Responses to “Detailed Comments” by S. Lovejoy on “Reconstruction of sub-daily rainfall sequences using multinomial multiplicative cascades”

L. Wang, C. Onof, S. Hsieh and Č. Maksimović

The authors would like to appreciate the referee’s insightful comments on this work and for generously providing useful references to help understand the deficiency of this paper. In the following, the authors would like to respond to the issues raised in the comments following the titles in the referee’s supplementary detailed comments.

1. Discrete scale ratios cascades versus continuous in scale cascades:

The long-term goal of this work, which was not clearly stated in the paper, is to develop a downscaling method which will be further used to generate high-resolution rainfall information as inputs in nearly real-time (NRT) flood modelling over urban areas. This means that, in addition to accuracy, efficiency is another factor needed to be considered when downscaling techniques are developed. Continuous in scale Universal multifractals (Tessier et al. 1993) provide a promising framework to generate more realistic and visually smooth rainfall distributions. To generate realistic rainfall distribution, fractional integration is however required to simulate the rainfall structures at all intermediate scales via numerical calculation. This suggests that, although the efficiency of the calculation has been improved (Lovejoy and Schertzer 2010a; b), the total simulation time may be somewhat too long for the current NRT flood modelling. Nevertheless, we agree that the involvement of the continuous in scale Universal multifractal cascades would certainly improve the accuracy and reliability of the state-of-the-art flood modelling.

Moreover, although the ability of the continuous in scale Universal multifractal cascades to satisfactorily reproduce the (multi-) scaling features of the observed rainfall has been validated, the ability to reproduce other (statistical) features is seldom explored (Bellone 2004). The latter ability however is very crucial for the uses of the corresponding hydrological modelling (e. g., ground runoff and sewer network simulation), and has been demonstrated in a number of previous works that the discrete-based cascade models are feasible techniques with this ability (Molnar and Burlando 2005; Onof and Arnbjerg-Nielsen 2009; Pathirana et al. 2003). Furthermore, discrete-based cascade models have already been applied to realistic rainfall prediction and modelling (Bowler et al. 2006).

In view of the reasons mentioned above, this work therefore employs the discrete-based cascade models and focuses mainly on the performance in reproducing certain statistics of observed rainfall data. The proposed method is a very simplified prototype model based upon merely the formation of an exact self-similar cascade. As the referee points out, it is to an extent an unrealistic (or “toy”) model with respect to the objective of satisfactorily illustrating the real rainfall distribution due to its oversimplified assumption. However, in view of previous researchers’ results with discrete-based cascade models and their associated applications, the authors believe that it is a rough yet practically acceptable model and could be used to generate rainfall efficiently within the forecasting framework of interest.

2. Weights versus fragmentation ratios:

Due to the assumption made in this work that rainfall time series could be simulated using an exact self-similar cascade with a finite number (λ_0 = b ≥ 2) of deterministic fragmentation ratios, one of the key tasks is to derive these fragmentation ratios from observed rainfall data. Before continuing, the authors would like to clarify some statements that may confuse the readers in the paper because of the insufficient explanation, and the associated clarifications will be updated in the final version of this work. In the paper, the authors use the "multinomial cascades" term because the discrete-based cascade model is employed. Although the empirical analyses shows a continuum of weights, merely a finite number (herein, λ_0 = b = 4) of values are optimally derived, which are further applied to generating the backbone pattern of rainfall sequences. According to this, the confusion between the fragmentation ratios and the weights may be clarified here insofar as the fragmentation ratios are in fact equivalent to those optimally-selected weights. Moreover, the misleading notation "*p*", which is conventionally used for "probabilities" rather than "weights," will be properly replaced by the standard "*w*" in the final proof.

The cascade model used in this work is shown in Fig. 1 and 2 in the paper. The process starts from a unit volume with unit scale and measure, and, for each subdivision among parent component and the associated child components between two successive levels, the sum of the fragmentation ratios is unity, due to the microcanonical assumption (the detailed description for the whole process is in section 4.1). In this work, the empirical method for estimating weights and the associated probability distribution from real rainfall data refers to previous work (Olsson 1998), where the weights are obtained from individually dividing the volumes of child components by those of the parent component. Instead of integrating rain rates (or fluxes) over scales (Lovejoy and Schertzer 2010a), the authors use the rainfall depth as the volume of a component. Eq. (1) in the detailed comments thus can be restated as:

$$p_0^{(1,d)} = (p_0^{(1)} p_{0,0}^{(2)} + p_0^{(1)} p_{0,1}^{(2)}) = p_0^{(1)} (p_{0,0}^{(2)} + p_{0,1}^{(2)}) = p_0^{(1)} = p_0^{(1,b)}$$

where the scale ratio (i.e., 1/2) in the original Eq. (1) is not necessarily being applied. Therefore, Eq. (5) in the paper follows when the exact self-similar (microcanonical) cascade model is employed. As mentioned above, this is an oversimplified model to simulate real rainfall data, and thus the validity of the original Eq. 5 is somewhat controversial, particularly in real world applications; however, the statistical results in this work and in the previous work (Olsson 1998) suggest that this type of method, to some extent, has an acceptable ability to reproduce real rainfall sequences.

3. The Log-Poisson model:

The reason that the authors employed the log-Poisson based model as benchmark is due to the availability of a reliable computer programme—Cascade programme. This programme has been used to generate high-resolution rainfall time series in several previous works (Onof and Arnbjerg-Nielsen 2009; Onof et al. 2005) and the associated theory has been well established (Deidda et al. 1999), which to some degree depicts its reliability. However, as mentioned by the referee, the major drawback of log-Poisson based models lies on the incapability of generating events of singularity stronger than a critical value, which suggests that they are not sufficient as benchmark, even though their overall performance has been shown to be very good. The suggestion from the referee that another model, e.g., the Log-Levy model be used, as well as the idea of further analysing sampling variability are very useful. However, although we plan to explore these issues in further work, due to the limited time, this is not possible before the deadline of the final version of the work.

4. Zeros and low rain rates:

As mentioned in the previous section, the fragmentation ratios are optimally derived from a continuum of weights, and the authors use them to produce the backbone pattern of rainfall sequences (including zero rain rates) by allowing some ratios to be zero-values. This is inspired by the construction of the Cantor set. To some extent, partially allowing zero-valued fragmentation ratios can be seen as a deterministic (yet rough) approach based upon scaling features of real rainfall data; however, one of the drawbacks of being deterministic is that the number of generated zero values is fixed once the fragmentation ratios are derived.

As the reviewer points out, there is, moreover, uncertainty as to what is a true zero rainfall. Some of the derived ratios (shown in Table 3) are found to be very small (nearly zero) values, so that the distinction zero/close to zero is a fuzzy one The introduction of zero fragmentation ratio is therefore not aimed at reproducing exactly the proportion of observed zeroes. , The authors therefore use thresholding (according to artefacts of the instruments) to carry out minor adjustments to the simulated rainfall. The method is therefore pragmatic, rather than based upon a claim that zero rainfall exhibits scaling.

References

Bellone, E. (2004). "Review of some multifractal models for rainfall." Department of Civil and Environmental Engineering Imperial College and Department of Statistical Science University College London, London.

Bowler, N. E., Pierce, C. E., and Seed, A. W. (2006). "STEPS: A probabilistic precipitation forecasting scheme which merges an extrapolation nowcast with downscaled NWP." *Quarterly Journal of the Royal Meteorological Society*, 132(620), 2127-2155.

Deidda, R., Benzi, R., and Siccardi, F. (1999). "Multifractal modeling of anomalous scaling laws in rainfall." *Water Resources Research*, 35(6), 1853-1867.

Lovejoy, S., and Schertzer, D. (2010a). "On the simulation of continuous in scale universal multifractals, part I: Spatially continuous processes." *Computers & Geosciences*, In Press, Corrected Proof.

Lovejoy, S., and Schertzer, D. (2010b). "On the simulation of continuous in scale universal multifractals, Part II: Space-time processes and finite size corrections." *Computers & Geosciences*, In Press, Corrected Proof.

Molnar, P., and Burlando, P. (2005). "Preservation of rainfall properties in stochastic disaggregation by a simple random cascade model." *atmospheric research*, 77(1-4), 137-151.

Olsson, J. (1998). "Evaluation of a scaling cascade model for temporal rainfall disaggregation." *Hydrology and Earth System Sciences*, 2(1), 19-30.

Onof, C., and Arnbjerg-Nielsen, K. (2009). "Quantification of anticipated future changes in high resolution design rainfall for urban areas." *Atmospheric research*, 92, 350-363.

Onof, C., Townend, J., and Kee, R. (2005). "Comparison of two hourly to 5-min rainfall disaggregators." *Atmospheric research*, 77, 176-187.

Pathirana, A., Herath, S., and Yamada, T. (2003). "Estimating rainfall distributions at high temporal resolutions using a multifractal model." *Hydrology and Earth System Sciences*, 7(5), 668-679.

Tessier, Y., Lovejoy, S., and Schertzer, D. (1993). "Universal multifractals: theory and observations for rain and clouds." *Journal of Applied Meteorology*, 32(2), 223-250.