Interactive comment on "Estimations of tidal characteristics and aquifer parameters via tide-induced head changes in coastal observation wells" by Y.-J. Chen et al.

Anonymous Referee #2

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The manuscript presents an approach for estimating tidal characteristics and aquifer parameters using the analytical solution of Jeng et al. (2005) and inverse modeling. The manuscript is well written and is appropriate for publication in Hydrology and Earth System Sciences Discussions. However, I have several comments that if addressed should improve the manuscript.

General Comments:

1. The authors make suggest that the method presented in the manuscript may provide a better estimation of aquifer parameters than the approach used in Nielsen (1990) for real world applications. Could application of the method of Nielsen (1990) and the simulated annealing optimization technique to the synthetic scenarios (1, 2, and/or 3) be used to demonstrate that method presented is clearly better? Obviously, the approach of Nielsen (1990) ignores the A2, δ2, and ω2 components but it may be instructive to demonstrate that the solution of the inverse problem, using synthetic data generated using the analytical solution of Jeng et al. (2005), is sensitive to the A2, δ2, and ω2 components.

Reply: Scenario 6 in Table 4 (shown below and also added in the revised manuscript) has the same synthetic WWL data as scenario 2. The parameter values shown in scenario 6 were estimated based on the simulated annealing approach; yet, Nielsen's solution (1990), in lieu of Jeng et al.'s solution (2005), was adopted to fit the WWL data. Note that the constraint on the shallow water parameter ε to be less than 0.6 is no longer required in scenario 6. The estimated values of A_1 and ω_1 are fairly close to the target values of main harmonic constituent of bichromatic tide. However, the estimated values of K/n_e and β are larger than their target values. Figure 2 (also given below) shows the synthetic heads and predicted heads in scenarios 2 and 6. The synthetic heads were generated based on Jeng et al.'s solution (2005) and the bichromatic-tide parameters given in Table 4. This figure shows that the predicted heads in scenario 2 match well with the synthetic heads. However, the predicted heads in scenario 6 significantly differ from the synthetic heads because Nielsen's solution (1990) only considers the monochromatic-tide effect.

Table 4 The results estimated based on Nielsen's solution (1990) with the synthetic WWL data generated from Jeng et al.'s solution (2005).

	Estimated Results										
	Aquifer Parameters				Tidal Characteristics						
	K / n_e (m/day)	β (rad)	β (degree)	D (m)	A_1 (m)	A_2 (m)	ω_1 (day^{-1})	ω_2 (day^{-1})	δ_2	RMSE (m)	CPU time (sec)
Target values	500	1.047	60	25	2	1	12.567	6.283	0.785	-	
scenario 6											
6a	583.962	1.336	76.546	25.039	1.931	-	12.566	-	-	0.584	13.96
6b	580.929	1.382	79.159	25.041	1.930	-	12.566	-	-	0.584	14.33
6c	578.870	1.377	78.871	25.042	1.932	-	12.566	-	-	0.583	13.93
6d	584.313	1.382	79.178	25.040	1.937	-	12.566	-	-	0.584	14.00
6e	578.516	1.312	75.153	25.037	1.933	-	12.566	-	-	0.586	13.84
Mean	581.318	1.358	77.781	25.040	1.932	-	12.566	-	-	-	-
SD	2.737	0.032	1.835	0.002	0.003	-	0.000	-	-	-	-
95% LLCI	577.920	1.318	70.580	25.037	1.929	-	12.566	-	-	-	-
95% ULCI	584.716	1.397	84.983	25.042	1.936	-	12.566	-	-	-	-

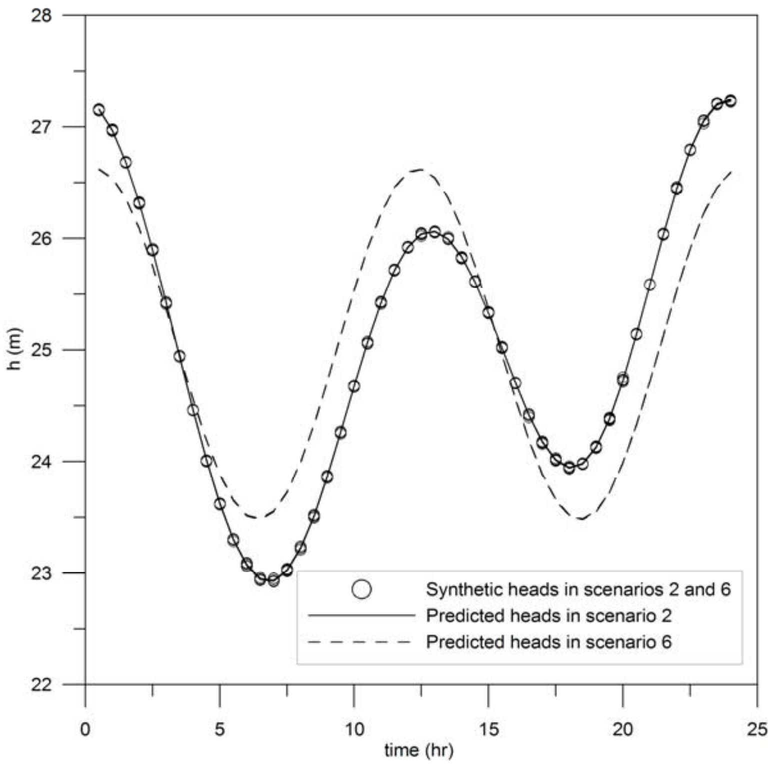

Figure 2 Comparisons of synthetic heads and predicted heads in scenarios 2 and 6. The synthetic heads in scenario 2 are analyzed based on Jeng et al.'s solution (2005) and those in scenario 6 are analyzed based on Nielsen's solution (1990).

2. Scenario 4 was configured to use a shallow water parameter (ε) of 1.772. Could the authors discuss why values converge to such different values? For example, is the maximum number of iterations exceeded in some cases but not others? Does the simulated annealing optimization approach fail for large ε values with different initial parameter values (i.e., is the solution sensitive to initial parameter values)?

Reply:

(1) The synthetic data in Scenario 4 are generated by the set of particular parameters to have a large value of shallow water parameter (ε = 1.772). However, we imposed the constraint on this parameter during the search of a set of trial solutions for k/n_e, ω_1, and D in SA, which makes it impossible to find out the target parameters. The constraint was adopted for the reason that the solution of water-table height was derived based on the perturbation approximation with two parameters, amplitude parameter and shallow water parameter, to be far less than unity.

(2) Two termination conditions are applied to the SA algorithm. The algorithm will be terminated when the best-so-far objective function value between two consecutive temperature is less than 10^{-6} for four consecutive times or the iteration number exceeds 2×10^7. In scenario 4, the SA algorithm was terminated when the first condition was met. The total iteration numbers in scenarios 4a to 4e are 196200, 199800, 201600, 203400, and 207000, respectively, which didn't exceed the maximum iteration number allowed in the algorithm.

(3) Table A shows the estimated results for scenario 4 while the search in SA algorithm starts with random parameter values within their lower and upper bounds. All the estimated results are similar to those in Table 3 (shown at the end of this reply), with the exception of scenario 4b in Table 3, which has significantly smaller values of k/n_e and β. Generally speaking, SA has the ability to escape from the local optimum solution. Different initial parameter values within the upper and lower bounds wouldn't affect the final solution as long as sufficient trial solutions have been made during the SA procedure.

Table A The target values and estimated results for scenario 4. The SA algorithm starts with random parameter values within the lower and upper bounds.

	Estimated Results									
	Aquifer Parameters				Tidal Characteristics					
	K / n_e (m/day)	β (rad)	β (degree)	D (m)	A_1 (m)	A_2 (m)	ω_1 (day^{-1})	ω_2 (day^{-1})	δ_2	RMSE (m)
Target values	50.000	1.047	59.989	25.000	2.000	1.000	12.566	6.283	0.785	-
scenario 4										
4a-IG	10000.000	1.571	90.000	28.082	0.509	3.337	12.566	1.346	2.769	0.266
4b-IG	9999.999	1.571	90.000	25.446	0.591	0.747	12.566	3.346	2.050	0.237
4c-IG	9999.999	1.571	90.000	25.440	0.593	0.740	12.566	3.365	2.043	0.238
4d-IG	9999.998	1.571	89.999	25.441	0.598	0.745	12.566	3.356	2.048	0.237
4e-IG	9999.997	1.571	89.999	25.432	0.593	0.738	12.566	3.389	2.038	0.237
Mean	9999.999	1.571	90.000	25.968	0.577	1.261	12.566	2.960	2.190	-
SD	0.001	0.000	0.000	1.182	0.038	1.160	0.000	0.902	0.324	-
95% LLCI	9999.997	1.571	89.999	24.501	0.530	-0.179	12.566	1.840	1.787	-
95% ULCI	10000.000	1.571	90.001	27.436	0.624	2.702	12.566	4.081	2.592	-
RE (%)	19899.997	50.028	50.028	3.873	-71.151	26.138	0.000	-52.885	178.791	

Table 3 The estimated results for the synthetic WWL data. Scenarios 4 and 5 have the same target parameter values and well location as scenario 2 except that K/n_e become 50 m/day and 5000 m/day, respectively, representing the cases of a shallow water parameter ε being 1.772 and 0.177.

	Estimated Results										
	Aquifer Parameters				Tidal Characteristics						
	K/n_e (m/day)	β (rad)	β (degree)	D (m)	A_1 (m)	A_2 (m)	ω_1 (day^{-1})	ω_2 (day^{-1})	δ_2	RMSE (m)	CPU time (sec)
scenario 4											
Target values	50	1.047	59.989	25	2	1	12.567	6.283	0.785	-	-
4a	9999.994	1.571	90.000	25.435	0.594	0.738	12.566	3.373	2.045	0.238	74.31
4b	441.079	0.113	6.477	24.539	2.671	2.038	12.566	11.679	3.006	0.209	88.94
4c	9999.999	1.571	90.000	25.440	0.593	0.740	12.566	3.365	2.043	0.238	77.88
4d	9999.999	1.571	90.000	25.441	0.598	0.745	12.566	3.356	2.048	0.237	76.96
4e	10000.000	1.571	90.000	25.432	0.593	0.738	12.566	3.389	2.037	0.237	80.39
Mean	8088.214	1.279	73.295	25.257	1.010	1.000	12.566	5.033	2.236	-	-
SD	4274.878	0.652	37.352	0.402	0.928	0.580	0.000	3.716	0.430	-	-
95% LLCI	2781.103	0.470	26.924	24.759	-0.143	0.279	12.566	0.420	1.701	-	-
95% ULCI	13395.326	2.089	119.667	25.756	2.162	1.720	12.566	9.645	2.770	-	-
RE (%)	16076.428	22.182	22.159	1.029	-49.507	-0.037	0.000	-19.904	184.667	-	-
scenario 5											
Target values	5000	1.047	59.989	25	2	1	12.567	6.283	0.785	-	-
5a	5019.124	1.046	59.905	25.000	2.000	1.000	12.566	6.284	0.785	2.76×10^{-4}	74.08
5b	5016.455	1.019	58.413	25.002	1.997	1.001	12.564	6.271	0.787	8.43×10^{-3}	76.85
5c	4920.108	0.958	54.893	24.998	2.002	0.999	12.566	6.289	0.782	8.04×10^{-3}	75.08
5d	4869.845	1.002	57.398	24.999	2.006	1.001	12.566	6.272	0.790	8.45×10^{-3}	76.16

5e	4944.421	0.972	55.670	24.996	2.001	1.003	12.566	6.292	0.784	8.76×10^{-3}	74.35
Mean	4953.991	0.999	57.256	24.999	2.001	1.001	12.566	6.281	0.785	-	-
SD	64.156	0.035	2.029	0.002	0.003	0.002	0.001	0.009	0.003	-	-
95% CI	4874.343	0.955	54.736	24.996	1.997	0.999	12.565	6.270	0.782	-	-
95% CI	5033.639	1.043	59.775	25.002	2.005	1.003	12.567	6.293	0.789	-	-
RE (%)	-0.920	-4.556	-4.574	-0.004	0.063	0.078	-0.004	-0.029	0.012	-	-

3. A relatively simple tidal forcing function composed of two harmonic constituents (bichromatic) has been used. In reality, the tide at Barrenjoey beach is more slightly more complicated (Figure 1). For example, using the harmonic constituents reported in the manuscript (9162 Lines 11-13) and equation 2 generally captures the rising and falling limbs of the tide but over- or under-predicts minimum and maximum tide levels. Although additional harmonic constituents cannot be accounted for in the method of Jeng et al. (2005), can the authors discuss how additional harmonic constituents might affect the ability to simulate observed groundwater levels at Barrenjoey beach?
Reply: The tide behavior is usually represented by a combination of several harmonic constituents using the sine or cosine functions. Figure 2, shown in the reply to the first comment, demonstrates the WWL oscillations in scenario 6 produced based on Nielsen's solution (1990) using a monochromatic tide and in scenario 2 generated from Jeng et al.'s solution (2005) with a bichromatic tide. The pattern of the predicted WWL data in scenario 6 can be depicted with a single cosine function. In contrast, the predicted WWL data in scenario 2 has the period characteristic of each monochromatic tide as well as a new period of resultant tide. In conclusion, Jeng et al.'s solution (2005), which considers additional harmonic constituents, might be more flexible to fit the complicated field data.

4. The analytical solution of Jeng et al. (2005) assumes the aquifer is homogeneous and incompressible. It is likely that the aquifer is heterogeneous and that this heterogeneity is contributing to the difference between aquifer parameters estimated using the optimization approach (simulated annealing) applied in this study. Can the authors comment on the role that heterogeneity may play in the ability to fit the observed data and under what conditions a method that can account for spatial heterogeneity would need to be applied to tidally-induced head problems (i.e., a numerical solution with a highly-parameterized inversion technique).
Reply:
We agree that it is likely the aquifer is heterogeneous in the real-world cases. If aquifer heterogeneity is significant, one may replace Jeng et al.'s solution (2005) with Chuang et al.'s solution (2010), which is suitable to describe the head fluctuation in a coastal leaky and heterogeneous aquifer system as shown in Figure A, in the work of parameter estimation. However, additional parameters such as the number of horizontal regions and the length of each region should be known in advance while the aquitard is considered to have the same hydrogeologic properties as the underlain confined aquifer.

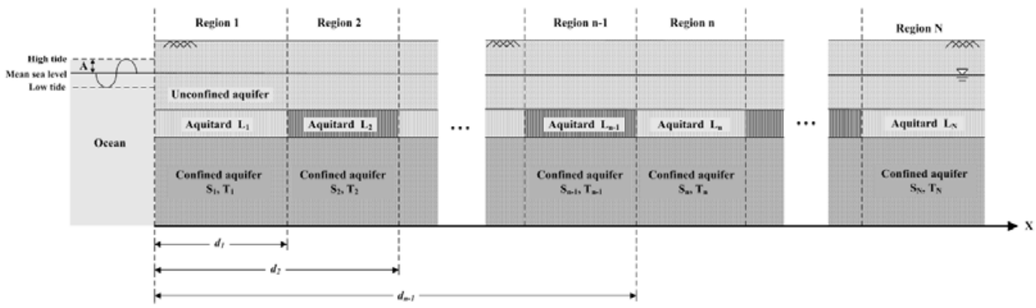

Figure A The heterogeneous coastal aquifer system considered in Chuang et al. (2010). (Source: Chuang et al., 2010, Figure 1)

Specific Comments:

1. 9158 Lines 16-20: It should be clearly stated that the solution of Jeng et al. (2005) neglects the effect that a seepage face would have on groundwater levels. Lines 18-19 indicate that the water table height at the boundary equals the tidal oscillation but readers may not be fully aware of the assumptions of Jeng et al. (2005).

Reply: Thanks for the suggestion. The sentence “The water table height at the boundary condition of ocean and coast equals tidal oscillation; that is,” in Lines 18-19 on Page 9158 is rewritten as “The water table height at the boundary of ocean and coast equals tidal oscillation (i.e., no seepage face); that is,”

2. 9159 Line 9: Suggest “effective porosity” rather than “soil porosity”

Reply: Thanks, we have made the correction.

3. 9161 Line 6: Suggest modifying “. . .1% for representing the accuracy of. . .” to “. . .1% and represents the accuracy of. . .”

Reply: Thanks, we have made the change.

4. 9171 Figure 2: It is difficult to read the figure legends and distinguish what each line represents on the printed version of the manuscript. Suggest the authors increase the size of the subplots, subplot text, and possibly use color.

Reply: Thanks for the comment. We have provided a revised version of this figure, which is Figure 3 in the revised manuscript. This new figure has the WWL predicted based on the result of a multi-well analysis, i.e., analysis using five sets of WWL data simultaneously.

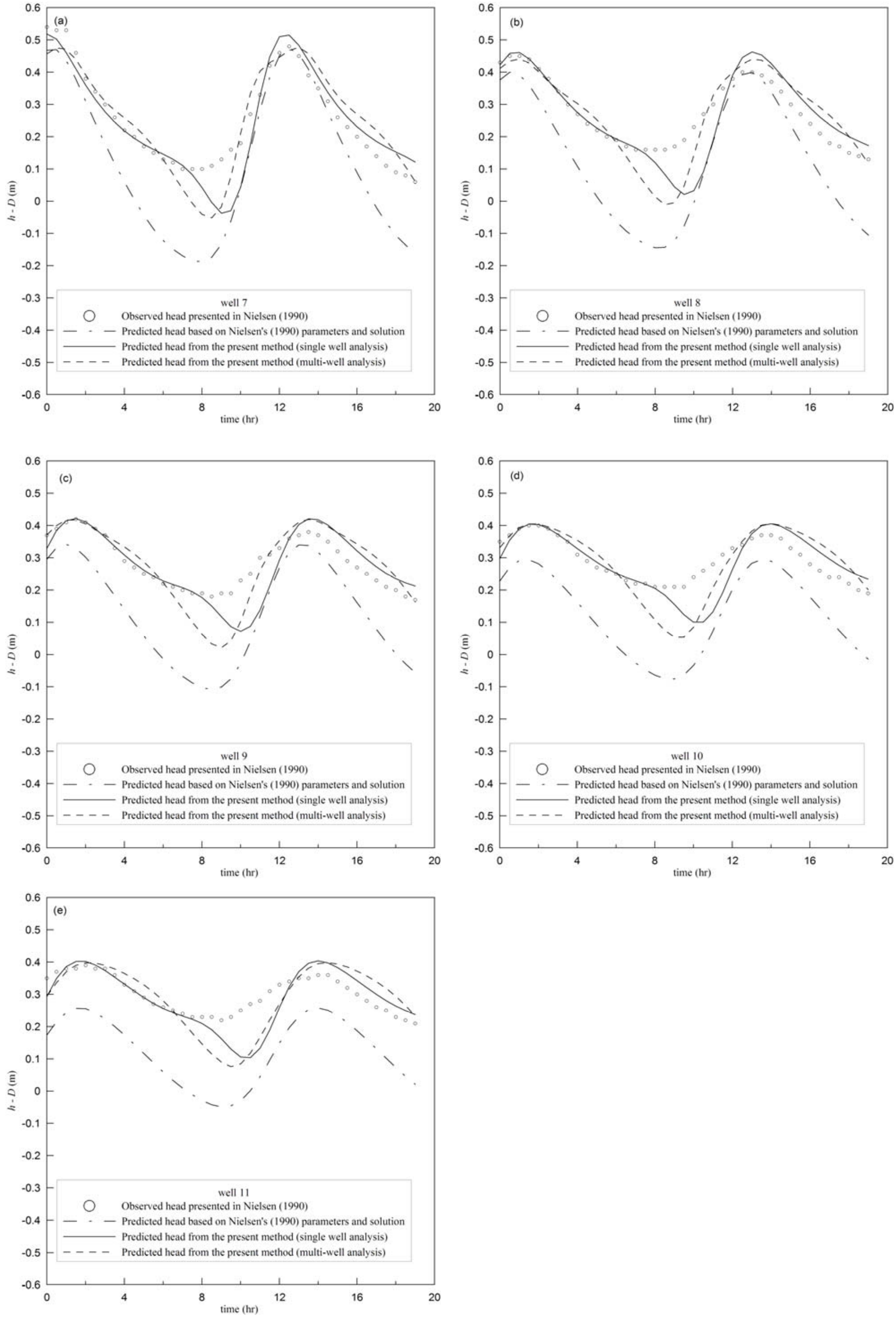
(a)
well 7
(b)
well 8
(c)
well 9
(d)
well 10
(e)
well 11
Observed head presented in Nielsen (1990)
Predicted head based on Nielsen's (1990) parameters and solution
Predicted head from the present method (single well analysis)
Predicted head from the present method (multi-well analysis)
h - D (m)
time (hr)

Figure 3 Plots of observed WWL given in Nielsen (1990), predicted WWL produced by Nielsen's parameter and solution (1990), and predicted WWLs produced by Jeng et al.'s solution (2005) with the parameters determined by the present method via single-well and multi-well analyses.

References:

Chuang, M.H., Huang, C.S., Li, G.H., and Yeh, H.D. (2010). "Groundwater fluctuations in heterogeneous coastal leaky aquifer systems." *Hydrol. Earth Syst. Sci.*, 14, 1819-1826.

Jeng, D.S., Mao, X., Enot, P., Barry, D. A., and Li, L. (2005). "Spring-neap tide-induced beach water table fluctuations in a sloping coastal aquifer." *Water Resources Research*, 41, W07026, doi:10.1029/2005WR003945.

Nielsen, P. (1990). "Tidal dynamics of the water-table in beaches." *Water Resources Research*, 26(9), 2127-2134.

Teo, H. T., Jeng, D. S., Seymour, B. R., Barry, D. A., and Li, L. (2003). "A new analytical solution for water table fluctuations in coastal aquifers with sloping beaches." *Advances in Water Resources*, 26(12), 1239-1247.