Referee Report
"Tidal propagation in an oceanic island with sloping beaches"
by Y.C. Chang, D.S. Jeng, and H.D. Yeh,
offered for publication to HESS

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1 General comments

The authors apply perturbation techniques with two parameters (ε and α) to supply an approximate solution for the tide-induced groundwater fluctions in an oceanic island with finite width and two different slopes of the beaches. Due to the nonlinear boundary condition (6) the problem is quite complicated. This paper is clearly an extension of the work by Teo *et al.* (2003), but the mathematics is much more involved. The authors explain their method in the Appendices where they derive their results for the first-order cases, leaving the derivations for the much more complicated second-order cases fully from this paper. The authors are helpful for the readers to derive the basic equations which were used by Teo *et al.* (2003) without derivation by these authors.

The whole character of this paper is more or less a mathematical exercise in perturbation techniques.

This reviewer found some errors which might crept in during the presentation and found some simplifications which might have been overlooked by the authors or by the formula manipulation package. If the authors have used some formula manipulation packages, that should be stated clearly. Checking all calculations by hand should be an enormous task and if the authors have used a formula manipulation package, one could have more confidence in their results. In view of the results in Table 1 this reviewer suspects that the authors have used indeed a formula manipulation package which failed to reduce the expressions found to a more concise form. See under the specific remarks below. This reviewer has checked the results of Appendix A and B, and he found some errors in the presentation of the derivation of Appendix A. The final results for Appendix A and B (H_{01}) are correct, but H_{01} can be written in a more concise form. This reviewer did not check the contents of Table 1, except for the values of a_1 ,a_2, δ_{11} and δ_{12}.

Summarizing, some more information about the derivation of the results is needed.

In their concluding remarks the authors menton the obvious fact that the differences between the zero-order and higher-order approximations increase as ε and α increase. It is advised to include a figure to show that the difference between the successive approximations become smaller for fixed ε and α. This will justify their perturbation approach.

The results of this paper are interesting, but only applicable if the small parameters (ε and $\alpha = A/D$) are indeed small. A remark about the parameter ε :

$$\varepsilon = \sqrt{\frac{D n_e \omega}{2K}},$$

(see also Teo *et al.* (2003), formula (6)) follows. For a normal tide $\omega = 2\pi/P \sim 2\pi/(1/2) = 4\pi$ [day^{-1}], $n_e \sim 0.3$, we have $\varepsilon \sim \sqrt{\frac{2D}{K}}$, which gives quite some restrictions for the applicability of their results in order ε to be small. This item is already discussed by Teo *et al.* (2003).

My recommendation is that the authors spend some more attention to the derivation of the results and that they check their constants carefully in Table 1.

2 Some specific remarks

Page 1410, line 9: The parameter A should be visible in Fig. 1.

Page 1411, line 9: $L = \sqrt{2KD/(n_e \omega)}$ in stead of $L = \sqrt{KD/(n_e \omega)}$.

Page 1411, line 16: In equation (11) the term $\varepsilon \Phi_x^2$ should be $\varepsilon \Phi_X^2$. There should be added "$z = H$".

Page 1412, line 16: For clarity, in equation (18a) the arguments of Φ_n should be added, so $\Phi_n(X, Z, T)$.

Page 1412, line 18: For clarity, in equation (18a) the arguments of H_n should be added, so $H_n(X, T)$.

Page 1419, line 12: Insert "leads to $C_2^* = 0$" after "(A6b)".

Page 1419, line 16: Delete "in Eq. (11)".

Page 1420, line 4: It should be remarked that one needs also the knowledge of Φ_{3Z} and Φ_{4Z}.

Page 1420, line 18: Change left-hand side into "$2(\Phi_{T_1} + \Phi_{X_1} \alpha \varepsilon \cot \beta_1 \sin T_1)$".

Page 1420, line 7: Change "$\varepsilon^4 H_{0X_1X_1}^2 H_0$" into "$\varepsilon^4 H_{0X_1X_1}^2 H_0^2$".

Page 1420, line 9: Change the first term into "$2H_0^2 H_{0X_1} H_{0X_1X_1X_1}$".

Page 1420, line 9: Delete the terms "$H_0 H_{0X_1}^2 H_{0X_1X_1}$" and "$H_0^2 H_{0X_1X_1}^2$".

Page 1420, line 9: Put a ")" after the term with the factor "$\frac{1}{3}$"; delete the ")" in front of the last "]".

Page 1420, line 15: Write that line as

$$H_{01} = \mathrm{Im}\left[(\Lambda_1 \exp((1+i)X_1) + \Lambda_2 \exp(-(1+i)X_1)) \exp\left(i\left(T_1 + \frac{\pi}{2}\right)\right)\right].$$

Reference could be made to the explanation for this in the following book directed to the field of hydrology: G.A. Bruggeman, "Analytical Solutions of Geohydrological Problems", Elsevier, 1999, p. 793-802. Otherwise, the authors should give another reference.

Page 1421, line 17: The term a_1 can be written simpler as: $a_1 = \frac{\cos X_R + \exp(-X_R)}{2(\cosh X_R + \cos X_R)}$.

Page 1421, line 19: The term a_2 can be written simpler as: $a_2 = \frac{\sin X_R}{2(\cosh X_R + \cos X_R)}$.

Page 1424, line 3: The first line in the expression for Δ_1 should read

$$\frac{1}{2}(\delta_{15} + \delta_{16}) \sin 2\sqrt{2} X_R,$$

Also line 4 can be reduced.

Page 1424, line 7: The first line in the expression for Δ_2 should read

$$-\frac{1}{2}(\delta_{16} + \delta_{18}) \sin 2\sqrt{2} X_R,$$

Also line 8 can be reduced.

Page 1422, line 4: "Dagan, D." -> "Dagan, G.".

Page 1422, line 19: "Ki, H." -> "Li, H.".

Page 1423, lines 3 and 4: The values for a_1 and a_2 should be modified; see above.

3 References

H.T. Teo, D.S. Jeng, B.R. Seymour, D.A. Barry, L.Li, "A new analytical solution for water table fluctuations in coastal aquifers with sloping beaches", Advances in Water Resources 26 (2003) 1239-1247.