Hydrol. Earth Syst. Sci. Discuss., 7, 9155–9171, 2010
www.hydrol-earth-syst-sci-discuss.net/7/9155/2010/
doi:10.5194/hessd-7-9155-2010

Hydrology and
Earth System
Sciences
Discussions

Estimations of tidal characteristics and aquifer parameters via tide-induced head changes in coastal observation wells

Y.-J. Chen[1], **G.-Y. Chen**[1], **H.-D. Yeh**[1], **and D.-S. Jeng**[2]

[1]Institute of Environmental Engineering, National Chiao Tung University, Hsinchu, Taiwan
[2]Division of Civil Engineering, University of Dundee, UK

Received: 27 October 2010 – Accepted: 15 November 2010 – Published: 3 December 2010

Correspondence to: H.-D. Yeh (hdyeh@mail.nctu.edu.tw)

Published by Copernicus Publications on behalf of the European Geosciences Union.

Abstract

The groundwater fluctuations affected by tidal variations at an observation well in a coastal aquifer can be used to determine the tidal characteristics and aquifer parameters without conducting an aquifer test. In this study, a method, comprised of Jeng et al.'s solution (2005) and simulated annealing (SA) algorithm, is developed to determine the coastal aquifer parameters (namely, hydraulic diffusivity, beach slope, and aquifer thickness) as well as the tidal characteristics (namely, bichromatic-tide amplitudes, bichromatic-tide wave frequencies, and tidal phase lag) from the analysis of the tide-induced well-water-level (WWL) data. Two data sets, i.e., synthetic WWL data generated from Jeng et al.'s solution (2005) with assumed parameter values and field data obtained from Barrenjoey beach in Australia, are analyzed. The estimated parameter values obtained from analyzing synthetic WWL data by the present method show good agreements with the previously assumed parameter values. The parameter estimation procedure may however fail in the case of a large shallow-water parameter which in fact violates the constraint on the use of Jeng et al.'s solution (2005). In the analysis of field WWL data, the results indicate that the estimated aquifer parameters from the present method are significantly different from those given in Nielsen (1990). Inspecting the observed WWL data and the WWL data predicted from Jeng et al.'s solution (2005) reveals that the present method may provide better estimations for the aquifer parameters than those given in Nielsen (1990).

1 Introduction

In coastal areas, groundwater levels of an aquifer fluctuate with tidal variations. The coastal aquifer parameters can be estimated from analyzing the well-water-level (WWL) data at an observation well without conducting conventional aquifer tests. Numerous investigations have been devoted to the study of hydraulics of tide-induced groundwater variation in coastal aquifers. Nielsen (1990) used a perturbation technique to obtain an analytical solution up to the second order of amplitude parameter ($\alpha = A/D$) for tidal dynamics in sloping sandy beaches. In his model, the shoreline boundary condition at the interface of the beach and the ocean was allowed to vary with the tide height, it however produced a seepage point deviated from the shoreline when the tide drops quickly. Li et al. (2000) overcame this conflict by introducing a moving shoreline condition and considered the tides to be bichromatic, namely, tides can be represented by two different wave frequencies. Both models however regard the beach slope as a part of the perturbation parameter, which restricts the applicability of the models to the case of aquifers with large beach slopes. Moreover, both models relied on the Boussinesq equation that was solved only by the zero-order approximation in shallow water expansion. On the other hand, Teo et al. (2003) developed a higher-order analytical solution based on the shallow water expansions for the water table fluctuations induced by the monochromatic tide in a sloping coastal aquifer. Jeng et al. (2005) further considered the effect of bichromatic tide in the development of solution for WWL fluctuations in a sloping coastal aquifer.

Identifying the aquifer parameters and tidal characteristics from the analysis of WWL data can be cast as a minimization problem. Simulated annealing (SA), first proposed by Metropolis et al. (1953), is a technique constructed on the statistical mechanics for solving the optimization problems. The concept of the algorithm is based on simulating the re-crystallization of a material in the process of annealing. SA has the ability to deal with the complicated problems involving multi-degrees of freedom and several local optima. In hydrological engineering, SA has been widely applied to solve various types of optimization problems (e.g., Marryott et al., 1993; Pardo-Iguzquiza, 1998; Huang and Yeh, 2007; Yeh and Chen, 2007; Chen and Yeh, 2009).

In this study, we propose a method to estimate the coastal aquifer parameters as well as tidal characteristics, including hydraulic diffusivity, beach slope, aquifer thickness, bichromatic-tide amplitudes, bichromatic-tide wave frequencies, and tidal phase lag, from analyzing the tide-induced WWL data. The analytical solution presented by Jeng et al. (2005) along with a set of chosen parameter values is adopted to generate the observed WWL data in the hypothetical case. Then, both synthetic and real field WWL data are analyzed to demonstrate the capability and the limitation of proposed method in the determination of the aquifer parameters and tidal characteristics.

2 Methodology

The method of least squares minimizes the sum of squared residuals between the observed WWL data and predicted WWL data from Jeng et al.'s solution (2005). If the observed WWL data in a specific well is analyzed, the objective function f being minimized can be expressed as

$$f = \sum_{i=1}^{n} \left[h_\mathrm{p}(t_i) - h_\mathrm{o}(t_i) \right]^2 \tag{1}$$

where n is the number of WWL data and $h_\mathrm{p}(t_i)$ and $h_\mathrm{o}(t_i)$ are the predicted and observed WWL data at time t_i, respectively. The SA algorithm is then applied to find the best estimates of the tidal characteristics and aquifer parameters that can minimize the objective function value.

2.1 Field data simulator

Figure 1 illustrates the groundwater fluctuations in response to tidal variations in a coastal aquifer. Jeng et al. (2005) assumed that the flow in a rigid porous medium is homogeneous and incompressible. The tides are assumed to be bichromatic, which can be synthesized by the superposition of two different wave frequencies. The water table height at the boundary of ocean and coast equals tidal oscillation; that is,

$$h(x_0, t) = D + A_1 \cos(\omega_1 t + \delta_1) + A_2 \cos(\omega_2 t + \delta_2) \tag{2}$$

where h is the tide-induced water table height; D is the average height of the water table, which can be regarded as the aquifer thickness; A_1 and A_2 are the amplitudes of bichromatic-tide variations; ω_1 and ω_2 are the bichromatic-tide wave frequencies; δ_1 and δ_2 are the tidal phases. In addition, the boundary located at x_0 is related to the tidal oscillation as:

$$x_0 = [A_1 \cos(\omega_1 t + \delta_1) + A_2 \cos(\omega_2 t + \delta_2)] \cot(\beta) \tag{3}$$

where β denotes the beach slope.

Jeng et al. (2005) gave the solution for the tide-induced water table height as

$$h = D[1 + \left(\alpha H_{01} + \alpha^2 H_{02}\right) + \varepsilon \left(\alpha H_{11} + \alpha^2 H_{12}\right) + \varepsilon^2 \alpha H_{21}] \tag{4}$$

where $\alpha = A_1/D$ is an amplitude parameter and $\varepsilon = \sqrt{n_e \omega_1 D/2K}$ is a shallow water parameter with n_e and K denoting the soil porosity and hydraulic conductivity, respectively. The coefficients, H_{01}, H_{02}, H_{11}, H_{12}, and H_{21}, are defined in Appendix A. Equation (4) is used either to calculate the values of the predicted WWL data $h_p(t_i)$ in Eq. (1) or to generate the observed WWL data $h_o(t_i)$ for the hypothetical cases.

2.2 Simulated annealing

The SA algorithm is applied to find a set of trial solution for the unknown parameters that minimize the objective function (i.e., Eq. 1). In SA, the Metropolis criterion (1953) describes the acceptation probability of the change from the current solution i to the trial solution j. The criterion making SA have the ability to escape from local optimum is expressed as

$$P(\text{accept trial solution } j) = \begin{cases} 1 & , \text{if } f(j) \le f(i) \\ \exp\left(\frac{-[f(j) - f(i)]}{T_e}\right) & , \text{if } f(j) > f(i) \end{cases} \tag{5}$$

where T_e represents the system temperature in SA. The acceptation probability of an inferior trial solution becomes smaller when the system cools down.

In the following, SA is used to analyze the aquifer parameters such as hydraulic diffusivity (K/n_e), beach slope and aquifer thickness as well as the tidal characteristics including tidal amplitudes, tidal wave frequencies, and tidal phase lag simultaneously. Note that the δ_1 is set as zero in all cases. Each parameter has its own lower and upper bounds; that is 1 m/day and 10^4 m/day for K/n_e, 0 and $\pi/2$ for β, 1 m and 100 m for D, 0.1 m and 10 m for A_1, 0 m and 10 m for A_2, 0 day^{-1} and 4π day^{-1} for ω_1, 0 day^{-1} and 4π day^{-1} for ω_2, and 0 and π for δ_2. The lower bound of A_1 is 0.1 m rather than zero to prevent the denominator of λ, which appears in the coefficients of Eq. (4) and defined in Appendix A as $\lambda = A_2/A_1$, being zero. Each parameter begins with the averaged value of the upper and lower bounds. Teo et al. (2003) indicated that the shallow water parameter ε is usually small in real environments and suggested its value ranged from 0.1 to 0.6 in their simulation. The trial solutions for K/n_e, ω_1, and D in each search are therefore constrained to ensure the value of shallow water parameter ε less than 0.6. Additionally, the SA algorithm starts at an initial temperature of 5. The system temperature reduces with a cooling rate of 0.85 after the searching of 600 trial solutions. The algorithm terminates when the difference of the best-so-far objective functions between two consecutive temperatures is less than 10^{-6} for four consecutive times or the iteration number exceeds 2×10^7.

3 Results and discussion

3.1 Analysis of hypothetical data

Table 1 shows the estimated results form the analyses of three scenarios for synthetic WWL data. Scenario 1 represents the problem that the WWL data being observed at well 1 located at 5 m away from the intersection point of the beach surface and mean sea level. Similarly, scenario 2 represents that taken at well 2 located at 10 m and scenario 3 represents that taken at well 3 located at 20 m. In each scenario, five cases named from cases a to e are analyzed. Case a analyzes the noise-free WWL data generated by Eq. (2), while case b to case e coped with the WWL data containing measurement errors produced by

$$h_\mathrm{m}(x,t) = h_\mathrm{ub}(x,t) + \varphi RN(o) \qquad (6)$$

where $h_\mathrm{ub}(x,t)$ and $h_\mathrm{m}(x,t)$, respectively, denote the noise-free WWL data used for case a and observed WWL data containing noise for other cases; φ is chosen to be 1% for representing the accuracy of field measurements on the order of centimeter; $RN(o)$ represents the random number of the order o. Four sets of random numbers being adopted in cases b to e are normally distributed and generated by the routine RNNOF of IMSL (2003).

Table 1 provides the root mean squared error (RMSE) between the predicted and observed WWL data as an index to examine the accuracy of the prediction, which is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(h_\mathrm{p}(t_i) - h_\mathrm{o}(t_i)\right)^2} \qquad (7)$$

Moreover, the relative error (RE) is also given at the bottom row of Table 1.

In Table 1, the WWL data were generated based on the parameters K/n_e, β, D, A_1, A_2, ω_1, ω_2 and δ_2 being given as 500 m/day, $\pi/3$ (1.047), 25 m, 2 m, 1 m, $4\pi\mathrm{day}^{-1}$, $2\pi\mathrm{day}^{-1}$, and $\pi/4$, respectively. The estimated results are fairly close to the target values. In addition, the RMSE value for case a is on the order of 10^{-4} m while those of cases b to e are on the order of 10^{-3} m. These results indicate good matches between the predicted and synthetic WWL data.

A small shallow water parameter ε is placed as the constraint on Jeng et al.'s solution (2005). Table 2 examines the effect of various ε on the estimated results. Scenarios 4 and 5 have the same target parameter values and well location as scenario 2 except that K/n_e become 50 m/day and 5000 m/day representing the cases of ε being 1.772 and 0.177, respectively. For the cases with an extremely large ε like scenario 4, the estimated results show large deviations on all of the parameter values and the RMSEs are on the order of 10^{-1} m. Such a poor result is due to the violation of the constraint of ε which should be smaller than 0.6. On the other hand, in scenario 5 with a small ε, the proposed method gives the estimations of the aquifer parameters and tidal characteristics close to their target ones.

3.2 Analysis of field data

Nielsen (1990) presented a hydraulic head solution in a coastal aquifer affected by the tide fluctuations and analyzed the WWL measurements observed at Barrenjoey beach in Australia. He provided the averaged WWL at wells 7 to 11 over 26 h in his Table 1. Wells 7–11 were located at 6.6 m, 9.1 m, 11.6 m, 14.1 m, and 16.6 m, respectively, away from the intersection of the beach surface and mean sea level. The aquifer thickness D was 0.51 m. The tidal amplitudes A_1 and A_2 were 0.516 m and 0.014 m, respectively, while the tidal wave frequencies ω_1 and ω_2 were 0.513 h^{-1} and 1.026 h^{-1}, respectively. Moreover, the tidal phases δ_1 and δ_2 were 0 and 1.303, respectively. The wave number (k), defined as $k=\varepsilon/D$, was determined by taking the average of the regression results of k in damping term and phase lags term in those wells to the hydraulic head solution. Thereafter, the value of K/n_e was obtained as 2076 m/day after inserting the known values of k, ω_1 and D to the formula of wave number. The estimated value of β was 0.09967, which was calculated from the difference in sand-level heights at wells 1 and 11 and divided by the distance between these two wells.

Table 3 shows the estimated results of aquifer parameters from the analysis of WWL data measured at Barrenjoey beach. The tidal characteristics are assumed to be the same as those given in Nielsen (1990). The estimated values of K/n_e and D based on the proposed method at all wells are significantly smaller than those given in Nielsen (1990) except that the β values are close to that given in Nielsen (1990). Table 3 also indicates that our estimated results with smaller RMSE values for the three aquifer parameters might be better than those given in Nielsen (1990). However, it is rather difficult to identify that at which well the WWL data analysis gives the most reasonable estimates of K/n_e, β, and D.

Figure 2 shows the observed WWL given in Nielsen (1990) represented by open circles. In addition, the figure also shows the predicted WWLs by Jeng et al.'s solution (2005) along with the parameters estimated by the present method are plotted as solid lines, while the WWL predicted by Jeng et al.'s solution (2005) along with aquifer parameters estimated by Nielsen (1990) are drawn as dashed lines. The figure indicates that the magnitudes of WWL fluctuations decrease landwards. At each well, it seems that the predicted data on the solid line is closer to the observed WWL data (in circle) than that on the dashed line, demonstrating the capacity of the present method in the estimation of tidal characteristics and aquifer parameters.

4 Conclusions

The method, based on coupling Jeng et al.'s solution (2005) with SA algorithm, is developed to simultaneously estimate the hydraulic diffusivity, beach slope, aquifer thickness, tidal amplitudes, tidal wave frequencies, and tidal phase lag. The method is used to analyze the synthetic WWL data generated by Jeng et al.'s solution (2005) and the field WWL data, collected from Barrenjoey beach in Australia, presented in Nielsen (1990). The estimated results of aquifer parameters and tidal characteristics from the present method are fairly close to those of target ones in the analysis of synthetic WWL data. In addition, Jeng et al.'s solution (2005) gives good predictions of the WWL fluctuations induced by the bichromatic tide in sloping coastal aquifer in the shallow-water cases. When analyzing the field WWL data, the aquifer parameters are estimated alone by setting the tidal characteristics as known. The estimated aquifer diffusivity and aquifer thickness are obviously different from those given in Nielsen (1990). The comparisons of the observed and predicted WWL data show that the present method gives better predictions to the observed WWL data than the predicted results based on the solution and parameter values given in Nielsen (1990).

Appendix A

Jeng et al.'s head solution (2005)

Jeng et al. (2005) provided the solution of water table fluctuations in response to bichromatic tides as

$$h(x,t) = D\left[1 + \left(\alpha H_{01} + \alpha^2 H_{02}\right) + \varepsilon\left(\alpha H_{11} + \alpha^2 H_{12}\right) + \varepsilon^2 \alpha H_{21}\right] \tag{A1}$$

with

$$H_{01} = e^{-X}\cos(\theta_1 + \delta_1) + \lambda e^{-\sqrt{\varpi}X}\cos(\eta_1 + \delta_2) \tag{A2}$$

$$\begin{aligned}
H_{02} = {} & \frac{1}{4}(1 - e^{-2X}) + \frac{\lambda^2}{4}(1 - e^{-2\sqrt{\varpi}X}) + \frac{1}{2}\left[e^{-\sqrt{2}X}\cos(\theta_2 + 2\delta_1) - e^{-2X}\cos 2(\theta_1 + \delta_1)\right] \\
& + \frac{\lambda^2}{2}\left\{e^{-\sqrt{2\varpi}X}\cos(\eta_2 + 2\delta_2) - e^{2\sqrt{\varpi}X}\cos[2(\eta_1 + \delta_2)]\right\} \\
& + \frac{\lambda(1 + \sqrt{\varpi})^2}{4\sqrt{\varpi}}\left[e^{-\sqrt{1+\varpi}X}\cos(\eta_3 + \delta_1 + \delta_2) - e^{-(1+\sqrt{\varpi})X}\cos(\theta_1 + \eta_1 + \delta_1 + \delta_2)\right] \\
& + \frac{\lambda}{2}\left[e^{-(1+\sqrt{\varpi})X}\cos(\theta_1 - \eta_1 + \delta_1 - \delta_2) - e^{-\sqrt{1+\varpi}X}\cos(\eta_4 + \delta_1 - \delta_2)\right] \\
& + \frac{\lambda(1 - \varpi)}{2\sqrt{\varpi}}\left[e^{-(1+\sqrt{\varpi})X}\sin(\theta_1 - \eta_1 + \delta_1 - \delta_2) - e^{-\sqrt{1-\varpi}X}\cos(\eta_4 + \delta_1 - \delta_2)\right]
\end{aligned} \tag{A3}$$

$$H_{11} = 0 \tag{A4}$$

$$\begin{aligned}H_{12}=\frac{1}{\sqrt{2}}\cot\beta\{&\frac{1}{\sqrt{2}}e^{-X}\cos(X-\frac{\pi}{4})+\lambda^2\varpi\left[\frac{1}{\sqrt{2}}-e^{-\sqrt{\varpi}X}\cos(\sqrt{\varpi}X-\frac{\pi}{4})\right]\\
&+\left[e^{-\sqrt{2}X}\cos(\theta_2+2\delta_1+\frac{\pi}{4})-e^{-X}\cos(\theta_3+2\delta_1+\frac{\pi}{4})\right]\\
&+\frac{\lambda^2}{2}\left[e^{-\sqrt{2\varpi}X}\cos(\eta_2+2\delta_2)-e^{-2\sqrt{\varpi}X}\cos 2(\eta_1+\delta_2)\right]\\
&+\lambda\sqrt{\varpi}\left[e^{-\sqrt{1+\varpi}X}-e^{-\sqrt{\varpi}X}\right]\cos(\eta_3+\delta_1+\delta_2+\frac{\pi}{4})\\
&+\lambda\sqrt{\varpi}\left[e^{-\sqrt{\varpi}X}\cos(T-\eta_1+\delta_1-\delta_2-\frac{\pi}{4})-e^{-\sqrt{1-\varpi}X}\cos(\eta_4+\delta_1-\delta_2-\frac{\pi}{4})\right]\\
&+\lambda\left[e^{-\sqrt{1+\varpi}X}\cos(\eta_3+\delta_1+\delta_2+\frac{\pi}{4})-e^{-X}\cos(\theta_1+\varpi T+\delta_1+\delta_2+\frac{\pi}{4})\right]\\
&+\lambda\left[e^{-\sqrt{1-\varpi}X}\cos(\eta_4+\delta_1-\delta_2+\frac{\pi}{4})-e^{-X}\cos(\theta_1-\varpi T+\delta_1+\delta_2+\frac{\pi}{4})\right]\\
&+\frac{\lambda\sqrt{\varpi}}{\sqrt{2}}\left[e^{-\sqrt{2\varpi}X}\cos(\eta_2+2\delta_2+\frac{\pi}{4})-e^{-\sqrt{\varpi}X}\cos(\theta_1+\varpi T+2\delta_2+\frac{\pi}{4})\right]\}\end{aligned}\tag{A5}$$

and

$$H_{21}=-\frac{\sqrt{2}}{3}\left[Xe^{-X}\cos(\theta_1+\delta_1-\frac{\pi}{4})+\frac{\lambda\varpi^2}{\sqrt{\varpi}}e^{-\sqrt{\varpi}X}\cos(\eta_1+\delta_2-\frac{\pi}{4})\right]\tag{A6}$$

where $\lambda=A_2/A_1$ is the ratio of tidal amplitudes; $\varpi=\omega_2/\omega_1$ is the ratio of tidal wave frequencies. Additionally, other variables are defined as $T=\omega_1 t$, $X=\sqrt{\frac{n_e\omega_1}{2KD}}x-\alpha\varepsilon\cot\beta[\cos(T+\delta_1)+\lambda\cos(\varpi T+\delta_2)]$, $\theta_1=T-X$, $\theta_2=2T-\sqrt{2}X$, $\theta_3=2T-X$, $\eta_1=\varpi T-\sqrt{\varpi}X$, $\eta_2=2\varpi T-\sqrt{2\varpi}X$, $\eta_3=(1+\varpi)T-\sqrt{1+\varpi}X$, and $\eta_4=(1-\varpi)T-\sqrt{1-\varpi}X$.

Acknowledgements. This study was supported by "Aim for the Top University Plan" of the National Chiao Tung University and Ministry of Education, Taiwan and the Taiwan National Science

Title Page
Abstract | Introduction
Conclusions | References
Tables | Figures
◀| |▶
◀ ▶
Back | Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

Council under the contract numbers NSC 98-3114-E-007-015, NSC 99-NU-E-009-001, and NSC 99-2221-E-009-062-MY3.

References

Chen, Y. J. and Yeh, H. D.: Parameter estimation/sensitivity analysis for an aquifer test with skin effect, Ground Water, 47(2), 287–299, 2009.

Huang, Y. C. and Yeh, H. D.: The use of sensitivity analysis in on-line aquifer parameter estimation, J. Hydrol., 335(3–4), 406–418, 2007.

IMSL: IMSL Fortran Library User's Guide Stat/Library Volume 2 of 2, version 5.0. Houston, Texas: Visual Numerics, 2003.

Jeng, D. S., Mao, X., Enot, P., Barry, D. A., and Li, L.: Spring-neap tide-induced beach water table fluctuations in a sloping coastal aquifer, Water Resour. Res., 41, W07026, doi:10.1029/2005WR003945, 2005.

Li, L., Barry, D. A., Stagnitti, F., Parlange, J. Y., and Jeng, D. S.: Beach water table fluctuations due to spring-neap tides: moving boundary effects, Adv. Water Resour., 23(8), 817–824, 2000.

Marryott, R. A., Dougherty, D. E., and Stollar, R. L.: Optimal groundwater management 2. Application of simulated annealing to a field-scale contamination site, Water Resour. Res., 29(4), 847–860, 1993.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., and Teller, E.: Equation of state calculations by fast computing machines, J. Chem. Phys., 21(6), 1087–1092, 1953.

Nielsen, P.: Tidal dynamics of the water table in beaches, Water Resour. Res., 26(9), 2127–2134, 1990.

Pardo-Iguzquiza, E.: Optimal selection of number and location of rainfall gauges for areal rainfall estimation using geostatistics and simulated annealing, J. Hydrol., 210(1–4), 206–220, 1998.

Teo, H. T., Jeng, D. S., Seymour, B. R., Barry, D. A., and Li, L.: A new analytical solution for water table fluctuations in coastal aquifers with sloping beaches, Adv. Water Resour., 26(12), 1239–1247, 2003.

Yeh, H. D. and Chen, Y. J.: Determination of skin and aquifer parameters for a slug test with wellbore-skin effect, J. Hydrol., 342, 283–294, 2007.

Title Page
Abstract Introduction
Conclusions References
Tables Figures
◀◀ ▶▶
◀ ▶
Back Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

Table 1. The estimated results for the synthetic WWL data. Scenarios 1, 2, and 3 denote the wells located at $x = 5$, 10, and 20 m, respectively. The target values of the parameters are $K/n_e = 500$ m/day, $\beta = 1.047$, $D = 25$ m, $A_1 = 2$ m, $A_2 = 1$ m, $\omega_1 = 4\pi$ day^{-1}, $\omega_2 = 2\pi$ day^{-1} and $\delta_2 = \pi/4$.

	Estimated Results									
	Aquifer Parameters				Tidal Characteristics					
	K/n_e (m/day)	β (rad)	β (degree)	D (m)	A_1 (m)	A_2 (m)	ω_1 (day^{-1})	ω_2 (day^{-1})	δ_2	RMSE (m)
Target values	500	1.047	59.989	25	2	1	12.567	6.283	0.785	–
scenario 1										
1a	498.807	1.047	59.989	25.046	2.000	1.000	12.566	6.283	0.785	2.61×10^{-4}
1b	535.708	1.046	59.906	23.805	1.992	0.996	12.566	6.278	0.787	8.32×10^{-3}
1c	502.620	1.032	59.145	24.636	2.000	0.998	12.566	6.283	0.784	8.14×10^{-3}
1d	515.027	1.031	59.072	24.293	2.003	0.999	12.566	6.270	0.791	8.40×10^{-3}
1e	550.440	1.041	59.649	24.873	1.987	0.999	12.559	6.289	0.782	8.91×10^{-3}
Mean	520.520	1.014	58.111	24.531	1.996	0.998	12.565	6.281	0.786	–
RE (%)	−4.104	3.130	3.130	1.878	0.179	0.167	0.017	0.038	−0.131	–
scenario 2										
2a	504.068	1.045	59.895	24.942	1.998	0.999	12.566	6.284	0.785	3.01×10^{-4}
2b	579.743	1.025	58.702	22.969	1.972	0.989	12.564	6.284	0.783	8.27×10^{-3}
2c	517.576	1.028	58.904	24.357	1.995	0.996	12.566	6.285	0.783	8.07×10^{-3}
2d	525.738	1.037	59.431	24.156	1.997	0.997	12.566	6.269	0.791	8.37×10^{-3}
2e	507.390	1.035	59.320	25.056	1.995	1.003	12.562	6.282	0.785	9.02×10^{-3}
Mean	526.903	1.034	59.250	24.296	1.991	0.997	12.565	6.281	0.785	–
RE (%)	−5.381	1.231	1.231	2.816	0.437	0.314	0.016	0.035	−0.040	–
scenario 3										
3a	506.623	1.045	59.890	24.838	1.995	0.998	12.566	6.285	0.784	2.87×10^{-4}
3b	664.062	1.027	58.825	21.218	1.914	0.973	12.557	6.327	0.755	9.70×10^{-3}
3c	547.173	1.025	58.740	23.611	1.971	0.989	12.566	6.289	0.778	8.07×10^{-3}
3d	526.065	1.030	59.029	24.251	1.989	0.996	12.566	6.266	0.790	8.47×10^{-3}
3e	493.028	1.038	59.482	25.278	2.004	1.006	12.562	6.275	0.790	8.93×10^{-3}
Mean	547.391	1.033	59.193	23.839	1.974	0.992	12.564	6.288	0.780	–
RE (%)	−9.478	1.326	1.326	4.643	1.277	0.750	0.027	−0.085	0.696	–

Table 2. The estimated results for the synthetic WWL data. Scenarios 4 and 5 have the same target parameter values and well location as scenario 2 except that K/n_e become 50 m/day and 5000 m/day, respectively, representing the cases of a shallow water parameter ε being 1.772 and 0.177.

	Estimated Results									
	Aquifer Parameters				Tidal Characteristics					
	K/n_e (m/day)	β (rad)	β (degree)	D (m)	A_1 (m)	A_2 (m)	ω_1 (day^{-1})	ω_2 (day^{-1})	δ_2	RMSE (m)
scenario 4										
Target values	50	1.047	59.989	25	2	1	12.567	6.283	0.785	–
4a	5739.842	1.571	90.000	27.614	8.731	10.000	3.314	3.314	2.664	3.20×10^{-1}
4b	2369.613	1.571	90.000	27.363	8.746	10.000	3.348	3.348	2.674	3.17×10^{-1}
4c	5936.256	1.571	90.000	27.623	8.733	10.000	3.313	3.313	2.664	3.19×10^{-1}
4d	2182.675	1.571	90.000	27.331	8.748	10.000	3.354	3.354	2.675	3.20×10^{-1}
4e	9999.999	1.571	90.000	25.917	0.578	1.222	12.566	2.823	1.892	2.85×10^{-1}
Mean	5245.677	1.571	90.000	27.170	7.107	8.244	5.179	3.230	2.514	–
RE (%)	−10391.354	−50.028	−50.028	−8.679	−255.352	−724.445	58.788	48.584	−220.213	–
scenario 5										
Target values	5000	1.047	59.989	25	2	1	12.567	6.283	0.785	–
5a	5019.780	1.044	59.838	25.000	2.000	1.000	12.566	6.283	0.785	2.75×10^{-4}
5b	5015.334	1.020	58.427	25.002	1.997	1.001	12.564	6.271	0.787	8.43×10^{-3}
5c	4921.880	0.959	54.922	24.998	2.002	0.999	12.566	6.289	0.782	8.04×10^{-3}
5d	4871.357	1.000	57.306	24.999	2.006	1.001	12.566	6.272	0.790	8.45×10^{-3}
5e	4945.235	0.972	55.667	24.996	2.001	1.003	12.566	6.292	0.784	8.76×10^{-3}
Mean	4954.717	0.999	57.232	24.999	2.001	1.001	12.566	6.281	0.786	–
RE (%)	0.906	4.595	4.595	0.004	−0.062	−0.078	0.009	0.027	−0.068	–

Table 3. The estimated results for the aquifer parameters from Nielsen (1990) and the proposed method based on the field WWL data at Barrenjoey beach in Australia.

		Estimated Aquifer Parameters				
	x (m)	K/n_e (m/day)	β (rad)	D (m)	RMSE[a] (m)	RMSE[b] (m)
Well 7	6.6	1241.774	0.109	0.447	6.19×10^{-2}	0.190
Well 8	9.1	1151.410	0.139	0.422	6.73×10^{-2}	0.169
Well 9	11.6	1265.603	0.171	0.363	5.91×10^{-2}	0.156
Well 10	14.1	1454.474	0.140	0.377	5.70×10^{-2}	0.149
Well 11	16.6	1958.721	0.121	0.389	5.25×10^{-2}	0.138
mean	–	1414.396	0.136	0.387	–	–
Nielsen (1990)	–	2076	0.1	0.51	–	–

[a] The RMSE values of the predicted WWL data with the parameters estimated based on our proposed method to the field data.

[b] The RMSE values of the predicted WWL data with the parameters given in Nielsen (1990) to the field data.

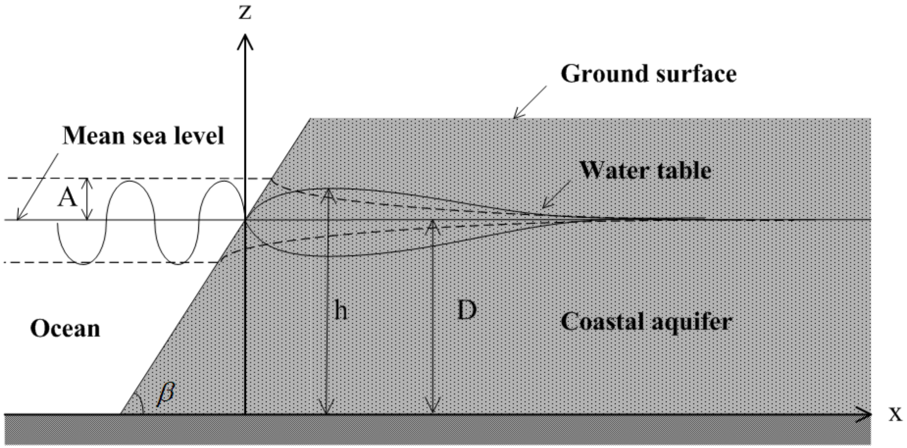
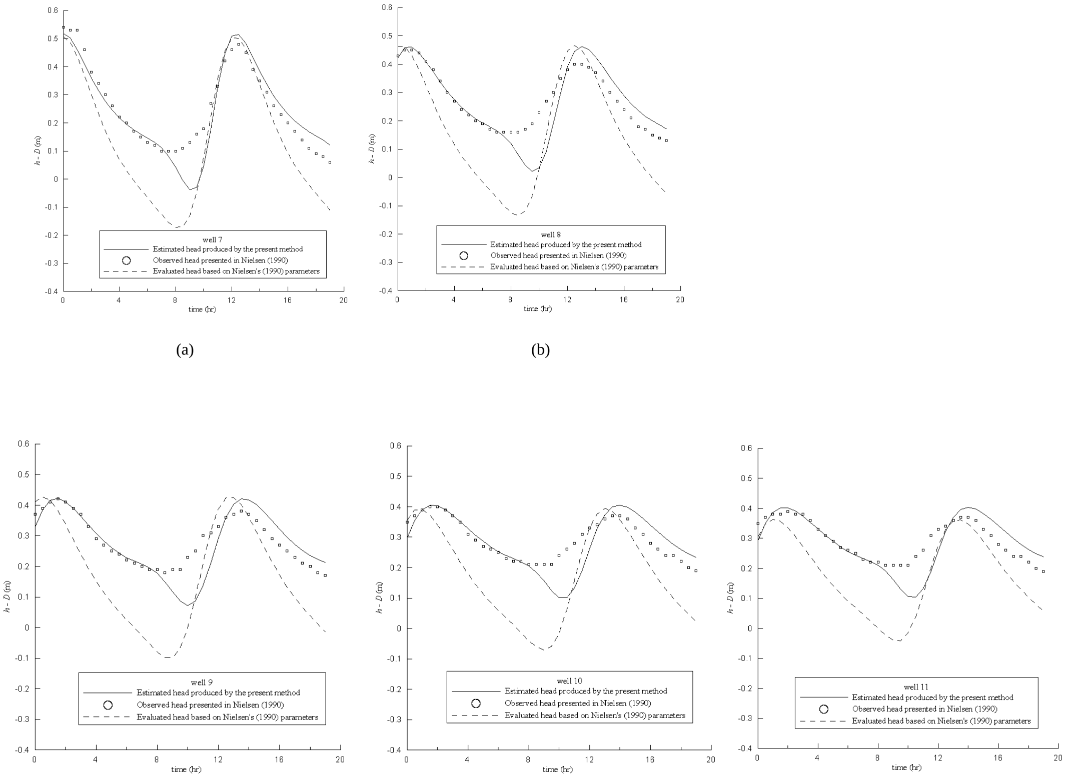


Fig. 1. Illustration of water table fluctuation in response to tidal variation in a coastal aquifer.



(a) (b)

(c) (d) (e)

Fig. 2. Comparisons of observed WWL given in Nielsen (1990) at observation wells 7–11 with the predicted WWL produced by Jeng et al.'s solution (2005) with the parameters determined by the present method and given in Nielsen (1990).