Extended Water Allocation System Model Formulation

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This supplemental material presents the complete optimization program for the extended Water Allocation System model in standard form: the objective function followed by nine primary and five additional constraints. Notation follows Fisher et al. (2005) and Rosenberg et al. (2008) . Model extensions include and allow return flows from agriculture, brine waste from desalination, multiple water quality types to meet a minimum in-stream flow requirement, fixed-increment infrastructure capacity expansions, and are shown in constraints 1and 6 – 9.

Objective Function

$$Max\,Z = \sum_e prob_e \cdot \left[\begin{array}{l} \displaystyle\sum_i \sum_d \frac{b_{ide}\left(\sum_q QD_{idqe}\right)^{\alpha_{ide}+1}}{\alpha_{ide}+1} - \sum_i \sum_q \sum_s cs_{iqse} QS_{iqse} - \sum_p \sum_{j \neq p} \sum_q ctr_{qpje} QTR_{qpje} \\ \displaystyle - \sum_i \sum_d \sum_q cr_{idqe} QTW_{idqe} - \sum_i \sum_d ce_{ide}\left(\sum_q QDC_{idqe}\right) \end{array} \right]$$
$$- \sum_p \sum_{j \neq p} \sum_q cxtr_{qpj} XTR_{qpj} - \sum_i \sum_q \sum_s cxs_{iqs} XS_{iqs} - \sum_i \sum_q \left(cxtw_{iq} XTW_{iq} + cxl_{iq} XL_{iq}\right)$$
$$- \sum_i \sum_d cxcon_{id} XCON_{id}$$

Subject to (constraints)

1. Continuity equation (mass balance) at each district for each quality in each event

$$\sum_d QDC_{idqe} = \left(\sum_s QS_{iqse} + \sum_d QTW_{idqe} + QB_{iqe} + \sum_p QTR_{qpie} - \sum_p QTR_{qipe} \right) \cdot \left(1 - dl_{0iq} - XL_{iq}\right), \ \forall i,q,e$$

2. Continuity equation (mass balance) at each node for each quality in each event

$$\sum_{p} QTR_{qpne} = \sum_{p} QTR_{qnpe}, \ \forall n, q, e$$

3. Treated waste-water comes from water demanded

$$\sum_{q} QTW_{idqe} = PR_{ide} \sum_{q} QDC_{idqe}, \ \forall i, d, e$$

4. Lower limit on demand for each water use sector in each district in each event

$$\sum_{q} QD_{idqe} \geq \left(\frac{p_{\max}}{b_{ide}} \right)^{\frac{1}{\alpha_{ide}}}, \ \forall i, d, e$$

5. User conservation reduces real water demanded without loss of economic benefit

$$QDC_{idqe} = QD_{idqe} \cdot \left(1 - pcon_{0\ id} - XCON_{id}\right), \ \forall i, d, q, e$$

6. Brine waste generated is a fraction of desalinated water

$$QB_{iqe} = \sum_{q_e \in dq_i(q)} bf_{iq_2} \cdot QS_{iq_2 se}, \ \forall i, q, e, s = desal$$

7. Supply expansions limited to fixed increments

$$XS_{iqs} = qs_{\text{int}\,erval\,iqs} \cdot XSLEV_{iqs}, \ \forall i, q, s$$

8. Conveyance expansions limited to fixed increments

$$XTR_{qpj} = qtr_{\text{int}\,erval\ \ qpj} \cdot XTRLEV_{iqs}, \ \forall q, p, j$$

9. Expected flow of one or more water quality types along a conveyance link must meet a minimum required flow

$$\sum_{e} \left(prob_e \cdot \sum_{q \in qt(p,j)} QTR_{qpje} \right) \geq qtr_{\text{minmq}\ pj}, \ \forall p, j$$

With the following bounds

$$QS_{iqse} \leq qs_{0\ \ iqs} + XS_{iqs}, \ \forall i, q, s, e \qquad qs_{0\ \ iqs} + XS_{iqs} \leq qs_{\max\ iqs}, \ \forall i, q, s$$

$$PR_{ide} \leq pr_{\max\ \ id}, \ \forall i, d, e \qquad QS_{iqse} \leq qs_{flow\,iqse}, \ \forall i, q, s, e$$

$$QTR_{qpje} \leq qtr_{0\ \ qpj} + XTR_{qpj}, \ \forall q, p, j, e \qquad qtr_{0\ \ qpj} + XTR_{qpj} \leq qtr_{\max\ \ qpj}, \ \forall q, p, j$$

$$QTR_{qpje} \geq qtr_{\min\ qpj}, \forall q, p, j, e \qquad pcon_{0\ id} + XCON_{id} \leq pcon_{\max\ id}, \forall i, d$$

$$\sum_{d} QTW_{idq} \leq qtw_{0\ iq} + XTW_{iq}, \forall i, q \qquad dl_{0\ iq} + XL_{iq} \leq dl_{\max\ iq}, \forall i, q$$

and all variables positive.

Variables are:

Z = net benefit from water in millions of dollars;

QB_{iqe} = brine waste volume generated in district *i* in event *e* of quality *q* in 10^6 m^3;

QS_{iqse} = volume supplied by source *s* of quality *q* in district *i* in event *e* in 10^6 m^3;

QD_{idqe} = volume of quality *q* demanded by sector *d* in district *i* in event *e* in 10^6 m^3;

QDC_{idqe} = volume demanded after conservation in 10^6 m^3;

QTR_{qpje} = volume of water quality *q* transferred from *p* to *j* in event *e* in 10^6 m^3;

QTW_{idqe} = sector *d* wastewater treated to quality *q* in district *i* in event *e* in 10^6 m^3;

PR_{ide} = percent of sector *d* wastewater treated in district *i* in event *e* in fraction;

XS_{iqs} = Size of supply expansions for source *s* of quality *q* in district *i* in 10^6 m^3;

$XSLEV_{i,q,s}$ = Number of source expansions implemented at district *i* of quality *q* for source *s* in integers [0, 1, 2, …];

XTR_{qpj} = Size of conveyance expansions from point *p* to *j* of quality *q* in10^6 m^3;

$XTRLEV_{qpj}$ = Number of conveyance expansions implemented of quality *q* from *p* to *j* in integers [0, 1, 2, …];

XTW_{id} = Size of wastewater reuse plant expansions in district *i* for quality *q* 10^6 m^3;

XL_{iq} = Leak reduction program expansion in district *i* for quality *q* in fraction;

$XCON_{id}$ = User conservation program expansion in district *i* for quality *q* in fraction;

Indices are:

p = points (districts and nodes);

i = district;

n = nodes;

d = water sector (urban, industrial, or agricultural);

s = supply source or step;

q = water quality type (fresh, recycled water);

e = events (water supply availability / demand)

Parameters are:

α_{ide} = exponent of inverse demand function for demand *d* in district *i* in event *e*;

b_{ide} = coefficient of inverse demand curve for demand *d* in district *i* in event *e*;

bf_{iq} = brine fraction that represents the volume of brine generated for each 1 m^3 of desalinated water produced [unitless];

ce_{ide} = unit environmental cost of water discharged by demand sector *d* in district *i* in event *e* in \$ m^{-3};

cr_{idqe} = unit cost to treat sector *d* waste in district *i* to quality *q* in event *e* in \$ m^{-3};

cs_{iqse} = unit cost to supply new water of quality *q* from source *s* in district *i* in event *e* in \$ m^{-3};

ctr_{qpje} = unit cost to transport water quality *q* from point *p* to *j* in event *e* in \$ m^{-3};

cxs_{iqs} = annualized capital cost to expand source *s* of quality *q* in district *i* in \$ m^{-3};

$cxtr_{iqs}$ = annualized capital cost to expand conveyance capacity of quality *q* from point *p* to *j* in \$ m^{-3};

$cxtw_{iq}$ = annualized capital cost to expand wastewater treatment capacity to quality *q* in district *i* in \$ m^{-3};

$cxcon_{id}$ = annualized capital cost to expand user conservation program in district *i* for sector *d* in \$ $fraction^{-1}$;

cxl_{iq} = annualized capital cost to expand leak reduction program in district *i* for quality *q* in \$ $fraction^{-1}$;

$dq_i(q)$ = set of source water quality types that, when desalinated in district *i*, generate brine of quality *q* [unitless];

p_e = probability of event *e* in fraction;

$p_{max\ id}$ = maximum price of water from demand sector *d* in district *i* in \$ m^{-3};

$pr_{max\ id}$ = maximum percent of waste from demand sector *d* in district *i* that can be treated in fraction;

$qs_{0\ iqs}$ = existing capacity of source *s* of quality *q* in district *i* in $10^6\ m^3$;

$qs_{flow\ iqse}$ = availability of source *s* of quality *q* in district *i* in event *e* in $10^6\ m^3$;

$qs_{interval\ iqs}$ = fixed interval to expand source capacity *s* of quality *q* in district *i* in $10^6 m^3$;

$qs_{max\ iqs}$ = maximum capacity for source *s* of quality *q* in district *i* in $10^6\ m^3$;

$qt(p,j)$ = set of water quality types whose flows can count towards the minimum required flow along the conveyance link from *p* to *j* [unitless];

$qtr_{0\ qpj}$ = existing conveyance capacity for quality *q* from point *p* to *j* in $10^6\ m^3$;

$qtr_{interval\ qpj}$ = fixed interval to expand conveyance capacity of quality *q* from *p* to *j* in $10^6 m^3$;

$qtr_{min\ qpj}$ = minimum required flow of quality *q* from point *p* to *j* in $10^6\ m^3$;

$qtr_{minmq\ pj}$ = minimum required flow from point *p* to *j* that multiple water quality types must satisfy on average in $10^6\ m^3$;

$qtw_{0\ iq}$ = existing capacity to treat water to quality *q* at district *i* in $10^6\ m^3$;

$pcon_{0\ id}$ = reduction in use associated with existing conservation programs for sector *d* in district *i* in fraction;

$pcon_{max\ id}$ = maximum reduction in use from conservation programs for sector *d* in district *i* in fraction;

$dl_{0\ iq}$ = existing leak rate for quality *q* in district *i* in fraction;

$dl_{max\ iq}$ = maximum reduction in leakage rate for quality *q* in district *i* in fraction;

Additional Constraints

10. Total demand consists of paid water and unaccounted-for losses

$$QDC_{idqe} = QD_{paid\ idqe} + r_{iqe}\, pr_{unpaid\ ide},\ \forall i, d, q, e.$$

11. Demand for certain water quality types must be less than a specified quantity

$$\sum_{d} QDC_{idqe} \leq q_{rec\max\ iq},\ \forall i, q, e$$

12. Demand for certain water quality types must be less than a specified percentage of total demand.

$$\sum_{d} QDC_{idqe} \leq p_{rec\max\ iq} \sum_{d,q2} QD_{id,q2,e},\ \forall i, q, e$$

13. Use from a pool of sources must be less than a specified quantity

$$\sum_{iqs} indcp_{ciqs} QS_{iqse} \leq q_{shared\ ce},\ \forall c, e$$

14. Minimum required allocation to each sector

$$\sum_{q} QDC_{idqe} \geq q_{required\ ide},\ \forall i, d, e$$

References

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