Interactive comment on “Estimations of tidal characteristics and aquifer parameters via tide-induced head changes in coastal observation wells” by Y.-J. Chen et al.

Anonymous Referee #1

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This paper presents an inverse modelling study in which head variations in observation wells induced by tidal variations are used to estimate aquifer parameters (and tidal characteristics in the synthetic study). Modelling is based on an analytical solution from Jeng et al. (2005). The aquifer parameters are optimized using a simulated annealing algorithm. The contribution that this paper claims to make is that is the first attempt to use the analytical expression in an inverse framework. It is my personal feeling that this is only a minor contribution and something that was obviously already intended when the analytical solution was derived. Therefore, I cannot currently recommend this paper for publication. Perhaps when the general comments listed below have been addressed, the contribution of the paper becomes clearer.

GENERAL COMMENTS

1. Simulated annealing is used as an optimization algorithm. This is a global optimization method that requires considerable computational resources. Is a global search method required for the current problem? Can the problem be solved with local search methodologies? Why did the authors choose to use simulated annealing instead of an optimization strategy from the family of genetic algorithms, which are much more established in the hydrological community and can be made more efficient than simulated annealing?

Reply:

(1) The computing time is mainly related to the complexity of the objective function and the severity of the convergence criterion. In calculating the objective function, the cosine and exponential functions in Jeng et al.’s solution (2005) can be easily and quickly calculated by personal computer nowadays. Tables 1, 3, and 4 provide the computing time (CPU time) for parameter estimation in each case by using a personal computer with Genuine Intel CPU 2140 @ 1.60 GHz and 1 GB RAM. Obviously, the computing task is not a problem since the search for the optimal results could be done within two minutes for all of the cases as demonstrated in the tables.

(2) In this study, we estimated eight parameters in the model that describes the tide-induced water table fluctuation. The problem of parameter estimation involves multi-degrees nonlinear optimization and may contain several local optima in the problem domain. A local search algorithm usually starts from an initial guess value,

referred to as the current solution, and moves to a neighbor solution according to the search methodology. Often, the result might end up with a local optimum. In contrast, the simulated annealing (SA) randomly chooses the neighbor solution by the method demonstrated in the next comment reply. The Metropolis criterion (Eq. (5) in the manuscript) decides whether the current solution moves to the neighbor solution or not. If the chosen neighbor solution has a better objective function value than the current one, it is then accepted as the new current solution. On the other hand, the current solution still has a chance to move to a worse neighbor solution based on the Metropolis criterion.
(3) SA has the ability to deal with the complicated problems which have high degrees of freedom. We have a lot of successful experiences in using SA to deal with various types of problem such as the THM forecast (Lin and Yeh, 2005), aquifer parameter estimation (e.g., Huang and Yeh, 2007; Yeh and Chen, 2007), pipe wall surface reaction rate (Yeh et al., 2008), and pumping source information (Lin and Yeh, 2008). This is the reason why we choose SA to handle this parameter identification problem.

2. In the description of simulated annealing, it is not described how new proposal points are generated. This should be included.
Reply: On page 9159, we add a paragraph to illustrate the method in creating a new trial solution as follows: "A new trial solution $TS_i^{'}$ for parameter i is randomly generated by the following equation:

$$TS_i^{'} = TS_i + (2 \times RD - 1)VM_i \tag{5}$$

where TS_i is the current solution for parameter i, RD is a random number generated from a uniform (0, 1) distribution, and VM_i is a step length vector of the parameter i. If the trial solution is out of specified upper and lower bounds, an alternative approach for creating a new trial solution within the bounds is

$$TS_i^{'} = LB_i + RD(UB_i - LB_i) \tag{6}$$

where LB_i and UB_i representing the lower and upper bounds for each parameter i."

3. The authors present a synthetic test case for parameter estimation. This can be interesting, but the focus typically is not on the optimal parameters. Instead, it is more interesting to discuss parameter uncertainty and correlation in this context. This can

be achieved by presenting confidence intervals for the parameters, and presenting 2D plots of the error landscape. In addition, it would be interesting to generate the data for the synthetic case study with a numerical model instead of the analytical solution. In this case, the relevance of the approximations in the analytical solution can also be evaluated.

Reply:

(1) We have added the standard deviation (SD) and 95% lower-limit and upper-limit of confidence interval (95% LLCI and 95% ULCI) for each parameter shown in Tables 1, 3, and 4. The LLCI and ULCI are calculated using the formula $\bar{y} \pm s_{\bar{y}} t_{4,0.025}$ where $\bar{y}$ is the mean value of estimated parameters from cases a to e; $s_{\bar{y}}$ is the estimated standard error of the mean; $t_{4,0.025}$ is t statistic with degrees of freedom equaling 4 and 95% confidence interval, obtained from a t-distribution table as 2.776. In Table 1, the target values of estimated parameters, except for the beach slope β in scenarios 2 and 3, are all within their 95% confidence interval and have small RE values on the order of 10^{-4} to 10^{-3}. However, the results of scenario 4 in Table 3, which represents the case of a large target value of shallow water parameter ε, indicate that the target values of estimated parameters are all out of their 95% confidence interval. This is because the estimated parameters should meet the constraint that the value of shallow water parameter ε is less than 0.6 as discussed in the 4th comment.

(2) Table 2 provides the correlation matrix of the eight estimated parameters in scenarios 1 - 3. Only the correlation coefficients for each pair parameters in the lower triangular matrix are listed since the matrix is symmetric. The table indicates high correlation in the pairs of β and D as well as ω_2 and δ_2 marked in red color in scenarios 1 and 2. Moreover, there are 3 pairs have high correlation in scenario 3, which are β and D, ω_2 and δ_2, and D and ω_2. It is not clear to us about the 2D plot of error landscape. Does it mean the plot of the different between synthetic head and predicted head versus time?

(3) A numerical model can do a good job in generating the synthetic data for the case study and making comparison with the analytical solution. However, our study focuses on presenting a method for the estimation of the coastal aquifer parameters and tidal characteristics rather than aims at developing an analytical solution. Therefore, we think that the use of the numerical model is beyond the scope of this study.

4. I could not follow the discussion related to Table 2. The analytical solution is valid for a small range of the shallow water parameter. If it becomes too large, the assumptions are violated, and the solution is not valid. However, if I use the analytical solution both for the forward and inverse simulation, I do not see how such model structural errors can affect the results. This needs more clarification.

Reply: The solution of water-table height was derived based on the perturbation approximation with two parameters, amplitude parameter and shallow water parameter, to be far less than unity. Therefore, the users are not allowed to apply the solution for the forward or inverse simulation if those two constraints are not hold. In our manuscript, on page 9160, we demonstrated that "Teo et al. (2003) indicated that the shallow water parameter ε is usually small in real environments and suggested its value ranged from 0.1 to 0.6 in their simulation. ~~The trial solutions for k/n_e, ω_1, and D in each search are therefore constrained to ensure the value of shallow water parameter ε less than 0.6.~~ Therefore, an additional constraint was imposed during the search of a set of trial solutions for k/n_e, ω_1, and D to ensure that the constraint on the shallow water parameter ε is not violated."

5. I think it would also be valuable to present the fit to all measured WWL data from different wells simultaneously. This could give an idea on how valid the assumption of a homogeneous medium is. Please add and discuss this result.

Reply: Thanks for the comment. We have made a composite analysis for the WWL data collected from the five wells. The results can be observed in Table 5 and Figure 3, i.e., Table 3 and Figure 2 in the previous manuscript, respectively. The estimated values of k/n_e, ω_1, and D obviously differ from those obtained from the single-well WWL data analysis or those given in Nielsen (1990).

6. The simulated results show very specific deviations around t = 10. What is the reason for this?

Reply: The greatest discrepancies between the real WWL data and predicted WWL data occur at low tide, i.e., around t = 8 hr to t =10 hr. Nielsen (1990, Figure 7) also addressed the same problem when comparing the real data (also used in this study) with the data generated by his analytical solution. He explained that the discrepancy might be due to the boundary condition at x_0, where the assumption of no seepage face contradicts the reality. Such a problem becomes more serious in the situations of flat beaches and/or large tidal range. Under such circumstances, the water table may emerge at the exit point some distance above the shoreline. In other words, the head drop between the tide and the water table makes the boundary condition of $h(x_0,t) = h_{tide} = D + A\cos\omega t$ at $x_0 = (h_{tide} - D)\cot\beta$ in Nelsen (1990) invalid. The same problem might occur when applying Jeng et al.'s solution (2005) to calculate the WWL data. The boundary condition for the spring-neap tide at $x_0 = [A_1\cos(\omega_1 t + \delta_1) + A_2\cos(\omega_2 t + \delta_2)]\cot\beta$ is represented as $h(x_0,t) = D + A_1\cos(\omega_1 t + \delta_1) + A_2\cos(\omega_2 t + \delta_2)$.

SPECIFIC COMMENTS

Page 9161, Line 14. The relative error is negative in Table 1 when the value is higher than the true value. This is not intuitive, and I propose to reverse the sign.

Reply: Thanks for pointing out the problem. The RE values given in Tables 1 and 3 have been corrected.

Table 1. Provide actual values as first line in the table instead of the caption. This makes the table easier to read.

Reply: Thanks for the suggestion. The sentence "The target values of the parameters are K/n_e = 500 m/day, β = 1.047, D = 25 m, A_1 = 2 m, A_2 = 1 m, ω_1 = 4π day^{-1}, ω_2 = 2π day^{-1} and δ_2 = π/4." has been removed from the caption. In addition, we have provided the "Target values" for each parameter in the first row (right above scenario 1) of Table 1.

OTHER MODIFICATIONS

The revised Tables and Figures are given at the end of this reply. Note that Table 4 and Figure 2 are new in this version to demonstrate the feasibility of using Nielsen's solution (1990), in lieu of Jeng et al.'s solution (2005), to analyze the synthetic WWL data in scenario 2.

References:

Lin, Y. C., and Yeh, H. D. (2005). "Trihalomethane species forecast using optimization method: genetic algorithm and simulated annealing." *Journal of Computing in Civil Engineering*, *ASCE*, 19(3), 248-257, doi:10.1061/(ASCE)0887-3801(2005)19:3(248).

Lin, Y. C., and Yeh, H. D. (2008). "Identifying groundwater pumping source information using optimization approach." *Hydrological Processes*, 22, 3010-3019, doi:10.1002/hyp.6875.

Huang, Y. C., and Yeh, H. D. (2007). "The use of sensitivity analysis in on-line aquifer parameter estimation." *Journal of Hydrology*, 335(3-4), 406-418.

Jeng, D. S., Mao, X., Enot, P., Barry, D. A., and Li, L. (2005). "Spring-neap tide-induced beach water table fluctuations in a sloping coastal aquifer." *Water Resources Research*, 41, W07026, doi:10.1029/2005WR003945.

Nielsen, P. (1990). "Tidal dynamics of the water-table in beaches." *Water Resources Research*, 26(9), 2127-2134.

Teo, H. T., Jeng, D. S., Seymour, B. R., Barry, D. A., and Li, L. (2003). "A new analytical solution for water table fluctuations in coastal aquifers with sloping beaches." *Advances in Water Resources*, 26(12), 1239-1247.

Yeh, H. D., and Chen, Y. J. (2007). "Determination of skin and aquifer parameters for a slug test with wellbore-skin effect." *Journal of Hydrology*, 342, 283-294.

Yeh, H. D., Wen, S. B., Chang, Y. C., and Lu, C. S. (2008). "A new approximate solution for chlorine concentration decay in pipes." *Water Research*, 42, 2787-2795, doi:10.1016/j.watres.2008.02.012.

Table 1 The estimated results for the synthetic WWL data. Scenarios 1, 2, and 3 denote the wells located at x = 5, 10, and 20 m, respectively. The target values of the parameters are K/n_e = 500 m/day, β = 1.047, D = 25 m, A_1 = 2 m, A_2 = 1 m, ω_1 = 4π day^{-1}, ω_2 = 2π day^{-1} and δ_2 = $\pi/4$.

	Estimated Results										
	Aquifer Parameters				Tidal Characteristics						
	K/n_e (m/day)	β (rad)	β (degree)	D (m)	A_1 (m)	A_2 (m)	ω_1 (day^{-1})	ω_2 (day^{-1})	δ_2	RMSE (m)	CPU time (sec)
Target values	500	1.047	60	25	2	1	12.567	6.283	0.785	-	-
scenario 1											
1a	502.888	1.046	59.915	25.000	1.999	1.000	12.566	6.283	0.785	2.61×10^{-4}	76.49
1b	482.076	1.048	60.022	25.002	2.002	1.003	12.566	6.266	0.790	8.46×10^{-3}	79.36
1c	491.280	1.016	58.239	24.998	2.003	1.000	12.566	6.288	0.782	8.09×10^{-3}	81.94
1d	488.153	1.036	59.354	24.999	2.008	1.003	12.566	6.269	0.790	8.46×10^{-3}	78.97
1e	516.780	1.008	57.734	24.996	1.996	1.002	12.564	6.296	0.781	8.73×10^{-3}	83.50
Mean	496.235	1.031	59.053	24.999	2.002	1.001	12.566	6.281	0.786	-	-
SD	13.754	0.018	1.022	0.002	0.004	0.001	0.001	0.013	0.004	-	-
95% LLCI	479.160	1.009	57.784	24.996	1.996	1.000	12.564	6.265	0.780	-	-
95% ULCI	513.311	1.053	60.321	25.002	2.007	1.003	12.567	6.296	0.791	-	-
RE (%)	-0.753	-1.560	-1.579	-0.004	0.092	0.133	-0.005	-0.041	0.045	-	-
scenario 2											
2a	500.520	1.047	59.982	25.000	2.000	1.000	12.566	6.283	0.785	3.07×10^{-4}	87.33
2b	488.703	1.049	60.111	25.002	2.003	1.003	12.566	6.263	0.792	8.50×10^{-3}	86.75
2c	496.319	1.007	57.725	24.997	2.003	0.999	12.566	6.292	0.781	8.03×10^{-3}	88.53
2d	494.530	1.034	59.231	24.999	2.008	1.002	12.566	6.267	0.792	8.42×10^{-3}	86.27

2e	505.278	1.005	57.565	24.995	1.998	1.002	12.565	6.299	0.781	8.79×10^{-3}	92.11
Mean	497.070	1.028	58.923	24.999	2.002	1.001	12.566	6.281	0.786	-	-
SD	6.251	0.021	1.215	0.003	0.004	0.002	0.001	0.016	0.006	-	-
95% CI	489.310	1.002	57.414	24.995	1.998	0.999	12.565	6.261	0.779	-	-
95% CI	504.831	1.055	60.431	25.002	2.007	1.003	12.567	6.300	0.793	-	-
RE (%)	-0.586	-1.777	-1.796	-0.005	0.124	0.128	-0.003	-0.041	0.095	-	-
scenario 3											
3a	500.151	1.048	60.038	25.000	2.000	1.000	12.566	6.283	0.785	2.73×10^{-4}	95.81
3b	490.044	1.064	60.991	25.005	2.007	1.006	12.566	6.248	0.798	8.51×10^{-3}	102.19
3c	498.577	0.995	56.998	24.996	2.003	0.998	12.566	6.300	0.778	8.05×10^{-3}	98.22
3d	498.298	1.037	59.418	25.000	2.008	1.002	12.566	6.259	0.794	8.49×10^{-3}	103.03
3e	504.058	0.983	56.322	24.993	1.998	1.001	12.564	6.308	0.779	8.68×10^{-3}	89.00
Mean	498.226	1.025	58.753	24.999	2.003	1.001	12.566	6.279	0.787	-	-
SD	5.118	0.035	2.006	0.005	0.004	0.003	0.001	0.026	0.009	-	-
95% CI	491.871	0.982	56.263	24.993	1.998	0.998	12.565	6.247	0.776	-	-
95% CI	504.580	1.069	61.243	25.004	2.009	1.005	12.567	6.311	0.798	-	-
RE (%)	-0.355	-2.059	-2.078	-0.005	0.159	0.133	-0.004	-0.061	0.211	-	-

Table 2 Correlation matrix of estimated parameters in scenarios 1- 3.

	K/n_e	β	D	A_1	A_2	ω_1	ω_2	δ_2
scenario 1								
K/n_e	1.000	—	—	—	—	—	—	—
β	-0.586	1.000	—	—	—	—	—	—
D	-0.752	0.933	1.000	—	—	—	—	—
A_1	-0.790	0.332	0.346	1.000	—	—	—	—
A_2	-0.281	0.176	0.202	0.266	1.000	—	—	—
ω_1	-0.747	0.640	0.671	0.663	-0.302	1.000	—	—
ω_2	0.852	-0.804	-0.835	-0.699	-0.569	-0.590	1.000	—
δ_2	-0.754	0.802	0.780	0.680	0.561	0.581	-0.973	1.000
scenario 2								
K/n_e	1.000	—	—	—	—	—	—	—
β	-0.561	1.000	—	—	—	—	—	—
D	-0.774	0.927	1.000	—	—	—	—	—
A_1	-0.673	0.278	0.360	1.000	—	—	—	—
A_2	-0.314	0.340	0.275	0.158	1.000	—	—	—
ω_1	-0.657	0.541	0.730	0.525	-0.302	1.000	—	—
ω_2	0.855	-0.808	-0.856	-0.704	-0.542	-0.578	1.000	—
δ_2	-0.766	0.770	0.774	0.699	0.652	0.451	-0.982	1.000
scenario 3								
K/n_e	1.000	—	—	—	—	—	—	—
β	-0.742	1.000	—	—	—	—	—	—
D	-0.891	0.962	1.000	—	—	—	—	—
A_1	-0.759	0.591	0.710	1.000	—	—	—	—
A_2	-0.713	0.655	0.713	0.539	1.000	—	—	—
ω_1	-0.570	0.603	0.659	0.577	0.000	1.000	—	—
ω_2	0.827	-0.892	-0.934	-0.853	-0.782	-0.562	1.000	—
δ_2	-0.783	0.862	0.897	0.808	0.849	0.448	-0.988	1.000

Table 3 The estimated results for the synthetic WWL data. Scenarios 4 and 5 have the same target parameter values and well location as scenario 2 except that K/n_e become 50 m/day and 5000 m/day, respectively, representing the cases of a shallow water parameter ε being 1.772 and 0.177.

	Estimated Results										
	Aquifer Parameters				Tidal Characteristics						
	K/n_e (m/day)	β (rad)	β (degree)	D (m)	A_1 (m)	A_2 (m)	ω_1 (day^{-1})	ω_2 (day^{-1})	δ_2	RMSE (m)	CPU time (sec)
scenario 4											
Target values	50	1.047	59.989	25	2	1	12.567	6.283	0.785	-	-
4a	9999.994	1.571	90.000	25.435	0.594	0.738	12.566	3.373	2.045	0.238	74.31
4b	441.079	0.113	6.477	24.539	2.671	2.038	12.566	11.679	3.006	0.209	88.94
4c	9999.999	1.571	90.000	25.440	0.593	0.740	12.566	3.365	2.043	0.238	77.88
4d	9999.999	1.571	90.000	25.441	0.598	0.745	12.566	3.356	2.048	0.237	76.96
4e	10000.000	1.571	90.000	25.432	0.593	0.738	12.566	3.389	2.037	0.237	80.39
Mean	8088.214	1.279	73.295	25.257	1.010	1.000	12.566	5.033	2.236	-	-
SD	4274.878	0.652	37.352	0.402	0.928	0.580	0.000	3.716	0.430	-	-
95% LLCI	2781.103	0.470	26.924	24.759	-0.143	0.279	12.566	0.420	1.701	-	-
95% ULCI	13395.326	2.089	119.667	25.756	2.162	1.720	12.566	9.645	2.770	-	-
RE (%)	16076.428	22.182	22.159	1.029	-49.507	-0.037	0.000	-19.904	184.667	-	-
scenario 5											
Target values	5000	1.047	59.989	25	2	1	12.567	6.283	0.785	-	-
5a	5019.124	1.046	59.905	25.000	2.000	1.000	12.566	6.284	0.785	2.76×10^{-4}	74.08
5b	5016.455	1.019	58.413	25.002	1.997	1.001	12.564	6.271	0.787	8.43×10^{-3}	76.85
5c	4920.108	0.958	54.893	24.998	2.002	0.999	12.566	6.289	0.782	8.04×10^{-3}	75.08

5d	4869.845	1.002	57.398	24.999	2.006	1.001	12.566	6.272	0.790	8.45×10^{-3}	76.16
5e	4944.421	0.972	55.670	24.996	2.001	1.003	12.566	6.292	0.784	8.76×10^{-3}	74.35
Mean	4953.991	0.999	57.256	24.999	2.001	1.001	12.566	6.281	0.785	-	-
SD	64.156	0.035	2.029	0.002	0.003	0.002	0.001	0.009	0.003	-	-
95% CI	4874.343	0.955	54.736	24.996	1.997	0.999	12.565	6.270	0.782	-	-
95% CI	5033.639	1.043	59.775	25.002	2.005	1.003	12.567	6.293	0.789	-	-
RE (%)	-0.920	-4.556	-4.574	-0.004	0.063	0.078	-0.004	-0.029	0.012	-	-

Table 4 The results estimated based on Nielsen's solution (1990) with the synthetic WWL data generated from Jeng et al.'s solution (2005).

	Estimated Results										
	Aquifer Parameters				Tidal Characteristics						
	K/n_e (m/day)	β (rad)	β (degree)	D (m)	A_1 (m)	A_2 (m)	ω_1 (day^{-1})	ω_2 (day^{-1})	δ_2	RMSE (m)	CPU time (sec)
Target values	500	1.047	60	25	2	1	12.567	6.283	0.785	-	
scenario 6											
6a	583.962	1.336	76.546	25.039	1.931	-	12.566	-	-	0.584	13.96
6b	580.929	1.382	79.159	25.041	1.930	-	12.566	-	-	0.584	14.33
6c	578.870	1.377	78.871	25.042	1.932	-	12.566	-	-	0.583	13.93
6d	584.313	1.382	79.178	25.040	1.937	-	12.566	-	-	0.584	14.00
6e	578.516	1.312	75.153	25.037	1.933	-	12.566	-	-	0.586	13.84
Mean	581.318	1.358	77.781	25.040	1.932	-	12.566	-	-	-	-
SD	2.737	0.032	1.835	0.002	0.003	-	0.000	-	-	-	-
95% LLCI	577.920	1.318	70.580	25.037	1.929	-	12.566	-	-	-	-
95% ULCI	584.716	1.397	84.983	25.042	1.936	-	12.566	-	-	-	-

Table 5 The estimated results for the aquifer parameters from Nielsen (1990) and the proposed method based on the field WWL data at Barrenjoey beach in Australia.

		Estimated Aquifer Parameters				
	x (m)	K/n_e (m/day)	β (rad)	D (m)	RMSE[a] (m)	RMSE[b] (m)
Well 7	6.6	1241.774	0.109	0.447	6.19×10^{-2}	0.166
Well 8	9.1	1151.410	0.139	0.422	6.73×10^{-2}	0.176
Well 9	11.6	1265.603	0.171	0.363	5.91×10^{-2}	0.177
Well 10	14.1	1454.474	0.140	0.377	5.70×10^{-2}	0.178
Well 11	16.6	1958.721	0.121	0.389	5.25×10^{-2}	0.183
mean	-	1414.396	0.136	0.387	-	-
Wells 7-11	-	795.999	0.041	1.536	6.73×10^{-2}	0.176
Nielsen (1990)	-	2076	0.1	0.51	-	-

[a] The RMSE values of the predicted WWL data with the parameters estimated based on our proposed method to the field data.

[b] The RMSE values of the predicted WWL data with the parameters given in Nielsen (1990) to the field data.

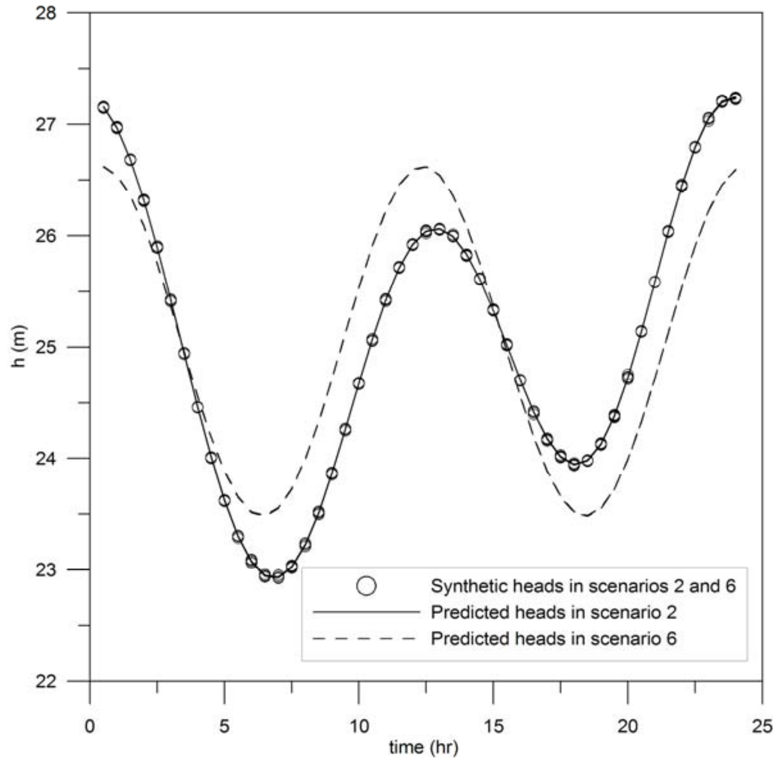

Figure 2 Comparisons of synthetic heads and predicted heads in scenarios 2 and 6. The synthetic heads in scenario 2 are analyzed based on Jeng et al.'s solution (2005) and those in scenario 6 are analyzed based on Nielsen's solution (1990).

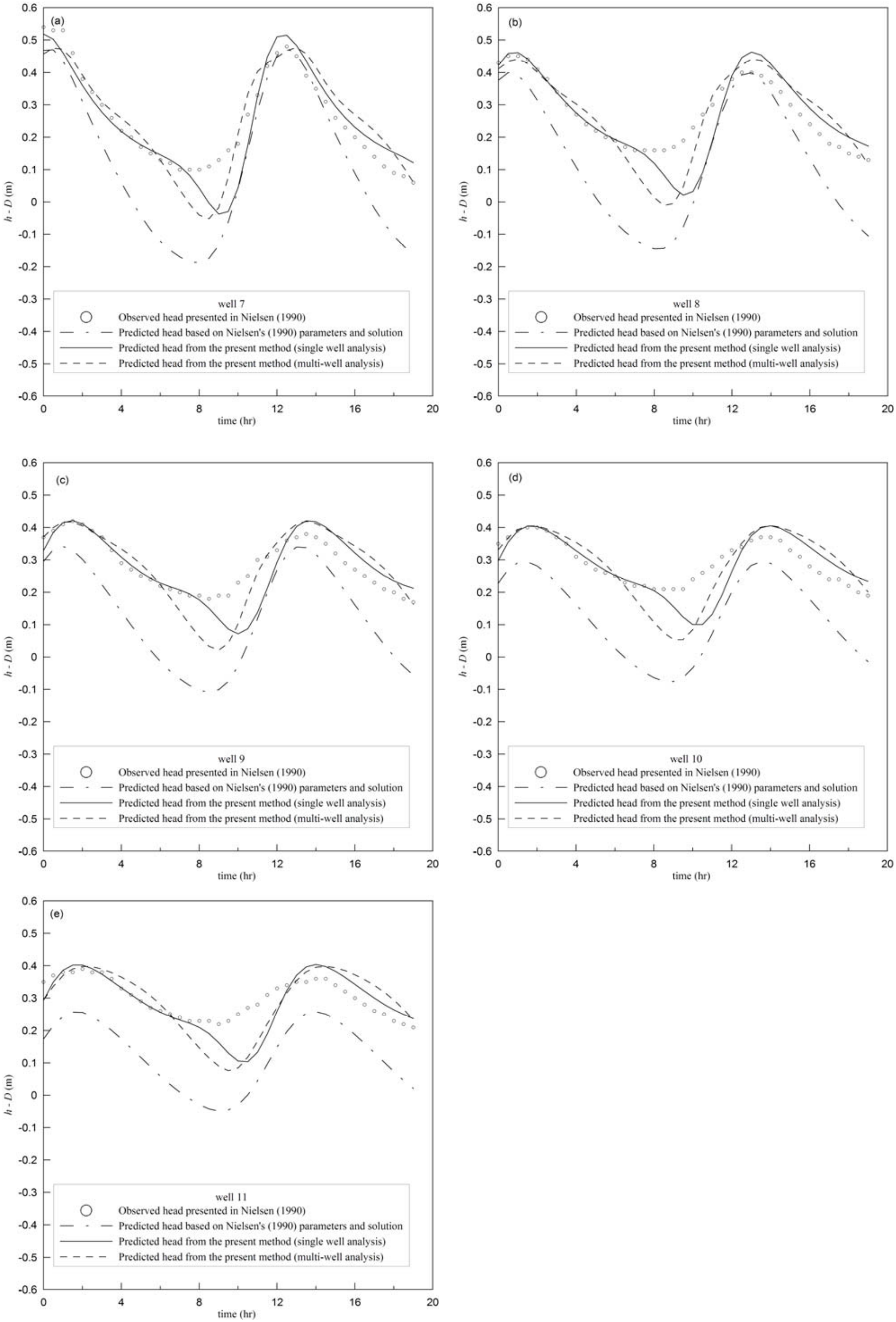
(a)
well 7
Observed head presented in Nielsen (1990)
Predicted head based on Nielsen's (1990) parameters and solution
Predicted head from the present method (single well analysis)
Predicted head from the present method (multi-well analysis)
h - D (m)
time (hr)
(b)
well 8
Observed head presented in Nielsen (1990)
Predicted head based on Nielsen's (1990) parameters and solution
Predicted head from the present method (single well analysis)
Predicted head from the present method (multi-well analysis)
h - D (m)
time (hr)
(c)
well 9
Observed head presented in Nielsen (1990)
Predicted head based on Nielsen's (1990) parameters and solution
Predicted head from the present method (single well analysis)
Predicted head from the present method (multi-well analysis)
h - D (m)
time (hr)
(d)
well 10
Observed head presented in Nielsen (1990)
Predicted head based on Nielsen's (1990) parameters and solution
Predicted head from the present method (single well analysis)
Predicted head from the present method (multi-well analysis)
h - D (m)
time (hr)
(e)
well 11
Observed head presented in Nielsen (1990)
Predicted head based on Nielsen's (1990) parameters and solution
Predicted head from the present method (single well analysis)
Predicted head from the present method (multi-well analysis)
h - D (m)
time (hr)

Figure 3 Plots of observed WWL given in Nielsen (1990), predicted WWL produced by Nielsen's parameter and solution (1990), and predicted WWLs produced by Jeng et al.'s solution (2005) with the parameters determined by the present method via single-well and multi-well analyses.