Reply to reviewer #1

General comment

This manuscript presents an analytical solution to describe the induced head fluctuations in a heterogeneous coastal aquifer system. The proposed analytical solution is used to analyze real and hypothetical cases. Unfortunately the objectives of these tests are not clearly defined and are difficult to understand. The whole manuscript needs language revision.

Reply: The manuscript will be edited by a native English speaker. In addition, the objectives for the test cases will be added as described below:

(1) In section 3.1, we insert a sentence as: "The objective of this case is to address the effect of the length of the semi-permeable aquitard on the amplitude and phase shift of head fluctuations." (line 11, page 4481)

(2) In Section 3.2, we add a sentence as: "The objective of this case is to investigate the effect of the value of d_1 on the amplitude and phase shift of head fluctuations." More detailed explanation is given in the reply 5.

(3) The objective for the test case in Section 3.3 (lines 6-9, page 4483) is given in a sentence as: "…. The underlying confined aquifer is considered to be homogeneous for the purpose of investigating the effect of aquitard heterogeneity." More explanation related to this objective is given in the reply 6.

Specific comments

1. The expression (8) seems not to be correct. If equation (6) is replaced in equation (1) a different expression for λ is obtained.

Reply: Thanks for the comment. Equation (8) is actually correct. Equation (1) has a typo and is corrected as

$$S_n \frac{\partial h_n}{\partial t} = T_n \frac{\partial^2 h_n}{\partial x^2} - L_n h_n \qquad (1)$$

2. It would be useful to include in Section 2.2 the system of equations to obtain coefficients c1n and c2n in matrix form (Ax = b).

Reply: Thanks for the suggestion. The original text (from line 11, page 4478 to line 9, page 4479) is rewritten and the new one including coefficients c1n and c2n in matrix form is shown below "

The equations for solving $c1_n$ and $c2_n$ can be expressed in matrix form as

$$D_{2N\times 2N} \times \begin{bmatrix} c1_1 \\ c1_2 \\ \vdots \\ c1_N \\ c2_1 \\ c2_2 \\ \vdots \\ c2_N \end{bmatrix}_{2N\times 1} = \begin{bmatrix} 1 \\ 0 \\ \\ \vdots \\ \\ 0 \end{bmatrix}_{2N\times 1} \quad (9)$$

with

$$D = \begin{bmatrix} I & I \\ B & C \\ E & F \\ G & H \end{bmatrix}_{2N\times 2N} , \quad (10)$$

$$I = [1 \quad 0 \quad \cdots \quad 0 \quad 0]_{1\times N} , \quad (11)$$

$$G = [0 \quad 0 \quad \cdots \quad 0 \quad 1]_{1\times N} , \quad (12)$$

$$H = [0 \quad 0 \quad \cdots \quad 0 \quad 0]_{1\times N} , \quad (13)$$

$$B = \begin{bmatrix} e^{\lambda_1 d_1} & -e^{\lambda_2 d_1} & 0 & \cdots & 0 & 0 \\ 0 & e^{\lambda_2 d_2} & -e^{\lambda_3 d_2} & 0 & \cdots & 0 \\ 0 & 0 & & & & \\ \vdots & \vdots & & \ddots & \vdots & \vdots \\ & & & & 0 & \\ 0 & \cdots & 0 & e^{\lambda_{N-2} d_{N-2}} & -e^{\lambda_{N-1} d_{N-2}} & 0 \\ 0 & 0 & \cdots & 0 & e^{\lambda_{N-1} d_{N-1}} & -e^{\lambda_N d_{N-1}} \end{bmatrix}_{(N-1)\times N} \quad (14)$$

and

$$E = \begin{bmatrix} T_1\lambda_1 e^{\lambda_1 d_1} & -T_2\lambda_2 e^{\lambda_2 d_1} & 0 & \cdots & 0 & 0 \\ 0 & T_2\lambda_2 e^{\lambda_2 d_2} & -T_3\lambda_3 e^{\lambda_3 d_2} & 0 & \cdots & 0 \\ 0 & 0 & & & & \\ \vdots & \vdots & & \ddots & \vdots & \vdots \\ & & & & 0 & \\ 0 & \cdots & 0 & T_{N-2}\lambda_{N-2} e^{\lambda_{N-2} d_{N-2}} & -T_{N-1}\lambda_{N-1} e^{\lambda_{N-1} d_{N-2}} & 0 \\ 0 & 0 & \cdots & 0 & T_{N-1}\lambda_{N-1} e^{\lambda_{N-1} d_{N-1}} & -T_N\lambda_N e^{\lambda_N d_{N-1}} \end{bmatrix}_{(N-1)\times N} \quad (15)$$

where both C and F are $(N-1)\times N$ matrixes and identical to matrixes B and E, respectively, except having a minus sign before the exponent in the exponential terms.

Based on Cramer's rule, the results for $c1_n$ and $c2_n$ can be expressed as

$$c1_n = \frac{\det D_n}{\det D} \quad (16)$$

and

$$c2_n = \frac{\det D_{N+n}}{\det D} \quad (17)$$

where *det* is an abbreviation for determinant; D_n and D_{N+n} can be obtained by replacing the *n*th and $(N+n)$th columns of matrix D, respectively, with a column matrix that the top element is one and the others are zero."

3. In Section 2.3 include a comparison of the proposed analytical solution and the solution derived by Guo et al (2010). This solution is a special case that can be obtained for N=2 and L1=L2=0.

Reply: Thanks for the suggestion. We added following text to describe the relationship between the present solution and Guo et al.'s solution (2010).

"Guo et al. (2010) presented an analytical solution to describe head fluctuations in a coastal aquifer consisting of two zones in horizontal direction. These two zones have different hydraulic properties. When N=2 and $u_1 = u_2 = 0$, the present solution reduces to Guo et al.'s solution (2010, Equations (7a) and (7b)) which is expressed, in our notation, as

$$h_1(x,t) = A\cdot c_1 \cdot \cos(\omega t - \phi_1) \quad (24)$$

$$h_2(x,t) = A\cdot c_2 \cdot \cos(\omega t - \phi_2) \quad (25)$$

where

$$c_1 = \sqrt{a_{X_1}{}^2 + b_{X_1}{}^2}, \quad c_2 = \sqrt{a_{X_2}{}^2 + b_{X_2}{}^2}, \quad \phi_1 = \arccos(\frac{a_{X_1}}{c_1}), \quad \phi_2 = \arccos(\frac{a_{X_2}}{c_2}) \tag{26}$$

$$a_{X_1} = \frac{\mu_a + \nu_a}{\chi}, \quad b_{X_1} = \frac{\mu_b + \nu_b}{\chi}, \quad a_{X_2} = \frac{\sigma_a + \tau_a}{\chi}, \quad b_{X_2} = \frac{\sigma_b + \tau_b}{\chi} \tag{27}$$

$$u_a = (a_1{}^2 T_1{}^2 - a_2{}^2 T_2{}^2)\cos[a_1(x - 2d_1)](e^{a_1(2d_1 - x)} + e^{a_1(2d_1 + x)}) \tag{28}$$

$$\nu_a = \cos(a_1 x)[(a_1 T_1 - a_2 T_2)^2 e^{a_1 x} + (a_1 T_1 + a_2 T_2)^2 e^{a_1(4d_1 - x)}] \tag{29}$$

$$\begin{aligned} \mu_b = {} & a_1{}^2 T_1{}^2 e^{-a_1 x}[e^{2d_1 a_1}(e^{2a_1 x} - 1)\sin[a_1(2d_1 - x)] + (e^{4d_1 a_1} - e^{2a_1 x})\sin(a_1 x)] \\ & + 2a_1 T_1 a_2 T_2 \sin(a_1 x)(e^{a_1(4d_1 - x)} + e^{a_1 x}) \end{aligned} \tag{30}$$

$$\nu_b = a_2{}^2 T_2{}^2 e^{-a_1 x}[(e^{2d_1 a_1} - e^{2a_1(d_1 + x)})\sin[a_1(2d_1 - x)] + (e^{4d_1 a_1} - e^{2a_1 x})\sin(a_1 x)] \tag{31}$$

$$\sigma_a = 2a_1 T_1(a_1 T_1 - a_2 T_2)\cos[d_1 a_1 + a_2(d_1 - x)]e^{d_1 a_1 + a_2(d_1 - x)} \tag{32}$$

$$\tau_a = 2a_1 T_1(a_1 T_1 + a_2 T_2)\cos[d_1 a_1 + a_2(x - d_1)]e^{3d_1 a_1 + a_2(d_1 - x)} \tag{33}$$

$$\sigma_b = 2a_1 T_1(-a_1 T_1 + a_2 T_2)\sin[d_1 a_1 + a_2(d_1 - x)]e^{d_1 a_1 + a_2(d_1 - x)} \tag{34}$$

$$\tau_a = 2a_1 T_1(a_1 T_1 + a_2 T_2)\sin[d_1 a_1 + a_2(x - d_1)]e^{3d_1 a_1 + a_2(d_1 - x)} \tag{35}$$

$$\chi = (a_1 T_1 - a_2 T_2)^2 + (a_1 T_1 + a_2 T_2)^2 e^{4d_1 a_1} + 2e^{2d_1 a_1}\cos(2d_1 a_1)(a_1{}^2 T_1{}^2 - a_2{}^2 T_2{}^2) \tag{36}$$

Accordingly, the solutions derived by Jiao and Tang (1999) and Guo et al. (2010) are considered as our special cases."

4. Section 3.1: note that the solution of Jiao and Tang (1999) can be approached using values of d1 smaller than 850m. It is suggested to include in figures 2 and 3 the solution for d1=300m.

Reply: Thanks for the suggestion. The curves of d_1=300m have been added in Figures 2 and 3 of the revised manuscript. The new figures are shown below.

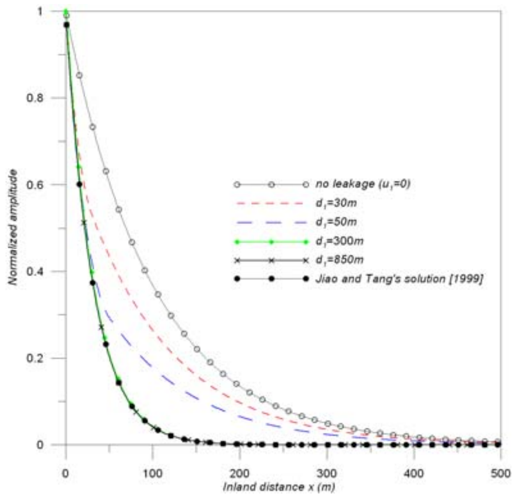

Figure 2

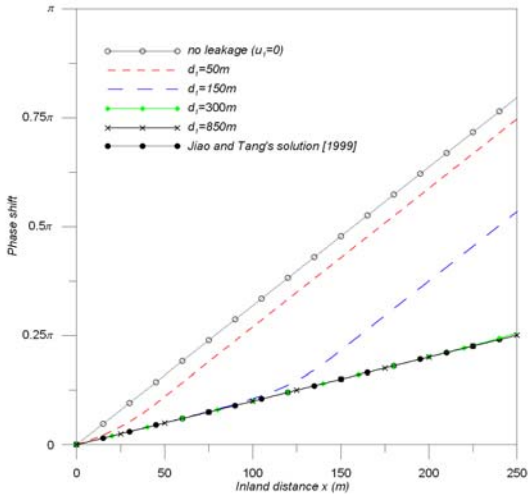

Figure 3

5. The objective of the test case presented in Section 3.2 is not clear. The proposed model has 3 regions (not 2). Please specify the parameters of region3 and include in the text the definition of the tidal intrusion distance.

   Reply: The "tidal intrusion distance" is defined as the farthest landward distance from the coastline to the location where the normalized amplitude of head fluctuations is less than 10^{-2}. This definition has been moved to Section 3.1. Section 3.2 has been rewritten and the objective of the test case is added as:
   "In contrast to the previous case, the aquitard in region 1 with a length d_1 is now treated as impermeable, i.e., $u_1 = 0$, while the aquitard in region 2 is semi-pervious with $u_2 = 5$. Both aquifer and aquitard in region 2 are considered to be of infinite extent. The objective of this case is to investigate the effect of the value of d_1 on the amplitude and phase shift of head fluctuations. Figures 4 and 5 demonstrate that the normalized amplitude and phase shift, respectively, of head fluctuations in the homogeneous confined aquifer versus the inland distance for $a_1 = a_2 = 1\times10^{-2}\,\text{m}^{-1}$ and d_1 = 0, 50, 100, or 350 m. Both figures show that the curve with $d_1 = 350\,\text{m}$ matches with the case of no leakage, indicating that the aquitard resided far away from the costal line does not have affects on the amplitude and phase shift of the head fluctuation in the coastal confined aquifer. In addition, the figures also show that those dashed lines approach the line with open circles at large d_1, indicating that the effects of the aquitard on the amplitude and phase shift of head fluctuations decrease with increasing d_1. Once the d_1 is larger than the tidal intrusion distance, the effects of the leakage on both amplitude and phase shift are negligible."

6. The description of Figure 7 is difficult to understand. Please define high-tide and

middle-tide conditions.

Reply: The high-tide and middle-tide conditions represent at the conditions of highest sea level and mean sea level, respectively. The original text regarding Figure 7 has been rewritten as

"Alluvial fans formed at the base of mountains usually have coarser sediment at the upper part of the fan and finer sediment near the coastal area. The formation of the coastal leaky aquifer may therefore exhibit the phenomenon of trending heterogeneity (Freeze and Cherry, 1979). The leaky aquifer system is divided into three regions for simulating trending heterogeneity of the aquifer (i.e., $T_1 = 10$ m^2/day, $T_2 = 50$ m^2/day and $T_3 = 100$ m^2/day). The leakages in these three regions are chosen the same for assessing the effect of aquifer heterogeneity. Figure 7 shows the spatial head distributions at $\omega t = 2\pi$ and $\omega t = 0.5\pi$ for the cases of homogeneous aquifer and trending heterogeneous aquifer. Following parameter values are used in the analyses: $u_1 = u_2 = u_3 = 5$, $S_1 = S_2 = S_3 = 10^{-4}$, $d_1 = 100$ m, $d_2 = 200$ m and $\omega = 2\pi / 0.5$ day^{-1}. The hydraulic conductivities are considered as $T_1 = T_2 = T_3 = 50$ m^2/day for the homogeneous aquifer and $T_1 = 10$ m^2/day, $T_2 = 50$ m^2/day and $T_3 = 100$ m^2/day for the trending heterogeneous aquifer. The figure indicates that the trending heterogeneous aquifer has a smaller tidal intrusion distance than the homogeneous one. This is because the region 1 has a smaller hydraulic conductivity. In addition, the slopes of the normalized head distribution are markedly different near $x = 100$ m and 200 m because the hydraulic conductivities in these regions are different. Obviously, the aquifer heterogeneity has an impact on the hydraulic head distribution."

7. Section 3.4: the solution obtained with the proposed analytical solution is similar to the one obtained by Jiao and Tang (1999) because the value of d1 is large. If more regions are defined near the coast (e.g. d1=50m, d2=100m, d3=150m…) you may probably obtain a better fit to the observed data.

Reply: Thanks for the suggestion. Consider the case that the aquifer near the coast has 6 regions which are divided at locations d_1=50m, d_2=100m, d_3=200m, d_4=250m and d_5=300m. The 6th region is considered to extend infinitely. Assume that the aquitard thickness decreases linearly from 1st to 3rd region and then increases linearly from 3rd to 5th region. The dimensionless leakage for these six regions is therefore considered as $u_1 = 6.24 \times 10^{-3}$, $u_2 = 7.83 \times 10^{-3}$, $u_3 = 9.38 \times 10^{-3}$, $u_4 = 7.83 \times 10^{-3}$, $u_5 = 6.24 \times 10^{-3}$ and $u_6 = 4.69 \times 10^{-3}$. Note that the dimensionless leakage is defined as $u_n = k'_n / (b_n \omega S_n)$ where ω is tide frequency and k'_n, b_n and S_n are hydraulic conductivity, thickness and storativity for the aquitard in *n*th region, respectively. The predicted head fluctuation in this case is added in Figure 9 listed below. This figure shows that predicted head fluctuations from the present solution with 6 regions is identical to the one with 2 regions ($u_1 = 9.38 \times 10^{-3}$ and $u_2 = 4.69 \times 10^{-3}$). Such a result is because the value of leakage is too small so that the effect of heterogeneous distribution of aquitard thickness on the head fluctuation is insignificant.

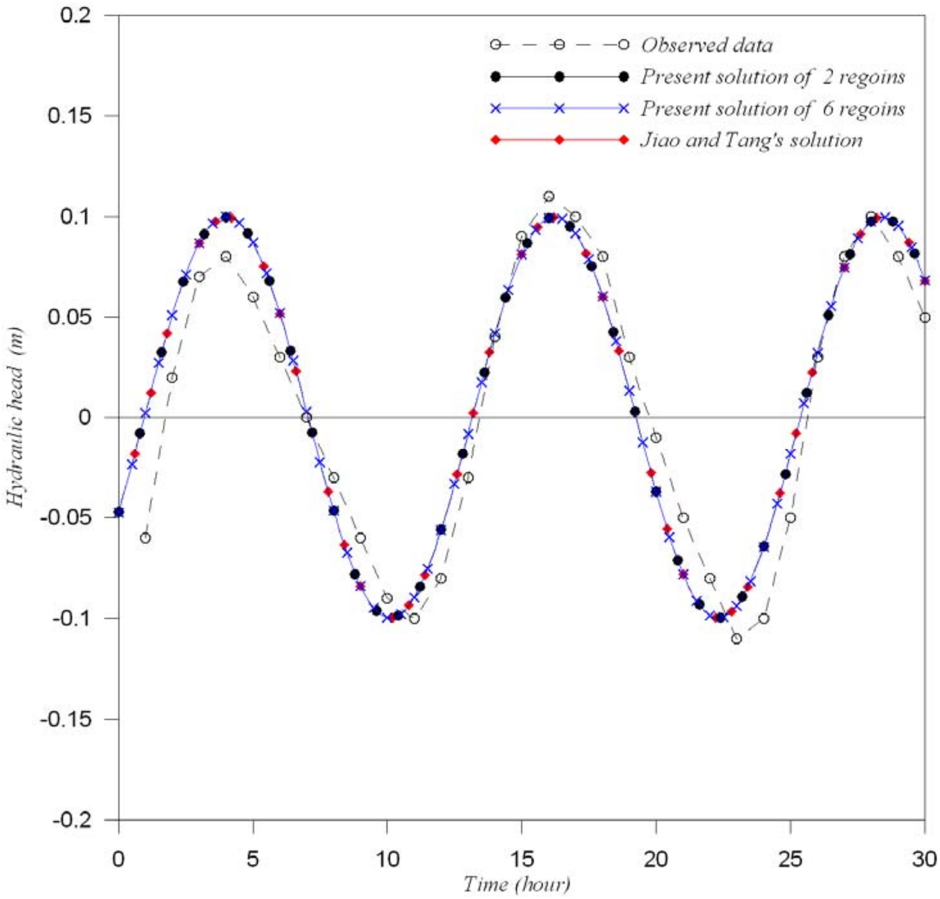

Figure 9

8. Parameter values used in Section 3.4 should be justified.

Reply: The parameter values used to plot Figure 9 in Section 3.4 are adopted from Jiao and Tang (1999).

9. Section 3.5 should be eliminated because the effect of heterogeneity in the aquitard is analyzed in section 3.3.

Reply: Section 3.5 has been removed.

References

Freeze, R. A. and Cherry, J. A.: Groundwater, Prentice-Hall, Inc., New Jersey, 1979.

Guo, H., Jiao, J. J. and Li, H.: Groundwater response to tidal fluctuation in a two-zone aquifer, J. Hydrol., 381(3-4), 364-371, doi: 10.1016/j.jhydrol.2009.12.009, 2010.

Jiao, J. J. and Tang, Z.: An analytical solution of groundwater response to tidal fluctuation in a leaky confined aquifer, Water Resour. Res., 35(3), 747-751, 1999.