

**Response to the comments of Anonymous Referee #2** by C-M Chang and H-D Yeh

We would like to thank Referee #2 for the thoughtful comments and suggestions. The following is our response.

1. The recharge term in Eq. (1), which denotes a source term, does not represent the surface recharge sketched in Figure1. This should be clarified and the motivation for this volume source term explained.

Reply

- (i) As suggested, we would add the clarification in section 2 as

“A schematic representation of seawater intrusion into an unconfined coastal aquifer is illustrated in Figure 1, **where  $W$  denotes the source term in Eq. (1).**”

- (ii) The following would be added in the Introduction as:

“Coastal areas often have moist climates and therefore may receive a large amount of rainfall. It is therefore of nature to characterize the interaction between freshwater and seawater with the consideration of recharge effect.”

2. The authors employ a continuous spectral approach. However, the model domain is finite. Thus, strictly speaking, a discrete spectral approach should be used. The authors should explain, why this is so.

Reply

The continuous spectral representation theorem of random functions in Fourier space assumes the existence of solution over infinite domains. In reality, groundwater flow fields are not over infinite domains. An infinite-domain assumption may reduce to a large finite-domain assumption if the correlation length of the random fields is much smaller than the domain size. A continuous spectral representation theorem of random functions over a finite domain is valid if the correlation length of the random fields is much smaller than the domain size.

3. In page 636, lines 11-12, it is mentioned that the mean elevation Eq (35) can be obtained from Eqs. (21) and (51). The authors should provide this explicit expression.

Reply

The sentence after Eq. (51) on page 11 has been modified as:

“In addition, the mean elevation of the interface can be explicitly determined

after substituting Eqs. (21) and (51) in to Eq. (35).”

4. Figures 2 and 4 indicate that the variance of the interface is decreasing towards the toe position and is largest at the seawater boundary. It would be good if the authors would extend the x-axis in Fig. 4 to include the toe position. Intuitively, I would think that the toe position should be subject to quite some variability due to spatial heterogeneity while the variability at the sea-boundary should be zero, because there the interface elevation is basically prescribed by the boundary condition.

Reply

- (i) As apparent from Eq. (52) that the variance of the interface is increasing with the heterogeneity ( $\sigma_f^2$ ), we agree that the toe position should be subject to quite some variability due to spatial heterogeneity. However, Figure 2 has been used to demonstrate the behavior of the variance of the interface and Figure 4 to illustrate the mean interface position with confidence intervals as a function of correlation scale of  $\ln K$  for fixed heterogeneity ( $\sigma_f^2$ ), not the function of heterogeneity. The features of the Albitar and Ababou (2005) numerical simulations qualitatively confirm the findings implied by our analytical results.
- (ii) To take the advantage of an analytic solution, although we do not neglect the boundary conditions of the mean model in the development of mean position of the salt-wedge tip, the boundary conditions of the perturbation equation in the development of the variance of mean position of the salt-wedge tip are however neglected. The position of the wedge tip is thus independent of the heterogeneity of the medium. In other words, the prediction of the reliability of the mean model near the medium boundary (the wedge tip) using our theoretical result would not be appropriate. However, it is expected that the perturbation-boundary effect is largely limited to a small zone next to the medium boundary.
- (iii) Figure 4 has been presented to show the mean interface position with confidence intervals as a function of correlation scale of  $\ln K$ . It indicates that the confidence intervals decrease with the position. To clearly illustrate the dimensionless mean interface position with confidence intervals, we therefore cut the dimensionless range to 10 in the x-axis.