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Hydrology and Earth System Sciences Discussions

# *Interactive comment on* "A stochastic design rainfall generator based on copulas and mass curves" *by* S. Vandenberghe et al.

**S. Vandenberghe et al.**

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We greatly acknowledge the comments of the referee. In this revision letter, we list all our responses to the comments made.

1. The authors present their model without taking any effort to validate their methodology. Testing the copula fit should not be considered as a validation measure for the presented model. That, in fact, is a requirement to use a specific type of copula. Given the large sample of data (105 years), the authors have a wide variety of methods (e.g., bootstrap technique) to validate their methodology.

   A recommendation to validate our method by means of a bootstrap technique is



   not fully understood by the authors. What is exactly meant with 'methodology'? If it concerns the fitting of the copula, we indeed firstly choose a specific copula family. There are no clear methods that allow for an objective choice among copula families. Given the choice of the copula family, a goodness-of-fit test is performed, which is based on a bootstrap.

2. Page 3624: 'For a specific season, the empirical copula is constructed. The storm with the largest empirical copula value (the highest point in a 3-D representation of the empirical copula) is thus the most extreme storm in the considered season out of a data set of 105 years.' I'm afraid the way the empirical return periods are derived may not be correct. If you are looking at the empirical distribution function of a specific season, you cannot claim that the highest value has a return period of 105 years, although the data is sampled from 105 years of data. Of course you can derive a return period, but I think you are altering the physical meaning of YEAR. Even if you are using the entire 105 year of data, the highest value will refer to a 105-year return period. However, the second value may not necessarily correspond to a return period of 52.5 years. The empirical return periods, used in practice, are obtained by sampling the annual maxima from the entire data set.

   We agree with the reviewer that the concept of multivariate return periods can easily get confusing. In a univariate context, the concept of a return period is quite well understood and used in practice. However, the extension to multidimensional phenomena is less common. In any dimension, the return period is an expected interarrival time of a specific 'extreme' event. Two issues are important to calculate the return period: the mean interarrival time of all the events considered and the probability that a more extreme event occurs. When considering an annual maxima analysis in the univariate context, the mean interarrival time is one year and the probability of occurrence of a more extreme event is calculated with the survival function of the univariate cumulative distribution function (CDF). Salvadori and De Michele (2010) provide a nice discussion on the extension of the return period to more than one dimension and state that the secondary return period, as used in this study, is the only valid multidimensional generalization from a theoretical point of view.

In our paper, we consider this bivariate return period in three different ways: theoretically, empirically and intuitively. First of all, no annual maxima are considered, but all storms are considered on a seasonal basis. The latter is done because of the assumption of stationarity needed for a valid application of copulas. So the mean interarrival time is less than one year. The probability of occurrence of a more extreme event is calculated by means of the $K_{\mathbf{C}}$ function. The theoretical secondary return period is calculated using the theoretical $K_{\mathbf{C}}$ function based on the fitted copula. The empirical secondary return period uses the empirical approximation of the $K_{\mathbf{C}}$ function: the empirical CDF of the copula-values. To incorporate the fact that a time series of 105 years is considered with a separate treatment of the seasons, the probability of occurrence of a storm in a specific season should be multiplied with the probability of a more extreme event.

The return period to which the reviewer refers is not the empirical return period, but concerns what we have called the 'intuitively derived return period'. The calculation of this return period is purely based on intuition, steered by the definition of the secondary return period. The highest point in the empirical copula gets a return period of 105 years, the second highest appears twice in 105 years and therefore, on average, gets a return period of 105/2 years, etc. This 'intuitively derived return period' is in fact the same as the empirical return period as calculated in the paper, if no seasonal analysis would be conducted:

$$\begin{aligned} \text{empirical } T_{\mathrm{SEC}} &= \frac{\frac{105}{n+1}}{1-K_{\mathbf{C}_n}(h)} \\ &= \frac{\frac{105}{n+1}}{1-\frac{R_h^{\mathbf{C}_n}}{n+1}} \end{aligned}$$



$$\begin{aligned} &= \frac{\frac{105}{n+1}}{\frac{n+1-R_h^{\mathbf{C}_n}}{n+1}} \\ &= \frac{105}{n+1-R_h^{\mathbf{C}_n}} \\ &= \frac{105}{n+1-(n-h+1)} \\ &= \frac{105}{h} \end{aligned}$$

In these equations, $R_h^{\mathbf{C}_n}$ is the rank of the $h$-th highest point in the empirical copula. In a dataset of $n$ storms, the rank of the $h$-th highest point in the empirical copula is $n-h+1$. The fraction $\frac{105}{n+1}$ is an approximation of the mean storm interarrival time (in years).

This thus illustrates that the concept of the secondary return period is intuitively very simple. We thus disagree with the reviewer that the return periods are incorrectly calculated.

3. At some parts the authors explain what they performed without sufficient discussion. For example: 'Figure 7 shows the outcome of the random generation of such a cumulative internal storm structure, together with the 10% and 90% percentile curves which serve as boundaries in the random generation.' Provide a detailed discussion for each figure. What does the figure mean? What does it represent?

   The caption of Figure 7 explains what is represented:

   'A cumulative randomly generated storm structure of a third quartile storm in summer. For each 5% time interval, a percentage of the total storm depth is assigned in the range of the 10% and 90% percentiles. The grey lines form the 10% and 90% percentile curves of the Huff curves. The points marked by × are the randomly chosen cumulative storm depths at 25%, 50% and 75% of the storm duration. The black line is the random generation using the 5% interval data.'

In the Introduction, the concept of Huff curves is explained as follows:

'Mass curves, often referred to as Huff curves, are representations of the non-dimensional cumulative time vs. the non-dimensional cumulative storm depth since the beginning of a storm. For specific time intervals, *i.e.* a 5% time interval, the empirical distribution of the normalized cumulative storm depth is then considered and often the 10%, 50% and 90% percentile of that distribution is then visualized. Furthermore, a classification of storms into quartile groups can be made.'

Preceding the reference to Fig. 7, details on the random generation are given:

'In order to obtain a random design storm generator, an algorithm for a random generation of a cumulative internal storm structure is developed. In first instance, the cumulative storm depths at the 25%, 50% and 75% of the total storm duration are randomly generated, constrained by the fact that these cumulative storm depths should lie between the 10% and 90% percentile Huff curve and that the cumulative storm depths may not decrease in time. Also, to assure that the design storm will respect the desired quartile, the maximum increase in cumulative storm depth should occur in that chosen quartile. Once the cumulative storm depths are chosen for each quartile, the rainfall in each quartile is further refined to each 5% time interval, based on the same Huff curves. Therefore, a random generator selects cumulative rainfall depths that fall within the 10% to 90% percentile curves, assuring that the total preset cumulative rainfall depth during the chosen quartile, as determined before, is obtained. Again, the algorithm respects the non-decreasing nature of the cumulative rainfall depth in time.

Figure 7 shows the outcome of the random generation of such a cumulative internal storm structure, together with the 10% and 90% percentile curves which serve as boundaries in the random generation.'

If the referee could indicate which of the above paragraphs need more elaboration in order to provide a clear understanding of Fig. 7, we would be happy to



incorporate more details.

4. Page 3622: 'An A12 copula (Vandenberghe et al., 2010a,b; Nelsen, 2006) is fitted to (W,D) for each season separately, resulting in four parameters.' When referencing to a model you should reference the original publication only and not every application. Vandenberghe et al., 2010a,b are not appropriate references for readers who want to learn about A12 copula; Nelsen, 2006 suffices.

   We referred to the Vandenberghe et al. papers because therein more details can be found on the fitting and goodness-of-fit evaluation of the A12 copula family, based on the same kind of data as used in the current paper. In the revised version of our paper we will put the references a bit differently, making the distinction between model- and application-info more clear.

5. I think it would be good to include the equation of A12 to make the manuscript more stand-alone.

   In the revised version of our paper we will include the equation of the A12 copula family.

6. Table 2: the copula parameter for winter, spring, summer, and autumn are estimated as 1.8622, 1.6953, 1.5485 and 1.7786, respectively. Are the differences statistically significant? Have you tried estimating the parameters with all data together? I think these are important issues that should be addressed.

   We treat the seasons separately to comply with the assumption of non-stationarity, which is needed because of the underlying copula theory (see also Comment nr. 2). The difference in parameter values can be evaluated on the basis of Table 2 in our paper. The 95% confidence intervals of the estimated parameters for winter, spring and summer do not overlap, indicating considerable differences between the parameters. The interval for the autumn storms lies somewhere in between the interval of the winter and spring storms. This will briefly be commented in the revised version of our paper.

7. Page 3623, Line 18, $T_{\mathrm{SEC}}$: Explain all notations although they may be obvious e.g., secondary return period ($T_{\mathrm{SEC}}$).

   This notation for the secondary return period is already introduced on page 3619 (lines 24-25) and page 3620 (Eq.(4)) and is therefore not repeated.

8. Equation 8: I think it is better to use I as indicator function instead of 1.

   In the revised manuscript we will incorporate this suggestion.

9. Please add an informative legend to Figure 4.

   The meaning of the dashed, full and dotted lines are explained in the caption of Figure 4. It is not visually attractive to add 20 times the same legend to the sub-figures.

10. Page 3615: Consider revising Lines 18-21.

    In the revised manuscript we will consider this.

11. 'Figure 4 shows different Huff curves which are constructed considering all storms in a specific season and quartile group, regardless of their return period.' Again, please provide a detailed discussion for each figure. What does the figure mean? What does it represent?

    The caption of Fig. 4 explains what is represented:

    'Huff curves for different quartile storms and different seasons. The 10%, 50% and 90% percentile curves are given by respectively the dashed, full and dotted line.'

    The concept of Huff curves is explained in the Introduction (See also comment nr. 3).

    The construction of the Huff curves, as given in Fig. 4, is given in the alinea preceding the reference to Fig. 4:



    'Each storm in the time series is classified as a first, second, third or fourth quartile storm, based on the occurrence of the largest rainfall amount in respectively the first ($[0, 0.25]$), second ($]0.25, 0.50]$), third ($]0.50, 0.75]$) or fourth ($]0.75, 0.5]$) quarter of the total storm duration (see also Sect. 2.1). If such classification cannot be made (e.g. a storm shorter than four 10-minute intervals) or the maximum is observed in two or more quartiles, then the storm is removed from further analyses. The construction of Huff curves is then based on the distribution of normalized cumulative rainfall amounts in 5% time intervals of the normalized storm duration. Here, the 10%, 50% and 90% percentile curves are analyzed, considering storms of different quartile groups and with specific secondary return periods.

    Figure 4 shows different Huff curves that are constructed considering all storms in a specific season and quartile group, regardless of their return period. The number of storms considered is given in Table 1.'

    If the referee could indicate which of the above paragraphs need more elaboration in order to provide a clear understanding of Fig. 4, we would be happy to incorporate more details.

12. Please revise the legend and/or line styles in Figure 5; it is quite difficult to distinguish one line from another.

    We will revise the legend in the revised manuscript which hopefully makes the lines more distinguishable.

13. Figure 5: I am not quite sure why the authors are showing the plots for return periods of 0.04 to 0.24 years. These return periods have no value in practical applications which is highlighted in the Abstract and Introduction. Please plot the figure for more meaningful and common return periods used for design purposes.

    The first alinea of Section 3.1 already discusses why such small thresholds for the return period are considered.

'In order to study the influence of the extremity of a storm on its internal storm structure, Huff curves can be constructed considering only storms having a return period larger than a specific threshold. To obtain reliable Huff curves a sufficient number of storms is needed. Therefore, these thresholds on the secondary return period should not be too large, as most secondary return periods are small (see Fig. 3). Figure 5 shows Huff curves for third quartile storms in summer, with thresholds on the secondary return period of 0.04, 0.08, 0.12, 0.16, 0.20 and 0.24 year. Note that these thresholds are not extreme, however, 68% of all 3rd quartile summer storms have a secondary return period smaller than 0.24 year (88 days). For these thresholds, respectively 299, 259, 185, 147, 117 and 97 storms are considered. As the percentile curves are very similar, this might indicate the independence of the internal storm structure and the extremity of a storm.'

14. Figure 6 is not discussed in the manuscript. I understand that the discussion on the bottom of Page 3626 is related to Figure 6. However, the authors are expected to explicitly discuss every figure in the text and explain every panel in enough details.

    In the revised version of our paper we will include the reference to Figure 6 in the corresponding discussion.

15. Please use Figure or Fig. consistently throughout the text.

    The guidelines for publication in HESS are to use Figure in the beginning of a sentence and Fig. in running text.

16. Page 3627: '...of the total storm duration are randomly generated, constrained by the fact that these cumulative storm depths should lie between the 10% and 90% percentile...' Please explain exactly how the above constrains on randomly generated rainfall are applied.



As explained in the Introduction (see also comment nrs. 3 and 11), the empirical distribution of the normalized cumulative storm depth is considered at each 5% time interval of the normalized cumulative storm duration. At each 5% time interval, the 10% and 90% percentiles of this empirical distribution are calculated. This thus results in two bounds for the normalized cumulative storm depth at the considered time interval. The algorithm then chooses uniformly at random a normalized cumulative storm depth in between these bounds. If the normalized cumulative storm depth in the preceding 5% time interval is higher than the lower bound given by the 10% percentile curve in the considered 5% time interval, then this preceding normalized cumulative storm depth forms the lower bound. Otherwise impossible decreases in normalized cumulative storm depths could occur.

In the revised manuscript we will make the explanation more clear.

17. Section 4: I think this part needs a major review. I don't understand why the authors select a secondary return period of 2.79 and a storm that occurred 100 years ago. I think you should choose a more common return period (e.g., 10, 20, 25). Furthermore, I think you need to provide the input and output ensemble to show how the model works. The provided figures do not give any idea about the ensembles.

    This response also covers the next two comments (nrs. 18 and 19).

    First of all, the choice of the storm is purely illustrative. The starting point of this section is to generate a set or an 'ensemble' of statistically identical storms based on a historical storm that caused problems in a catchment. The use of a common return period is not the objective. It is easy to understand that a storm with a smaller return period can also cause problems. This is very much influenced by the antecedent soil moisture conditions of the cathment. Therefore, the same train of historical storms is used in the simulation excercise in order to ensure similar antecedent conditions. To have an idea of the considered storm and its preceding storms, Figure 9 is given. Additionally, as stated before, the bulk of the storms have a 'small' return period.

With respect to the input and output ensemble, there might be some confusion. It should be clear that the ensemble as presented here can not be seen as a one-dimensional ensemble which one can validate by means of a Talagrand diagram. The latter is indeed often used to validate ensembles. However, these ensembles are not the same as what we call 'ensemble' in our paper. If we would have perturbed the rainfall pattern of the historical storm at each time step with e.g. a normal distributed noise and routed all these rainfall patterns through the PDM model, then we would have obtained an ensemble of the discharge that could be evaluated by means of a Talagrand diagram. The core of the latter, in very simple words, is to evaluate whether the spread of the ensemble at each time step captures the observed discharge. Our 'ensemble' contains storms that are statistically identical, but which do not necessarily follow the same pattern as the observed storm, nor have the same length. This ensemble could not be seen as 'one-dimensional' as several statistical aspects are considered in the production of the ensemble: the dependence between storm duration and storm depth and the internal storm structure. We therefore believe that a representation and validation of the 'ensemble' as suggested by the reviewer does not makes sense here. In fact, Figure 10 provides some kind of visualization and validation of the ensemble. If e.g. the historical $Q_{\max}$ would not fall into the spread of the $Q_{\max}$ for the ensemble, the proposed generator would be biased.

To illustrate that our methodology also works for more recent storms with a higher return period, the same analysis as in the paper is done for an observed 2nd quartile storm in summer with a return period of 26.96 years. Figures 1 and 2 are the same as the corresponding Figs. 9 and 10 in our paper. The possible discharges are of course in a higher range, as the secondary return period is higher than the storm considered in the paper. In the revised version of our



manuscript we will use this example.

18. Figure 9: I think it is better to remove the gray area and show a clear figure of the storm (black line) and the ensemble bounds. Figure 9 in its current form is not informative at all.

    See comment nr. 17.

19. Since you are presenting an ensemble generator you need to validate your ensemble too. A typical way to evaluate and validate one-dimensional ensembles is to derive the rank histogram (also referred to as Talagrand diagram). Please provide the rank histogram of your generated ensemble.

    See comment nr. 17.

20. The authors are expected to provide a compelling conclusion that indicates the work is worthwhile. However, the current version of Conclusions is more like a summary/abstract of the paper.

    In the revised manuscript, the conclusion will be rewritten, indicating the importance and limitations of the work.

21. The Conclusions does not acknowledge the limitations of the study. Please discuss the limitations of the proposed model.

    See previous comment (nr. 20).

22. Many of the blow references are irrelevant to this manuscript. They are applications based on the same data used in this manuscript. Please refer to one or two of them and remove those that are not relevant: Willems, 2000; Demaree, 1985; Laurant, 1976; Ntegeka and 10 Willems, 2008; De Jongh et al., 2006; Blanckaert and Willems, 2006; Vaes et al., 2002; Gellens, 2000; Schmitt and Nicolis (2002); Schmitt et al. (1998); Vaes et al., 2000, 2001; Vaes and Berlamont, 2000, 2001; Vaes, 1999.

In the revised manuscript we will remove some references, keeping only the most important and recent ones.

23. I think the work can benefit from a comprehensive literature review. Many relevant publications on application of copulas in simulation of rainfall fields are not acknowledged in the manuscript. Some of which are listed below. Please conduct a careful literature review and discuss relevant works: (Bárdossy and Pegram, 2009), (Serinaldi, 2009), (AghaKouchak et al., 2010), (Serinaldi, 2009), (Villarini et al., 2008).

    The reason why these references were not included is that the scope of the current paper is point rainfall, whereas the mentioned papers refer to copula-applications in spatial rainfall simulation. In the revised version, we will provide some extra references in the conclusions in the discussion of the relevance of spatial rainfall generators for designing purposes.

## References


AghaKouchak, A., Bárdossy, A., and Habib, E.: Conditional simulation of remotely sensed rainfall data using a non-Gaussian v-transformed copula, Advances in Water Resources, 33, 624–634, doi:10.1016/j.advwatres.2010.02.010, 2010.

Bárdossy, A. and Pegram, G. G. S.: Copula based multisite model for daily precipitation simulation, Hydrol. Earth Syst. Sci., 13, 2299–2314, 2009.

Salvadori, G. and De Michele, C.: Multivariate multiparameter extreme value models and return periods: a copula approach, Water Resour. Res., doi:10.1029/2009WR009040, in press, 2010.

Serinaldi, F.: Copula-based mixed models for bivariate rainfall data: an empirical study in regression perspective, Stoch. Environ. Res. Risk Assess., 23, 677–693, doi:10.1007/s00477-008-0249-z, 2009.

Serinaldi, F.: A multisite daily rainfall generator driven by bivariate copula-based mixed distributions, J. Geophys. Res. - Atmospheres, 114, doi:10.1029/2008JD011258, D10103, 2009.





Villarini, G., Serinaldi, F., and Krajewski, W. F.: Modeling radar-rainfall estimation uncertainties using parametric and non-parametric approaches, Adv. Water Resour., 31, 1674–1686, doi:10.1016/j.advwatres.2008.08.002, 2008.


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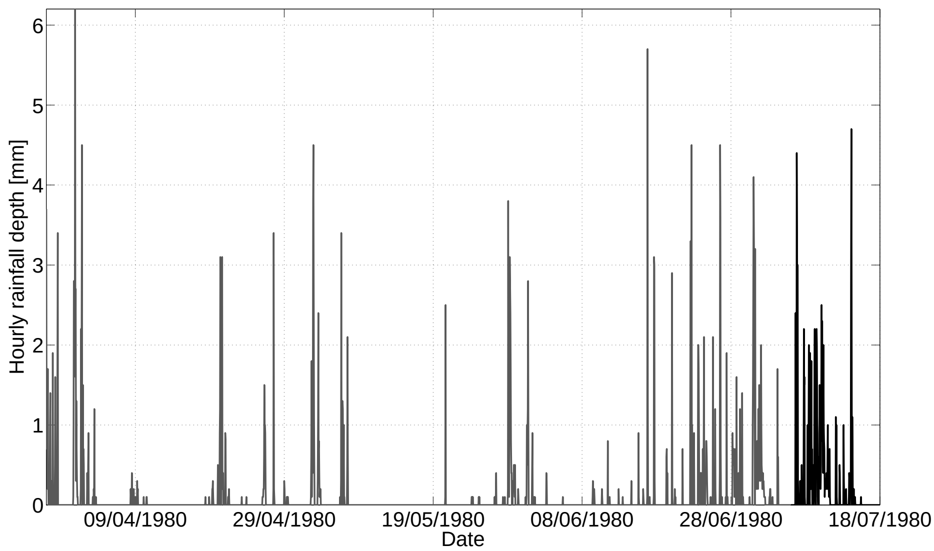


**Fig. 1.** The grey storms determine the antecedent soil moisture conditions and the black storm is the historical storm which is replaced by 10 000 randomly generated design storms.



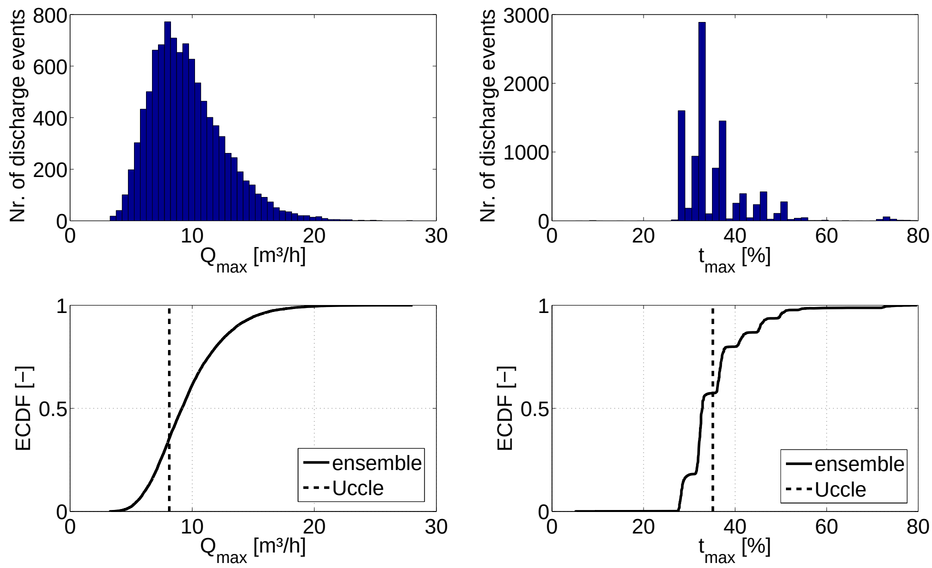


**Fig. 2.** The ensemble of design storms gives an idea of the distribution of the maximum peak discharge and its relative time of occurrence. These properties are also indicated for the historical storm.