the number of available concentration data, N that of the daily discharge data and T the

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time period of data collection. Assuming sufficiently regular measurements, n/N thus corresponds to the sampling frequency of the concentration data. Eight n/N (sampling frequency) values of 1, 2, 3, 5, 10, 20, 50 and 80% were considered together with five T values of 1, 4, 8, 12 and 15 yr. Twenty replicates of each (n/N, T) combination for each of the twenty-one USGS stations support the subsequent analysis.

Besides the effects of the density (frequency) and duration of the available records, the purpose of this study is to gain insights on the transmission of initial uncertainties on Q and c measurements, treating exhaustiveness and quality of the data as separate issues. The latter is simply addressed by applying systematic relative perturbations to all available raw data, increasing or decreasing Q and c values by predetermined percentages. This deterministic method bounds the results expected from random perturbations on Q and c measurements over the same predetermined relative ranges and allows easier interpretation.

As the Q_i and c_i series endure systematic modifications, useful changes of variables are $Q' = (Q_1, \ldots, Q_N)$ and $c' = (c_1, \ldots, c_n)$. Consequently, dropping the "temporal" (n, N, T) components reduces Eq. (5) to:

$$c_i^{(f)} = c_i^{(f)}(Q', c') \tag{6}$$

Fitted concentration values result from adjustments of $c(Q)$ laws, for example in the reference $c = aQ^b$ form where (a, b) is the parameter set, expecting $a(Q', c')$ and $b(Q', c')$. In the general case of laws involving the $(p_1, \ldots, p_q)$ parameter set, each fitted p_i parameter may be anticipated as $p_i(Q', c')$. The expansion of $c_i^{(f)}$ is thus:

$$c_i^{(f)} = c_i^{(f)}\left(p_1(Q', c'), \ldots, p_q(Q', c')\right) \tag{7}$$

After Eq. (3), the most explicit expression of $M_{N,f}$ is:

$$M_{N,f}\left(p_1(Q', c'), \ldots, p_q(Q', c')\right) = \sum_{i=1}^{N} Q_i c_i^{(f)}\left(p_1(Q', c'), \ldots, p_q(Q', c')\right) \delta t_i \tag{8}$$

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The left-hand term of Eq. (8) clearly indicates a multi-stage process which we may consider in the following manner:

$$(Q',c')\xrightarrow{f}(p_1,...,p_q)\xrightarrow{E}M_{N,f} \quad (9)$$

where the first arrow (f) refers to the fitting procedure (inverse calculation) and the second arrow (E) to the extrapolation procedure (direct calculation). Alternatively, the whole process may be summarized by:

$$(Q',c')\xrightarrow{\varphi}M_{N,f} \quad (10)$$

where the φ function short-circuits the intermediate stage in determining sediment fluxes. The propagation of uncertainties is studied under the angle of sensitivity to relative perturbations (RQ', Rc') of the initial (Q', c') raw data (Fig. 2) responsible for relative variations in fitted parameter values (Rp_i) and predicted sediment fluxes ($RM_{N,f}$).

3 Results and discussion

3.1 Effects of the infrequence and number of concentration data on the predicted sediment fluxes

Figure 3a targets the evolution of the precision ($M_{N,f}/M_N$ ratio) of the $f=1$ method for increasing values of the sampling frequency (n/N). It also shows the positive impact of increasingly long periods of data collection (T values) for given sampling frequencies. By contrast, a clear increase in dispersion occurs for sampling frequencies less than 20% and time periods less than 4 yr where effects of the decrease in sampling frequency are incompletely compensated by these of increased time periods. Accordingly, the typical (monthly) 3% sampling frequency associated with an 8-yr period seems the leftmost combination over which a too large set of values lies outside the gross [0.5,5] interval and endangers relevancy of the fittings. Figure 3b was

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drawn from the $c = aQ^b + a_1\delta S$ law ($f = 4$ fitting) for comparison. It starts with slightly less dispersed values for low sampling frequencies but exhibits more oscillations before achieving obvious convergence. The main difference between Fig. 3a and b is the highest dispersion in values for time periods of 4 and 8 yr, which extends towards the 20% and 5% sampling frequencies, respectively. Those modifications indicate the complex influence of the $a_1\delta S$ term added to the power law, plausibly allowing a more dynamic though less controlled solution outside dense data conditions.

A complementary view is given by Fig. 3c and d that target the evolution of the median of the absolute relative error on prediction in function of the number of available concentration data (n), using overlapping n values issued from various sampling frequencies and time periods. The noticeable continuity and similarity in Fig. 3c between curves corresponding to different time periods allows reasoning in terms of number of data only: $n = 150$ takes the error under 20% and $n = 300$ is required for errors lower than 10%. Equivalently, if one disposes of 3% sampling frequencies (monthly measures) then 13.7 yr is the minimum time period to fulfill the 20% error criterion when resorting to the $f = 1$ fitting. By contrast, the trends in Fig. 3d are less uniform as error values exhibit a clear dependence on the time periods, questioning analyses only involving the number of data. Combined criteria appear therefore more relevant, leading for example to impose a number of data $n > 200$ and a time period $T \geq 8$ yr to confine the error under 20%. For monthly samplings, the n threshold alone corresponds to $T > 18$ yr, which *de facto* verifies the second inequality. This result tends to indicate that the condition on the number of data is the most restrictive and could still be kept alone.

Figure 3e and f depicts the evolution of the statistics of the absolute relative error in sediment fluxes predicted from the $f = 1$ and $f = 4$ fittings, focusing on the monthly (3%) sampling frequencies. The monotonous and exponential-like decrease of the error in Fig. 3e emphasizes the regular gain in stability of the $c = aQ^b$ method for increasingly long data collection periods. Figure 3f shows the same trend, with few irregularities, maybe thresholds in the decrease of the error for the $c = aQ^b + a_1\delta S$

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method. The definition chosen for the storage term (S) and its variations (δS) plausibly requires dense enough data for significant gains in precision, with potentially better performances than the $c = aQ^b$ method but more restrictive conditions of application.

3.2 Effects of the uncertainties in discharge-concentration data on the predicted sediment fluxes

Figure 4a indicates the negative sensitivity of the predicted fluxes to variations in discharge data, as a systematic decrease in Q values ($RQ' < 0$) causes an increase in the predicted flux ($RM_{N,f} > 0$) and conversely, with slightly stronger effects on the left side of the graph. The trend nevertheless remains approximately linear and even sub-linear for most of the stations, as the absolute magnitude of the effect remains proportional or inferior to that of the cause. Figure 4b demonstrates the strict linear dependence of the predicted fluxes on variations in concentration data. Figure 4c,d exhibits clearly additive sensitivity trends, as Fig. 4c may be seen as "Fig. 4a + b" and Fig. 4d as "Fig. 4a–b", in response to the $R1 = RQ' + Rc'$ and $R2 = RQ' - Rc'$ variations, respectively.

The slight dissymmetry of the curves between regions $RQ' < 0$ and $RQ' > 0$ may be due to distant or attenuated asymptotic effects: further reductions of Q values would take Q closer to zero, in regions where small variations are expectedly associated with large influences, slowly approaching a vertical asymptote. By contrast, the same small relative variations in Q values seem to have decreasing influences for higher Q values, slowly approaching a horizontal asymptote. Nevertheless, these explanations silence possible changes in nature of the $c(Q)$ relation for drastic variations in Q ranges. The $c = aQ^b + a_1\delta Q/Q + a_2\Delta Q/\Delta t$ law reproduces the major sensitivity trends of the reference $c = aQ^b$ law but shows a little less dispersion and "regularizes" all curves inside narrowed envelops. A plausible hypothesis is that additional terms involving transformations of Q, especially $\delta Q/Q$, tend to homogenize sensitivities to variations in discharge data whatever the nominal values and ranges of Q.

The $f = 3$ and $f = 4$ fittings also exhibit near-linear and weakly-dispersed sensitivities to variations in discharge or concentration data only but exhibit specific reactions

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to combined discharge and concentration variations, as illustrated in Fig. 5 for the $c = aQ^b + a_1\delta S$ law. A close look at Fig. 5b nevertheless indicates that the previous "additive sensitivity rule" still holds over the middle range of the considered variations whereas non-linearity prevails near the boundaries of the tested intervals, where corrections due to the free $a_1\delta S$ term overcompensate the expected variations in predicted flux. As displacements along the $R2$ axis (Fig. 2) create the concomitant conditions of a strong sensitivity, the curves in Fig. 5b find themselves in a well-defined envelop. By contrast, displacements along the $R1$ axis (Fig. 2) create a conflict between the negative sensitivity to variations in discharge data and the positive sensitivity to variations in concentration data, which enhances dispersion and complicates the interpretation of Fig. 5a without the help of Fig. 5b. Having freed the last terms of the $f = 3$ and $f = 4$ fittings yields completely different results than in the $f = 1$ and $f = 2$ cases where all parameters adjust at once. The inclusion of feedback terms in a two-stage adjustment designs more dynamic but less controlled yet procedures.

Finally, Tables 3 and 4 summarize the above results into user-oriented recommendations and their simple theoretical counterparts:

i. The fifth column of Tables 3 and 4 recalls that uncertainties in concentration data are transmitted almost unaffected by the two-stage (fitting and extrapolation) process whatever the fitting method, the USGS station and the magnitude of discharge and concentration values. This result certainly meets the expectations: a relative change in source concentration data is equivalent to a change in offset of the prediction. In other terms the scale-invariance is respected, which in turn means that the $c = aQ^b$ component of each fitting plays a dominant role, at least in the indicated ranges.

ii. The fourth column of Tables 3 and 4 indicates that the propagation of uncertainties in discharge values remains sub-linear in most of the cases though somewhat variable between stations: for fixed concentration data, a relative decrease in discharge data gives a lesser relative increase in the predicted sediment fluxes. As

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the predicted $M_{N,f}$ fluxes depend on the $Q_i c_i^{(f)}$ products in Eq. (3), a decrease in Q_i values results in a decrease in the prediction unless fitted $c_i^{(f)}$ values increase enough to maintain the same prediction. In the present cases, the relative variations of $c_i^{(f)}$ even overcompensate these of Q_i. The practical implication is that underestimations of discharge lead to overestimations of sediment fluxes, provided concentration data suffer limited uncertainty.

iii. Columns six and seven in Table 3 confirm the nearly additive trends in average sensitivity scores when combining variations in both discharge and concentration. Consequently, even problematic cases of overestimation of discharge and underestimation of concentration (or conversely) do not result in multiplicative errors regarding the assessment of sediment fluxes.

iv. Table 4 exhibit strong discrepancies between methods in which all parameters are fitted at once ($f = 1$ and $f = 2$) and "feedback" methods ($f = 3$ and $f = 4$) especially for simultaneous increases or decreases in both discharge and concentration data, associated with opposite effects on the predicted fluxes. In such cases, the feedback methods induce noticeably higher dispersions which forbid predicting whether variations in discharge or in concentration will have dominant effects. In absence of further arguments, high dispersion scores practically mean that underestimating (or overestimating) both discharge and concentration data may result either in underestimation or overestimation of the sediment fluxes. As seen in Fig. 5a though, the committed error remains generally weak.

3.3 Transmission of uncertainties throughout the fitting-extrapolation process

This subsection deals with "composite effects" involved in the formal description of Eq. (9) and adds a level of detail to Sect. 3.2 referring to Eq. (10). As expected from the scale-invariance of power laws, almost all adjustments in parameter values of the $c = aQ^b$ fitting concern the a prefactor, leaving b values unchanged, to the exception

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of datasets involving the most widely-dispersed Q and c values (mainly stations 3, 5, 13 and 20 in Table 1). The analysis of parameter variations therefore focuses on the prefactor in the $f=1$ fitting.

Figure 6 (showing relative variations of a) exhibits very similar trends as Fig. 4 (showing variations of the predicted flux) but this time with more significant dispersion. For this reason, the bold dotted lines trace the median of the results in Fig. 6 while they represented averages in Fig. 4, in attempting to identify a characteristic (or representative) sensitivity trend. The three noticeable outliers on the left of Fig. 6a correspond to cases in which b values were not correctly fitted whereas the various trends on the lower right of Fig. 6a rather traduce specificities of the collected data and plasticity (or adaptability) of the fitting method.

Considering for example the RQ' variation and the $f=1$ fitting, the overall sensitivity score of a given fitting method is $RM_{N,1}/RQ'$, relating the variation in predicted flux to the variation in discharge data.

The associated composite sensitivities are $RM_{N,1}/Ra$ and Ra/RQ' whose product is the overall sensitivity. In this example, Ra/RQ' is the sensitivity of the fitting stage and $RM_{N,1}/Ra$ that "transmitted" during the extrapolation stage. Whereas Ra/RQ', Ra/Rc', $Ra/R1$ and $Ra/R2$ are clearly-defined quantities, the $RM_{N,1}/Ra$ ratio a priori has different values whether testing the RQ', Rc', $R1$ or $R2$ variations. Remarkably, the $RM_{N,1}/Ra$ stays very close to unity for fittings involving negligible adjustments of the b exponent, indicating that sensitivity is gained during the fitting stage then transmitted as before.

Eventually, this result does not explain the reduction of dispersion between the lower right regions of Figs. 4a and 6a, which are clues of $RM_{N,1}/Ra<1$ ratios. In cited regions, stations 3, 5, 13 and 20 in the list of Table 1 give Ra/RQ' ratios between -1.5 and -3 to compare with $RM_{N,1}/RQ'$ ratios between -0.9 and -1.1. These stations indeed have in common that $f=1$ fittings caused non-negligible adjustments of the b exponent in the $c=aQ^b$ power law, which leads to damped sensitivities during the extrapolation stage.

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The study of composite sensitivities for the $f = 2$ fitting method leads to the conclusion that "damping" or transmission ratios $RM_{N,2}/Ra$, $RM_{N,2}/Ra_1$ and $RM_{N,2}/Ra_2$ are not unique and depend on the selected variations in the (RQ', Rc') plane. The transmission ratios generally remain slightly less than unity, at least when focusing on median behaviors with typically negligible variations of the b parameter. For cases of significant b variations (of the order of 5–10%) noticeable dispersion and station-specific behaviors are induced regarding increased parameter sensitivities, but the extrapolation stage tends to reverse this trend, as was the case for the $f = 1$ fitting.

Again, the feedback methods ($f = 3$ and $f = 4$) require dedicated attention as their mechanisms of parameter adjustment are conceptually different than these of the "at once" methods ($f = 1$ and $f = 2$). Figure 7 gathers some noticeable results and is preferentially read in columns. Discrepancies between Fig. 7a, c and e refer to the differences in roles played by the $\delta Q/Q$, $\Delta Q/\Delta t$ and δS terms, but all have in common the fanned out curves at both ends of the tested interval of $R2$. The median curves show near-linear sensitivities to $R2$ in their middle range. Their numerical values are unity for a_1 (Fig. 7a) and slightly greater in absolute magnitude than the overall sensitivity for a_2 (Fig. 7c). By contrast, the median sensitivity score of a_1 in Fig. 7e is limited to $2/3^{rd}$ of the overall sensitivity for the $f = 4$ fitting (for small $R2$ variations) but this score drastically increases in the outer regions of the graph to reach more than twice the overall sensitivity. The same score holds for a_2 as $|R2|$ tends towards 25% in Fig. 7b, increasing inasmuch the influence of the feedback terms.

Figure 7b, d and f especially attempt at an estimation of the reactivity or "plasticity" of the $f = 3$ and $f = 4$ fittings by tracking changes in the Ra_1/Ra and Ra_2/Ra ratios where Ra is the relative adjustment of a in the $f = 1$ fitting and Ra_1 (or Ra_2) that of a_1 (or a_2) in the $f = 3$ or $f = 4$ fittings. Intuition indicates that ratios of high absolute magnitude are responsible for aliasing of the sensitivity in Fig. 5b, attributable to overcompensation effects. More precisely, overcompensation is achieved when the horizontal asymptote $RM_{N,f} = 0$ is reached in Fig. 5b, that is for $|R2|$ values high enough to suppress the variations of the predicted flux. For such $|R2|$ values, the feedback terms

$a_1\delta Q/Q + a_2\Delta Q/\Delta t$ ($f=3$) or $a_1\delta S$ are of same magnitude but opposite sign as the "main" aQ^b term.

The general trend in study of composite sensitivities is that most of the uncertainty in calculated sediment fluxes is acquainted in the fitting stage then hardly reduced during the extrapolation stage, not without differences between methods:

i. The a prefactor is responsible for almost the entire sensitivity score and trends of the $c = aQ^b$ method ($f=1$ fitting). In somewhat marginal cases the b exponent is significantly adjusted too. In such cases the extrapolation stage tends to reduce the uncertainties present in the fitting and previously inherited from imperfect source data.

ii. In the $c = aQ^b + a_1\delta Q/Q + a_2\Delta Q/\Delta t$ method ($f=2$ fitting) all parameters but b respond to variations in discharge and concentration data, with similarities in relative variations of a_2 and a. A plausible explanation is that, b being close to unity, relative changes in Q affect Q^b and $\Delta Q/\Delta t$ in similar ways, provoking similar adjustments of their prefactors. By contrast, a_1 exhibits specific trends as the $\delta Q/Q$ term is of another kind. The interplay between parameters a, a_1 and a_2 preserves the dominant role of the aQ^b term. It also gives slightly lower dispersion of the results between stations, as if the method was both stabilizing and a little bit normalizing, despite the additional terms intended to provide variety.

iii. The feedback methods $c = aQ^b + a_1\delta Q/Q + a_2\Delta Q/\Delta t$ ($f=3$) and $c = aQ^b + a_1\delta S$ ($f=4$) use pre-fitted a and b parameters but allow free adjustment of the additional parameters. Both methods provide strong and very dynamic adaptability of the a_1 and a_2 coefficients governing the influence of the correction terms, at the risk of overcompensation phenomena when using too wide variations in source data. The plasticity of these methods appears in the dispersion between fittings performed for different data stations, potentially allowing very specific calculations. The recommended use of discharge units associated with comparable

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magnitudes of the (a, Q, b) triplet would maybe ensure a more stable hierarchy between the terms of these last two methods, as would adaptations in the definition of the storage capacity (S) term.

3.4 Confronting uncertainties in French sediment budgets

All the previous considerations allow the characterization of uncertainties for sediment budget assessments. In the French sediment budget proposed by Delmas et al. (2011b) for the major rivers to the sea, the calculations have been performed from monthly data (the 3% sampling frequency) which span more than 25 years. As the number of paired discharge-concentration data is $n > 300$, the uncertainty due to the sampling frequency is lower than 10%. Additionally, hypothesized errors of 20% on the discharge and concentration measurements induce relative variations in the calculated fluxes ranging between −20 to +20% with the $f = 4$ fitting. This uncertainty analysis thus allows the description of the sediment budget giving intervals of values for the sediment exports as indicated in Fig. 8.

4 Conclusions

This study focused on the uncertainties in the calculation of sediment fluxes, either arising from the questionable data quality regarding discharge and concentration measurements, or the sampling frequencies and the duration of the data collection period. To analyze the transmission of uncertainties from the discharge and concentration measurements to the sediment flux calculation, the exposed mathematical development also integrates facilities to describe the role played by separate or combined uncertainties in discharge (Q) and concentration (c) values. From a technical point of view, fittings of the reference $c = aQ^b$ power law reproduced the expected scale-invariance: variations in discharge affected the a prefactor only, leaving the b exponent almost unchanged, unless wide dispersion was present in the (Q, c) source data which met

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another known theoretical result. The other tested fittings involved corrections and feedbacks added to the main aQ^b term, inducing noticeable changes in response to variations in Q data, but keeping almost always a linear dependence on variations in c data. Finally, the effect of uncertainties in source data is represented by the sensitivity of the fittings to variations in (Q, c) data. Any relative variation in c data results in the same relative variation in predicted sediment fluxes, whereas the trend is more complicated and variable among methods regarding the influence of discharge variations. Nevertheless, combined discharge and concentration variations produce additive effects and therefore never create diverging predictions.

Considering the sediment export calculations and the application of rating curve approaches, the typical problematic case which is reported is the sampling frequency. Thus, classical concentration data produced for the water quality monitoring of water agencies, where daily discharge data are available while suspended matter concentration is measured once a month only, are generally considered as non valid. The uncertainty analysis here shows that when the sampling frequency is as low as 3% of the daily data (monthly concentration data), a time period of $T = 8$ yr is required to predict sediment fluxes within the 20% error interval. This analysis has also shown that despite fluctuations with imposed sampling frequency and fitting methods considered, $n = 150$ to $n = 300$ concentration data always seems to be a sufficient input for reliable flux estimations which provides an alternative to descriptions in terms of sampling frequencies only. In France, the monitoring of water quality for the major rivers to sea has begun in the 70's, yielding about 30 years of monthly data disposable for calculation of mean sediment fluxes. As over 300 paired discharge-concentration data are available in most cases, the uncertainty linked to the sampling frequency falls below 10%. Moreover, if one has to establish the best compromise between costs linked to more numerous concentration measurements and performances of the methods, the 10% sampling frequency seems a good optimization, involving one concentration measurement each ten days.

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Table 1. Station number, name and location, drainage area, upper base-flow limit and period of data collection for the 21 selected USGS stations.

Nr	USGS station	Drainage area	Base-flow limit	Data period
	Name and Location	km^2	$L\,s^{-1}$	yr
1	Rappahannock River at Remington, VA	1603	17 429	1951–1993
2	Roanoke River at Randolph, VA	7682	72 632	1968–1981
3	Dan River at Paces, VA	6700	64 420	1954–1981
4	Yadkin River at Yadkin College, NC	5905	68 668	1951–1989
5	Muskingum River at Dresden, OH	15 522	89 056	1952–1974
6	Muskingum River at McConnelsville, OH	19 223	111 709	1978–1991
7	Hocking River at Athens, OH	2442	24 253	1956–1965
8	Scioto River at Highby, OH	13 289	121 621	1953–1982
9	Little Miami River at Milford, OH	3116	39 785	1978–1989
10	Great Miami River at Sydney, OH	1401	9953	1967–1975
11	Stillwater River at Pleasant Hill, OH	1303	14 455	1963–1975
12	Maume River at Waterville, OH	16 395	95 003	1950–2003
13	Upper Iowa River near Dorchester, IA	1994	9132	1975–1981
14	Iowa River at Iowa City, IA	8472	21 166	1959–1987
15	Des Moines River near Saylorville, IA	15 128	41 484	1961–2004
16	Illinois River at valley City, IL	69 264	3638	1980–2008
17	Kaskakia River at Cooks Mills, IL	1225	5804	1979–1997
18	Kaskakia River near Venedy Station, IL	11 378	35 820	1980–1997
19	Mississipi River at St. Louis, MO	1 805 222	3 808 615	1980–2008
20	Salinas River near Spreckels, CA	10 763	16 367	1969–1979
21	Sacramento River at Sacramento, CA	60 883	358 208	1957–1979

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Table 2. Fitted discharge-concentrations laws, median parameter values and coefficients of determination obtained for the 21 USGS stations listed in Table 1.

Fitting	Fitted law	Fixed parameters	a	b	a_1	a_2	R
$f=1$	$c=aQ^b$	–	4.30	0.77	–	–	0.58
$f=2$	$c=aQ^b+a_1\delta Q/Q+a_2\Delta Q/\Delta t$	–	6.16	0.78	21.46	0.66	0.65
$f=3$	$c=aQ^b+a_1\delta Q/Q+a_2\Delta Q/\Delta t$	(a,b)	4.30	0.77	27.70	0.67	0.65
$f=4$	$c=aQ^b+a_1\delta S$	(a,b)	4.30	0.77	−1114	–	0.62

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Table 3. Slopes of the approximate linear relations between mean relatives changes ($RM_{N,f}$) in calculated sediment fluxes and relative changes (RQ') in discharge data, (Rc') in concentration data or combined changes ($R1$ and $R2$) in both discharge and concentration data (see Fig. 2 for details).

Fitting	Fitted law	Fixed parameters	$RM_{N,f}/RQ'$	$RM_{N,f}/Rc'$	$RM_{N,f}/R1$	$RM_{N,f}/R2$
$f=1$	$c=aQ^b$	–	−0.80	1.00	0.23	−1.77
$f=2$	$c=aQ^b+a_1\delta Q/Q+a_2\Delta Q/\Delta t$	–	−0.85	1.00	0.23	−1.77
$f=3$	$c=aQ^b+a_1\delta Q/Q+a_2\Delta Q/\Delta t$	(a,b)	−0.85	1.00	0.19*	−1.80*
$f=4$	$c=aQ^b+a_1\delta S$	(a,b)	−0.85	1.00	0.13*	−1.85*

* Acceptable on a restricted interval only (see text).

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Table 4. Dispersion of the sensitivity results shown in Table 3, represented by the mean relative difference between sensitivity results at a station and the average linear trend.

Fitting	Fitted law	Fixed parameters	$RM_{N,f}/RQ'$	$RM_{N,f}/Rc'$	$RM_{N,f}/R1$	$RM_{N,f}/R2$
$f=1$	$c=aQ^b$	–	38.7%	0.1%	117%	24.0%
$f=2$	$c=aQ^b+a_1\delta Q/Q+a_2\Delta Q/\Delta t$	–	21.3%	1.7%	70.4%	14.6%
$f=3$	$c=aQ^b+a_1\delta Q/Q+a_2\Delta Q/\Delta t$	(a,b)	36.8%	0.2%	163%*	19.1%*
$f=4$	$c=aQ^b+a_1\delta S$	(a,b)	37.1%	0.2%	245%*	19.2%*

* Acceptable on a restricted interval only (see text).

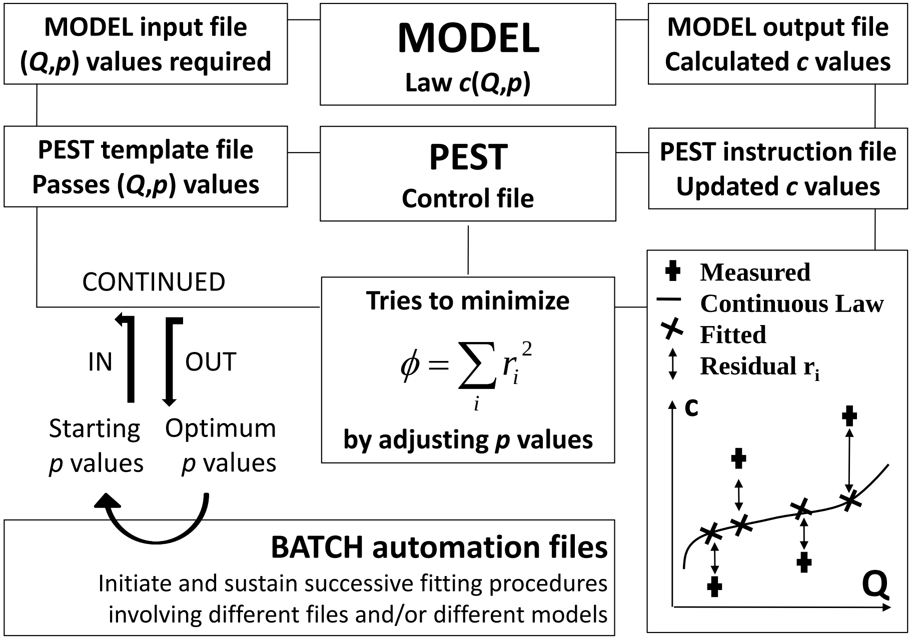

Fig. 1. Automation of the fitting procedure. Starting parameter values are fed to PEST which runs the "MODEL" the form of a series of FORTRAN codes including various treatments. PEST then analyzes the residuals between predictions and expectations and tries to minimize the associated cost function by adjusting parameter values. Batch files ensure execution of all fitting methods over all data stations.

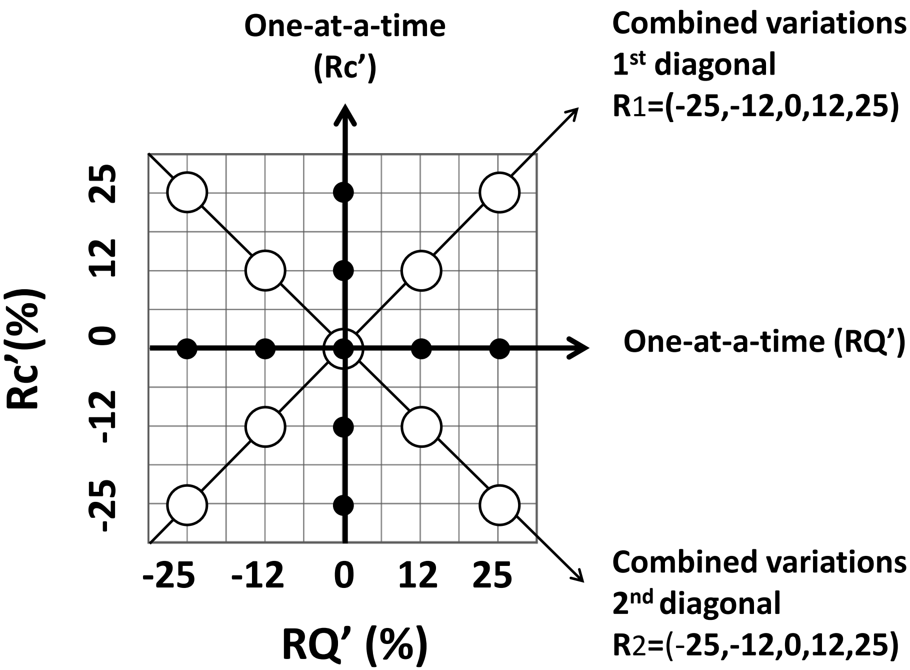

Fig. 2. Screening strategy and coordinates in the (RQ', Rc') plane where RQ' denote the relative variations of discharge data and Rc' that of concentration data. Graduated trajectories $R1$ and $R2$ indicate combined variations in RQ' and Rc, either identical ($R1$) or of opposite signs ($R2$).

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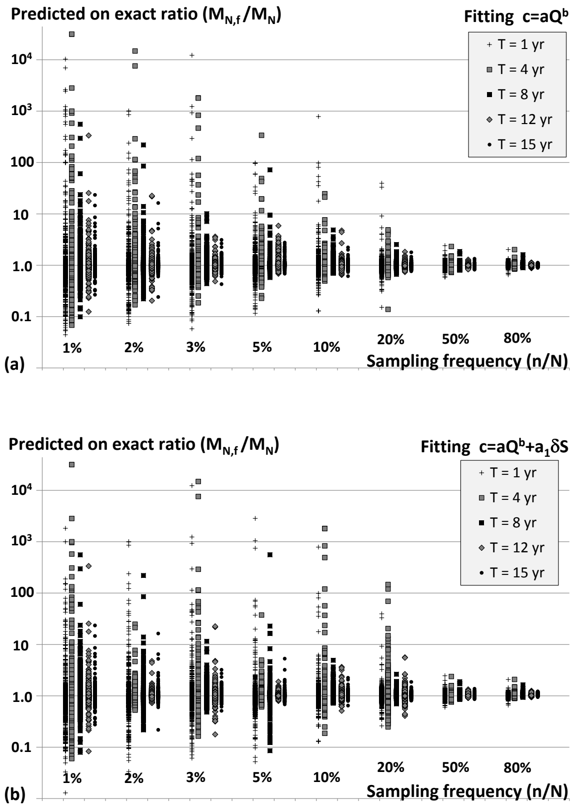

Fig. 3. See caption at the end of the figure.

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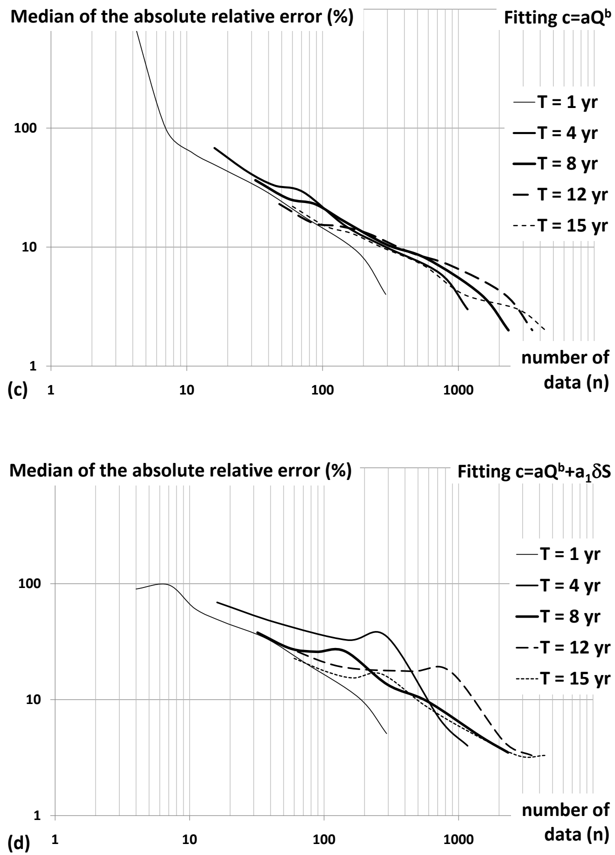

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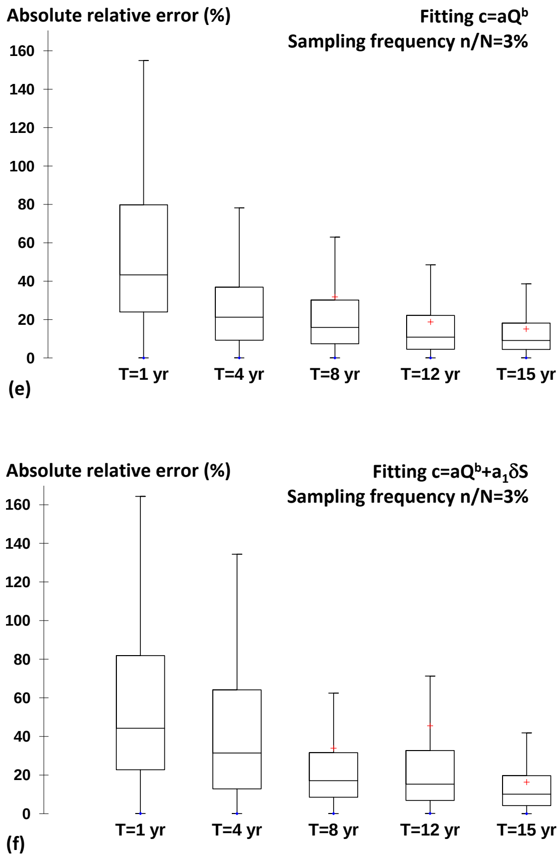

Fig. 3. How the number n and sampling frequency n/N of the concentration data relative to the discharge data N collected over the same time period T influence the reliability of the sediment flux predicted from methods $f = 1$ **(a, c, e)** and $f = 4$ **(b, d, f)**. The predicted/exact ratio **(a, b)** and the median of the absolute relative error **(c, d)** are possible criteria, as are statistics on the absolute relative error **(e, f)** when focusing on the difficult and typical $n/N = 3\%$ case of monthly concentration data.

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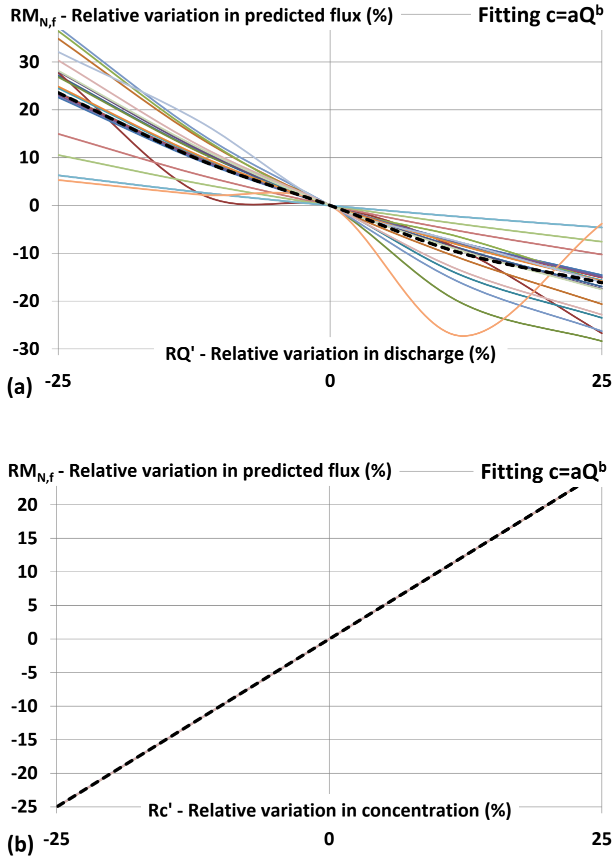

Fig. 4. See caption on next page.

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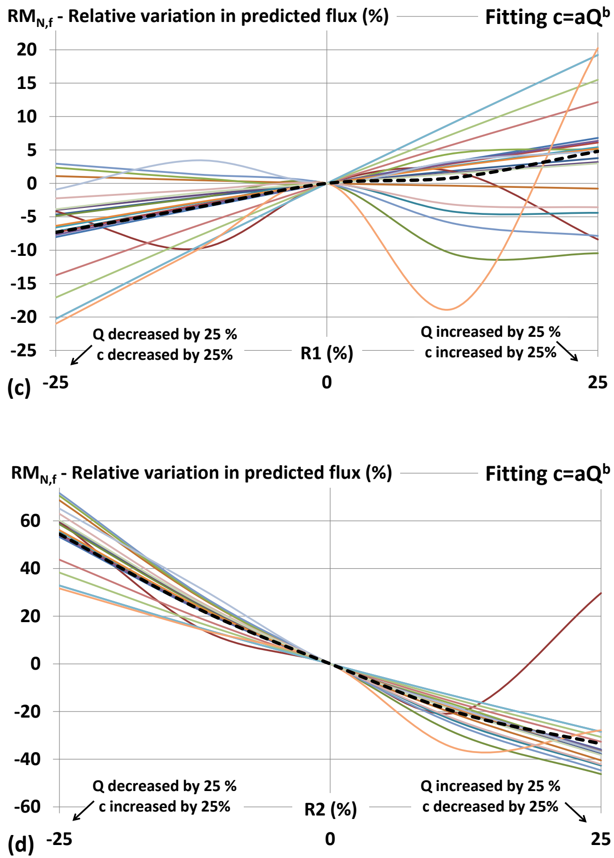

Fig. 4. Relative variations $RM_{N,f}$ of the predicted sediment flux obtained from the $c = aQ^b$ fitting under relative variations RQ' in discharge only **(a)**, Rc' in concentration only **(b)** or combined relative variations $R1 = RQ' + Rc'$ **(c)** and $R2 = RQ' - Rc'$ **(d)** detailed in Fig. 2. Each thin line is the result associated with one of the 21 USGS stations in Table 1. Bold dotted lines are their averages.

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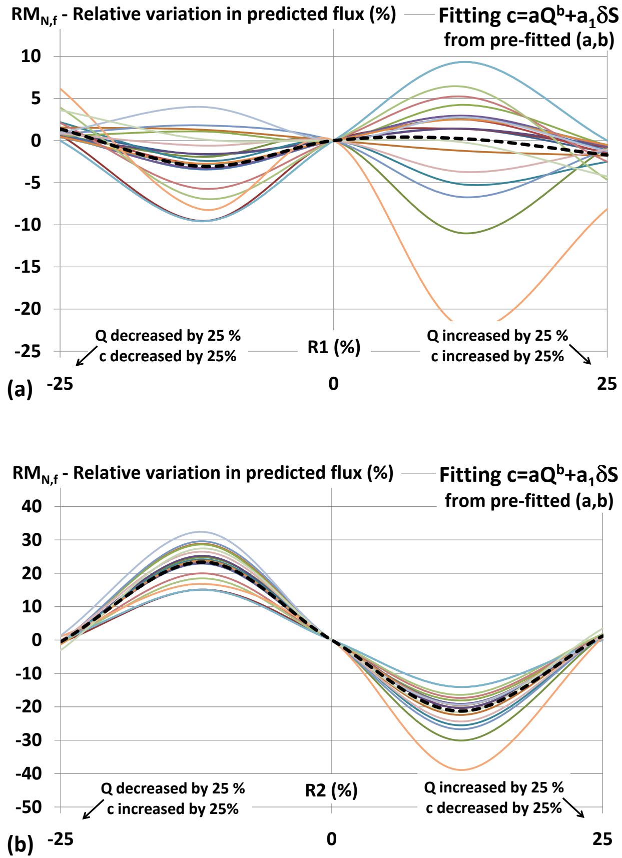

Fig. 5. Relative variations $RM_{N,f}$ of the predicted sediment flux obtained from the $c = aQ^b + a_1\delta S$ fitting under combined relative variations $R1 = RQ' + Rc'$ **(a)** and $R2 = RQ' - Rc'$ **(b)** detailed in Fig. 2. Each thin line is the result associated with one of the 21 USGS stations in Table 1. Bold dotted lines are their averages.

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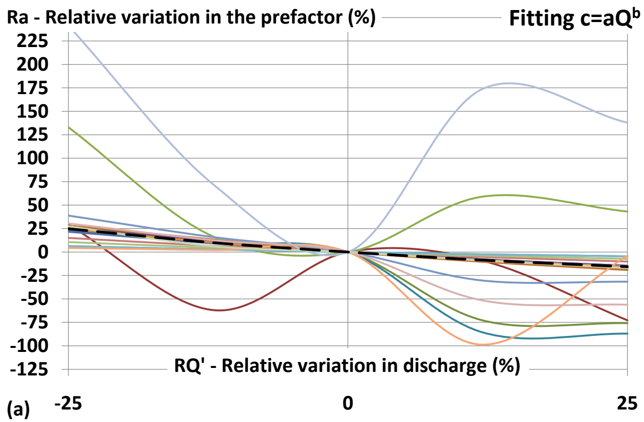

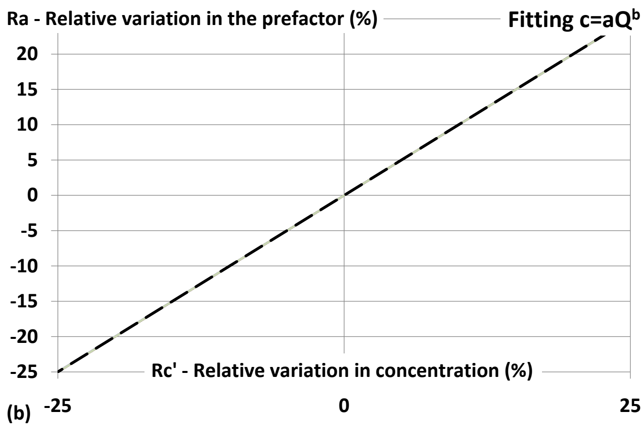

Fig. 6. See caption on next page.

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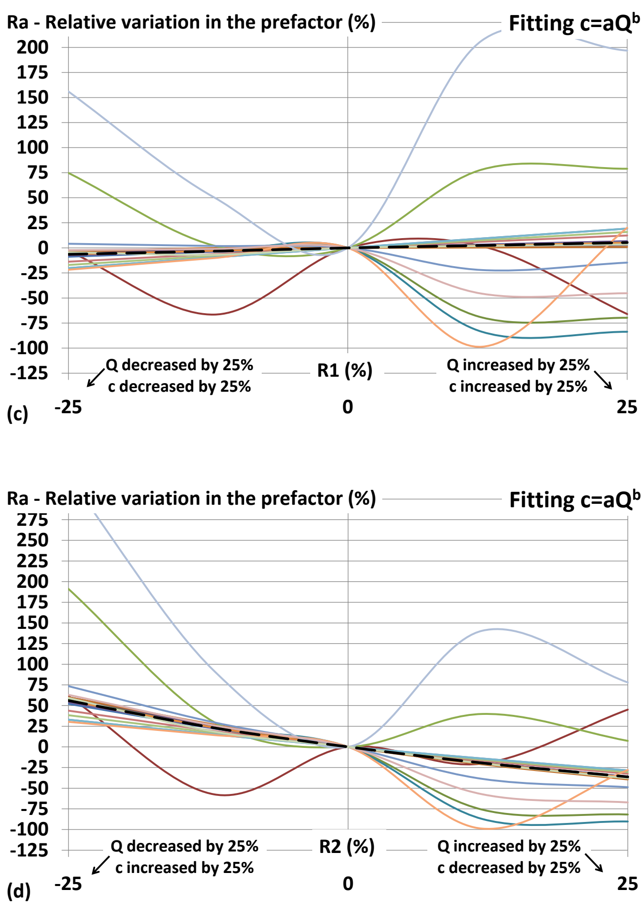

Fig. 6. Relative variations Ra in the fitted value of the prefactor in the $c = aQ^b$ law, for relative variations RQ' in discharge only **(a)**, Rc' in concentration only **(b)** or combined relative variations $R1$ **(c)** and $R2$ **(d)** detailed in Fig. 2. Bold dotted lines figure median Ra values.

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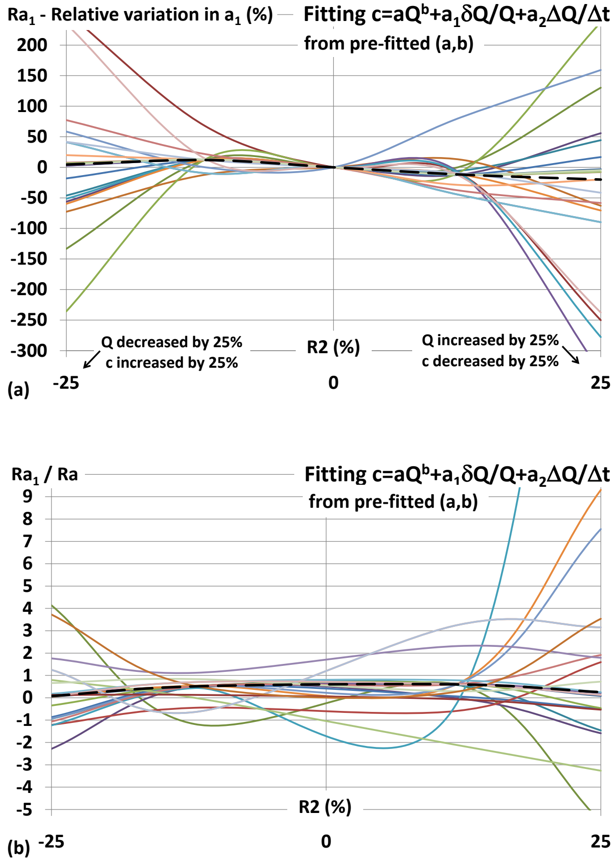

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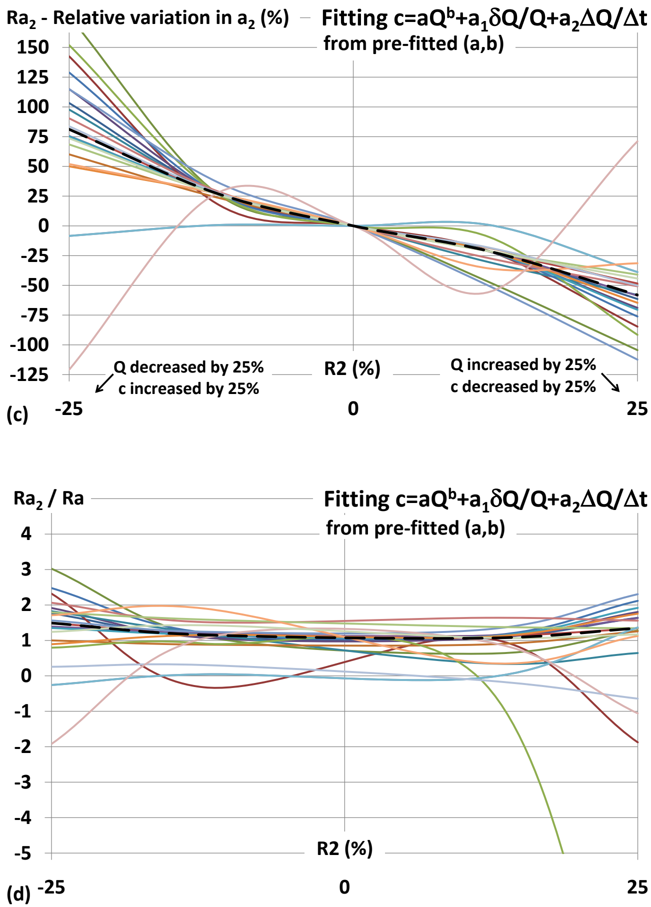

Fig. 7. See caption on next page.

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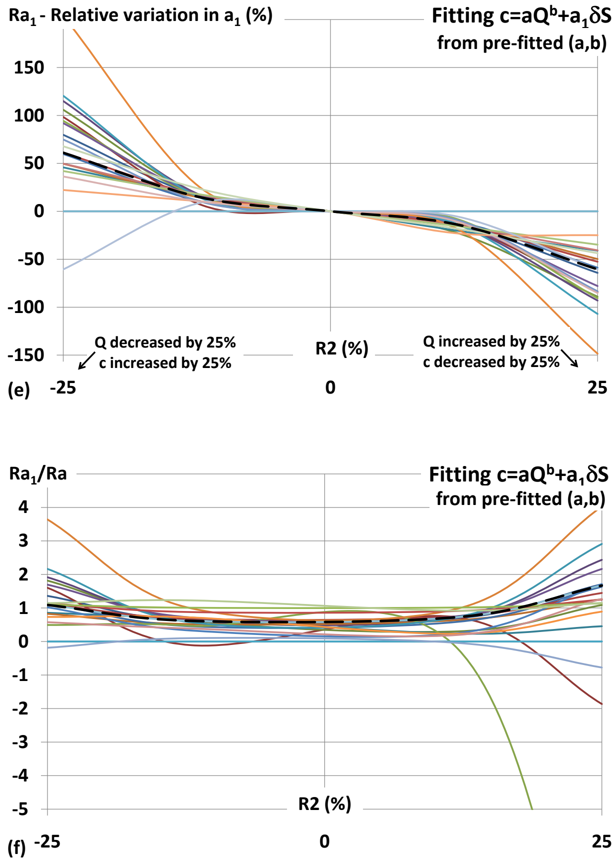

Fig. 7. Relative variations of the prefactors governing the correcting terms in fittings $f = 3$ **(a, c)** and $f = 4$ **(e)** for combined discharge and concentration variations of opposite signs, on the $R2$ axis detailed in Fig. 2. Relative variations of these prefactors compared to that of the a parameter in the reference $c = aQ^b$ fitting, for the same variations along the R2 axis **(b, d, f)**.

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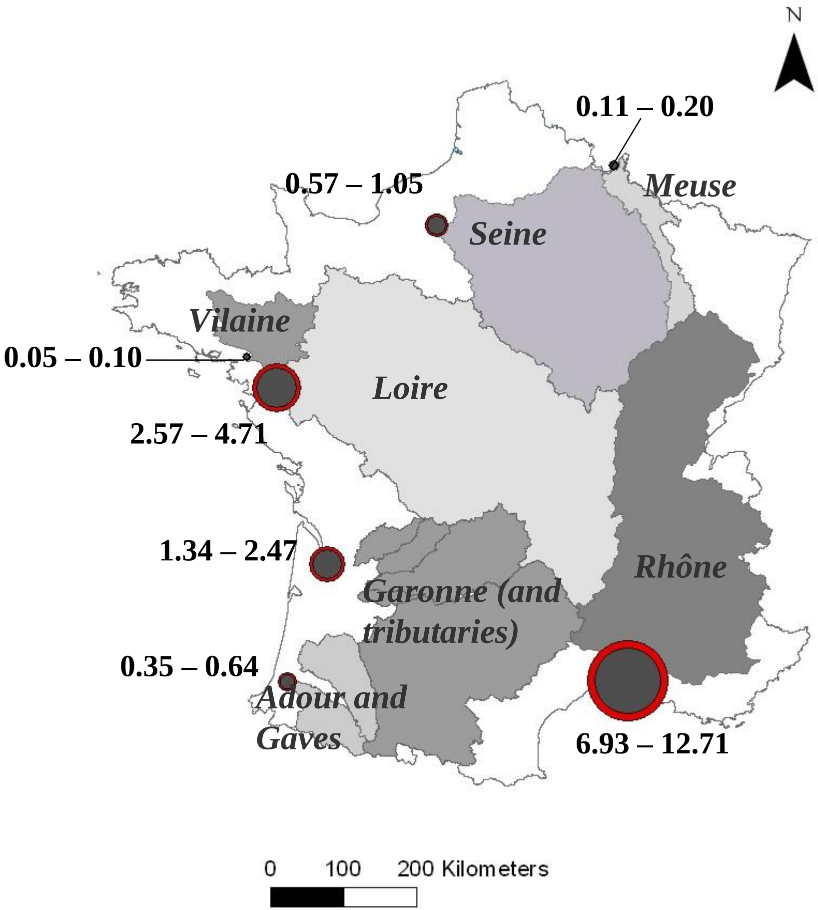

Fig. 8. Map of the sediment budget for the major French rivers to the sea, having considered the uncertainties arising from insufficient sampling frequency and those issued from the source discharge-concentration data. Values are given in Mt/yr.

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