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Big and small: menisci in soil pores affect water pressures, dynamics of groundwater levels, and catchment-scale average matric potentials

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Abstract

Soil water is confined behind the menisci of its water-air interface. Catchment-scale fluxes (groundwater recharge, evaporation, transpiration, precipitation, etc.) affect the matric potential, and thereby the interface curvature and the configuration of the phases. In turn, these affect the fluxes (except precipitation), creating feedbacks between pore-scale and catchment-scale processes. Tracking pore-scale processes beyond the Darcy scale is not feasible. Instead, for a simplified system based on the classical Darcy's Law and Laplace-Young Law we i) clarify how menisci transfer pressure from the atmosphere to the soil water, ii) examine large-scale phenomena arising from pore-scale processes, and iii) analyze the relationship between average meniscus curvature and average matric potential. In stagnant water, changing the gravitational potential or the curvature of the air-water interface changes the pressure throughout the water. Adding small amounts of water can thus profoundly affect water pressures in a much larger volume. The pressure-regulating effect of the interface curvature showcases the meniscus as a pressure port that transfers the atmospheric pressure to the water with an offset directly proportional to its curvature. This property causes an extremely rapid rise of phreatic levels in soils once the capillary fringe extends to the soil surface and the menisci flatten. For large bodies of subsurface water, the curvature and vertical position of any meniscus quantify the uniform hydraulic potential under hydrostatic equilibrium. During unit-gradient flow, the matric potential corresponding to the mean curvature of the menisci should provide a good approximation of the intrinsic phase average of the matric potential.

1 Introduction

Unsaturated porous media can generally be described by the configurations of the solid, liquid, and gas phases. When we exclude physical rearrangement of the solid phase by such mechanisms as land slides, erosion/sedimentation, soil tillage, root

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5 growth, animal burrowing, etc., the solid phase is largely rigid, although organic matter and the configuration of clay particles in swelling and shrinking soils can vary, and some elastic deformation will occur. The liquid phase has a volume that depends only minimally on pressure, while the gas phase is fully expandable. Natural conditions
10 often lead to situations where the liquid phase in an extended area (a field of several hectares or a catchment of several square kilometers) is confined behind its solid-liquid and liquid-gas interfaces, possibly with enclosed pockets of the gas phase trapped within it. The mobility of the gas-liquid interface nevertheless allows ample movement of and exchange between subsurface and atmospheric water through infiltration and
15 evapotranspiration. Still, the bulk of the gas phase is above the bulk of the liquid phase, and most of the gas phase is generally well-connected to the atmosphere.

In soils, the potential energy of the soil solution governs such catchment-scale fluxes as groundwater recharge, transpiration, and evaporation. In return, exchanges of water between the soil and the groundwater, and between the soil and the atmosphere
20 (directly or via the vegetation) affect the energy status of the soil solution, which is evidenced in the wide variation of the matric potential with time. Changes in the matric potential cause changes in the curvature of the menisci, and through this curvature, the configuration of liquid and gas in the pore space. Thus, large-scale processes such as rainfall and evapotranspiration from vegetated areas interact with pore-scale water distribution and meniscus curvatures.

The relationship between the liquid-gas interfaces in pores and the overall energy status of the soil solution on the field scale and larger scales merits attention. The interaction between the architecture of the pore space, the curvature of the menisci, the distributions of the phases, and the potential energy status of the soil solution leads to
25 some intriguing phenomena at the field scale and larger scales. Among the most well-known of this is the extreme response of the hydraulic head and the discharge generation in fine-textured soils to rainfall: a small amount of rainfall can cause a groundwater level rise that is much larger than is to be expected from the soil's porosity (e.g., Rosenberry and Winter, 1997; Sklash and Farvolden, 1979). Zehe et al. (2006) were among

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the first to attempt to quantify and interpret the potential energy of bodies of subsurface water at the catchment scale by volume-integrating local values. They postulated that the volume-averaged unsaturated zone pore water matric potential should reflect the pressure drop over the gas-liquid interface averaged over the unsaturated zone of an entire catchment, thereby implying a very direct link between the two.

Among the most comprehensive treatments of the interaction between the various phases, their interfaces, and the contact lines between the interfaces, is the thermodynamic analysis by Hassanizadeh and Gray (1990). The analysis produces 28 equations and unknowns, and moves from the pore scale to the Darcian scale (in the order of several hundreds of pore diameters). Hassanizadeh and Gray (1990) demonstrated that the generally accepted proportionality between the curvature of a meniscus and the pressure jump across it is only valid at equilibrium. They also offered a modified version of Darcy's Law that accounts for the masses and energies of the interfaces in situations where these move and deform rapidly. The analysis targets the Darcy scale, but most issues of societal interest arise at the much larger field and regional scales that are the focus of this paper.

Gray and Hassanizadeh's (1990) full analysis is difficult to carry out beyond the Darcy scale. Even at the Darcian scale it is not (yet?) feasible to measure the areal density and other properties of the interfaces, and at the larger scales that we focus on, it may likely never be possible. Therefore, the development of a full analysis at the scale of aquifers, fields, or catchments is forfeited here. Instead the system is simplified to a degree that still allows several natural phenomena at these larger scales to be explained and upscaling issues to be addressed. The three phases and their interactions are simplified according to Sposito (1981), which implies that all phases are uniform in composition and do not mix or react with one another. Furthermore, we consider an isothermal system with a rigid solid phase, an inelastic liquid phase, and a continuous gas phase (no entrapped soil air unless stated otherwise). The three surface tensions are assumed constant, and flows are slow enough to make the kinematic energy negligible. The potential energy of the liquid at any location is assumed to be fully

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quantified by its gravitational potential and its pressure potential (below the phreatic level) or matric potential (between the soil surface and the phreatic level). The formal unit for potential energy is J kg^{-1} (potential energy per unit mass), but for clarity the potential energy per unit volume is used here ($\text{J m}^{-3} = \text{Pa}$). As Sposito (1981) points out, these definitions are interchangeable, and the potential should not be considered a true pressure, even if it has the same dimensions. In the following, the terms “potential” and “potential energy” refer to the volume-based quantities unless stated otherwise. For actual liquid or gas pressures the term “pressure” is used; potentials expressed in units of pressure carry the qualifier “potential” (e.g., pressure potential). An important difference is the absolute nature of pressure (cannot be negative) and the relative nature of the pressure potential and matric potential, both of which equal zero when the liquid is at atmospheric pressure. Matric forces make the matric potential negative, so we deploy the potential instead of pressure when discussing situations in which negative pressures could otherwise occur.

In view of the simplifying assumptions, the use of the Laplace-Young Law (e.g., Brutsaert, 2005; Jury et al., 1991; Or and Wraith, 2000) is permitted to determine the pressure jumps across the liquid-gas interfaces:

$$\Delta P_{\text{lg}} = \gamma_{\text{lg}} \left(\frac{1}{r_1} + \frac{1}{r_2} \right), \quad (1)$$

with ΔP_{lg} (Pa) the positive pressure jump from the wetting to the non-wetting phase, γ_{lg} (N m^{-1}) the liquid-gas surface tension, and r_1 and r_2 (m) the principle radii of curvature of the interface. Similarly, flow is described by the classical form of Darcy’s Law. The use of these classical equations restricts the analysis at this time to problems in which the energy changes and movements of the interfaces are negligible, i.e., saturated flows or unsaturated flows in which the liquid-gas interfaces move only slowly. This is in line with Roth’s (2008) finding that phenomena with small characteristic times (e.g., infiltration fronts) lend themselves poorly to upscaling. Furthermore, we will not consider the forces exerted upon the soil solution by the porous matrix, such as Van

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der Waals forces and hydrogen bonds, other than through their combined effect as expressed in the matric potential.

The objectives of this paper are i) to clarify the role of menisci in transferring pressure from the atmosphere to subsurface water; ii) to explore the consequences at the field and the catchment scale of the pore-scale behavior of the interfaces for a number of cases of practical interest; and iii) to examine the relationship between the volume-averaged potential energy of subsurface bodies of water and the curvatures of the gas-liquid interfaces, thereby advancing and refining the analysis by Zehe et al. (2006). The response of the pressure potential to addition or removal of water will be treated in some detail. To arrive at large-scale expressions, local quantities are frequently volume-averaged. Terms such as “average” and “averaging” used below are implied to refer to volume-averaging over volumes of subsurface water, unless otherwise stated.

2 Theory

2.1 Hydrostatic pressure potential below a liquid-gas interface

In soils, the capillary fringe above the phreatic level is defined here as the region where all pores, except macropores like cracks and biopores, are liquid-saturated (except for possible pockets of enclosed air). Thus, the capillary fringe extends to the depth where the largest capillary pore empties and a continuous path of soil gas extends from that pore to the atmosphere. The elevation of the liquid-gas interface of that pore, and thus of the top of the capillary fringe is denoted x_{lg} [L]. For a continuous liquid phase below the top of the capillary fringe in a satiated porous medium, the pressure at hydrostatic equilibrium is given by:

$$P(x_3) = \rho g (x_{lg} - x_3) + P_{\text{atm}} \pm \gamma_{lg} \left(\frac{1}{r_1} + \frac{1}{r_2} \right), \quad (2)$$

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with x_3 (m) the vertical coordinate (positive upwards), P (Pa) the pressure in the liquid, P_{atm} (Pa) the gas pressure above x_{lg} , ρ (kg m^{-3}) the density of the liquid phase, and g (m s^{-2}) the gravitational acceleration. The sign before the last term is positive for non-wetting liquids (in which case the capillary fringe does not exist) and negative for wetting liquids. If a small quantity of water dV [m^3] is added or removed and x_{lg} changes as a result, we have:

$$\frac{dP(x_3)}{dV} = \rho g \frac{dx_{\text{lg}}}{dV} \pm \gamma_{\text{lg}} \frac{d}{dV} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) = \rho g A_{\text{lg}}^{-1} \pm \gamma_{\text{lg}} \frac{d}{dV} \left(\frac{1}{r_1} + \frac{1}{r_2} \right), \quad (3)$$

where A_{lg} (m^2) is the liquid-occupied horizontal area at x_{lg} . Thus, the pressure change in the liquid phase is equal to the change in gravitational potential of the water at the liquid-gas interface plus its pressure change caused by changes in the curvature of the interface.

From Eq. (3) it follows that the pressure at an arbitrary depth in a vertical liquid-filled column with a cross-sectional area of A m^2 (open at the top; Fig. 1a) can be reduced by $\rho g \Delta L$ Pa by removing $\Delta L A$ m^3 of liquid (with ΔL [m] the height of the removed liquid layer). If the column is completely filled and then closed at the top by a lid perforated by a wettable capillary tube with radius r (m) and a capillary rise of ΔL (m), a meniscus develops in the capillary, and the pressure also drops by $\rho g \Delta L$ Pa (Fig. 1b). To return the pressure to its original value, $\Delta L \pi r^2$ m^3 of liquid needs to be added to raise the liquid level in the capillary by ΔL m. If the capillary only extends downward into the liquid but does not extend above the lid (Fig. 1c), the pressure at any depth below the interface can be increased again by $\rho g \Delta L$ Pa by adding the minute amount of liquid (at most $2\pi r^3/3$ m^3 , for a contact angle of zero) that suffices to flatten the curved liquid-gas interface at the top of the capillary.

Thus, it is possible to effect similar pressure changes in the column by adding or removing zero, minute, sizeable, or large volumes of liquid. In the case of zero additional liquid, the interface was changed from flat to curved, in the case of minute addition, from curved to flat, for a sizeable addition, the interface remained curved but moved,

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and for the large added volume, the interface remained flat but moved. This reveals the two mechanisms by which the pressure in a static liquid column can be manipulated: by changing the height of the liquid column, or by changing the curvature of the interface. (An obvious third method would be to change the pressure in the gas phase above the column.)

The sometimes very substantial changes in the potential energy of large bodies of liquid caused by adding or removing small quantities of liquid seem to suggest that energy is added and dissipated in quantities that are only determined by the volume of liquid experiencing these changes in pressure potential, not by the possibly very much smaller volume of liquid that brings these changes about. It should be noted that in all cases discussed above the potential energy of the liquid at the interface is changed, either by a change in the position of the interface in the gravitational field (affecting the gravitational potential), a change in the interface curvature (affecting the pressure potential), or both. To restore hydrostatic equilibrium after such a perturbation, the potential energy of the rest of the liquid must follow suit. Since the location of the liquid is fixed, only the matric/pressure potential is available to accommodate the required change.

Pressure changes thus serve as signal carriers that convey and effectuate changes in hydraulic potential throughout a body of liquid in a fixed position. How effectively the increased potential energy can be converted to work depends on the geometry of the vessel or the porous medium in the vicinity of x_{lg} . According to Eq. (3), if the increased pressure in a body of liquid results in a displacement of liquid out of its original confines (e.g., by flow through a confining layer into another aquifer, or flow into a well, or water trickling through a leak in a vessel wall caused by the increased pressure), the pressure will drop inversely proportionally to the liquid-filled horizontal cross-section at the position of the liquid-gas interface if the curvature of the menisci remains the same. For small A_{lg} , the pressure drop will be large enough to rapidly diminish the pressure gradients that drive the flow. This too is consistent with the change of the gravitational potential of the liquid at the receding liquid-gas interface and

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the transfer of the changing hydraulic potential at the interface throughout the liquid. Similarly, if the liquid displacement is minute but sufficient to change the curvature of the interface, the pressure throughout the liquid will be affected predominantly through the second term of Eq. (3). As a practical example, relatively rapid pressure changes as a result of water extractions can occur in a confined aquifer with a small phreatic intake area, where the aquifer's only air-water interface exists. Such an aquifer can only sustain small transfers of water to a deeper aquifer before the hydraulic potential drops to equilibrium with the neighbor and flow ceases. And if the aquifer over some extent is artesian, too many artesian wells may cause the hydraulic head to drop below the soil surface and the wells will then run dry.

The interpretation of Eq. (3) in a porous medium is more problematic than for open water. In the following, the water below a phreatic level at some vertical position x_p (m) is termed groundwater, that above x_p soil water or soil solution. The groundwater at any depth rests upon water below it, as in the vessel discussed above, creating an overburden pressure there. The water above x_p can be viewed as hanging from its menisci at the top of the capillary fringe. Although its pressure potential still continuously drops with elevation in the same fashion as the groundwater, it is pushed up into the capillaries by the pressure in the non-wetting gas phase. At equilibrium, the curvatures of the menisci are such that the resulting subatmospheric pressure exactly cancels the overburden pressure of the soil water and thus does not add an overburden pressure to the groundwater. Since only the water below x_p contributes to the pressure potential in the groundwater, a change in the pressure potential typically requires a change in x_p for the near-equilibrium conditions that tend to prevail in aquifers. This can be accomplished by inducing a downward flow through the capillary fringe, or by supplying water from below (e.g., from infiltration drains or a deeper aquifer). For the trivial case of a very coarse matrix (gravel) without significant capillary fringe and virtually no water present above x_p , the liquid-gas interface is at x_p , and A_{lg} represents the horizontal cross-section of the pore space, which will be equal to the porosity when expressed as a fraction of the area. The more realistic case of fine-textured media is

considerably more complicated and treated in various illustrative cases below.

2.2 The role of menisci

The liquid-gas interface of a body of subsurface water consists of a multitude of menisci. This interface can be viewed as a port that passes on the pressure signal from the atmosphere to the liquid phase with an offset determined by the direction and magnitude of the curvatures of the interfaces, which in turn depend on the pore architecture, the various surface tensions, which determine the wetting angle (Koorevaar et al., 1983), and the detailed liquid-gas configuration (the degree of wetting of the pore walls). If the gas phase is well connected to the atmosphere, the reservoir providing the gas pressure effectively consists of the atmosphere of the entire planet.

The liquid-gas interface essentially is a one-way port: the atmosphere modifies the pressure in the liquid phase, but not the other way around. This has various reasons: the sheer size of the Earth's atmosphere makes the effect of water movement in the lithosphere below it inconsequential for the atmospheric pressure. Furthermore, the volume of the soil and the groundwater solution hardly changes with pressure. Thus, considerable pressure changes brought about by changes in interface curvatures lead to minute volume changes of those solutions, and therefore do not influence the volume that the gas phase needs to occupy in the subsurface. An already very large gas reservoir therefore only experiences insignificant volume changes, leading to negligible pressure changes.

Below the interface, the liquid phase superimposes upon this prescribed pressure potential at the interface its own hydrostatic pressure/matric potential at hydrostatic equilibrium (Eq. 2) or its hydrodynamic potential field during flow. In deep aquifers with slow flow, the observed pressure potential field will deviate little from the pressure potential distribution at hydrostatic equilibrium, but in regions of high flow (near pumping wells in aquifers, and near the soil surface during infiltration), the deviations can be substantial. In case the infiltration occurs in a dry soil, the degree of saturation and the shape and positions of the air-water interfaces change very rapidly, and this case

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is therefore outside the scope of the analysis presented here.

In the unsaturated zone, the larger pores are filled with air (with the soil solution possibly present as a film on the pore walls), while smaller pores can be entirely filled with water. In very dry soils water is predominantly present in pendular rings around the contact point between soil particles and possibly in liquid films on the solid particle surfaces. Soil solution films in dry soil experience different forces than the solution behind a meniscus in filled capillaries. Therefore the degree of curvature of their liquid-gas interfaces not only depends on the liquid-gas surface tension, but also on the matric forces acting upon the water molecules. The matric potential may still be an adequate vehicle for describing the energy status of the soil solution in those films, but the relationship between pressure and gas-liquid interface curvatures becomes ambiguous (Iwata et al., 1995). The energy status and flow-conducting properties of these water films are less easily defined than those of water sufficiently far away from the solid surface to be free of matric forces.

In saturated small pores and pendular rings, there is little if any flow (often driven by condensation and evaporation at the interface). The matric potential in these pockets of water is entirely governed by the air-water interface. Its curvature determines the matric potential directly behind the interface and – through the hydrostatic equilibrium – anywhere else in the body of water at a sufficient distance from the solid phase to be unaffected by any forces it exerts on the water molecules. For a hypothetical pore with a single exit in which the meniscus is located (Fig. 2), the intrinsic phase average (Whitaker, 1986, see below) of the matric/pressure potential when no flow occurs is:

$$\begin{aligned} \langle \psi_m \rangle_l &= \int_{-\infty}^{\infty} A(x_3) [(P_g - P_{\text{atm}} \pm \Delta P_{\text{lg}}) + \rho g (x_3^* - x_3)] dx_3 \left(\int_{-\infty}^{\infty} A(x_3) dx_3 \right)^{-1} \\ &= P_g - P_{\text{atm}} \pm \Delta P_{\text{lg}} + \int_{-\infty}^{\infty} A(x_3) \rho g (x_3^* - x_3) dx_3 \left(\int_{-\infty}^{\infty} A(x_3) dx_3 \right)^{-1} \end{aligned} \quad (4)$$

where $\langle \psi_m \rangle_l$ (Pa) is the intrinsic phase average matric potential in the pore, x_3^* (m) is the vertical position of the meniscus, $A(x_3)$ (m²) is the horizontal liquid-filled cross-

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section of the pore at x_3 , P_g (Pa) is the pressure in the gas phase immediately in front of the interface (often equal to P_{atm}), and ΔP_{lg} is given by Eq. (1). For wetting liquids ΔP_{lg} acquires a minus sign, for non-wetting liquids a plus sign. It is interesting to note that the liquid pressure behind the interface affects and even determines the matric/pressure potential everywhere in the pore, but yet the average matric potential is determined by both the liquid pressure behind the interface and the geometry of the pore.

For a pocket of liquid enclosed by multiple liquid-gas interfaces, the pressure in the surrounding continuous gas phase will differ much less between the menisci than the liquid pressure because of the density contrast. In the pore system the menisci will be located in places where the curvatures allowed by the pore geometry and contact angles produce pressure jumps that vary with position of the menisci according to:

$$\Delta P_{lg}(x_3) = \Delta P_{lg}(x_3^*) \pm (\rho - \rho_g)g(x_3 - x_3^*) \approx \Delta P_{lg}(x_3^*) \pm \rho g(x_3 - x_3^*) \quad (5)$$

where x_3^* (m) now is the vertical position of a reference meniscus for which the pressure jump is known and ρ_g (kg m^{-3}) is the density of the gas phase (assumed constant over the vertical extent of the pocket of liquid). The signs of the second terms are positive for wetting liquids and negative for non-wetting liquids. Equation (4) still holds for this case, as long as the liquid is stagnant. Any interface with a known pressure jump can serve as the reference interface that defines x_3^* .

The pocket of liquid for which Eq. (5) describes the pressure jumps at its interfaces is in principle unlimited in size, as long as the gas phase is continuous. The body of liquid can therefore be conceived to be the entire connected body of subsurface water in a catchment (groundwater and the soil solution), with the menisci obviously only present between the soil surface and the top of the capillary fringe. Equation (5) does not apply to menisci enclosing entrapped air bubbles because the gas pressure inside these bubbles is not determined independently from the menisci but the result of the interplay between the amount of gas, liquid pressure, pore space architecture, and interfacial tensions.

Particularly in the unsaturated zone (for which it is most relevant), the assumption of hydrostatic equilibrium will limit the direct application of Eq. (5) to such large scales. If the matric potential field is known, Eq. (5) can be generalized for non-equilibrium conditions:

$$\Delta P_{lg}(\mathbf{x}) = \Delta P_{lg}(\mathbf{x}^*) - \psi_m(\mathbf{x}) + \psi_m(\mathbf{x}^*), \quad (6)$$

where $\mathbf{x}(m)$ is the location vector, ψ_m (Pa) is the matric potential, and \mathbf{x}^* (m) denotes the location of the reference meniscus. (For clarity, only the version for wetting liquids is given.)

2.3 Averaging potentials

The values of the matric or pressure potential and the gravitational potential at a point are well defined, but if one is interested in the average potential energy of non-zero liquid volumes, these point values need to be volume-integrated and averaged. Several methods have been applied. Zehe et al. (2006) used direct volume-averaging:

$$\langle \psi \rangle_V = \frac{1}{V} \int_V \psi dV, \quad (7a)$$

where ψ (Pa) can represent the gravitational, matric/pressure or hydraulic potential, and V (m^3) is the averaging volume within the porous domain (in the case of Zehe et al., 2006, V is the subsurface volume of an entire catchment). Whitaker (1986) only considered the volume of the phase of interest within the averaging volume and defined the phase average of the potential as:

$$\langle \psi \rangle = \frac{1}{V_l} \int_{V_l} \psi dV, \quad (7b)$$

where the integration is only carried out over the volume occupied by the phase of interest within V . Since we are interested here in the liquid phase this volume is de-

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noted by V_l (m^3). Whitaker (1986) also defined the intrinsic phase average, in which the averaging is performed over V_l only:

$$\langle \psi \rangle_l = \frac{1}{V_l} \int_{V_l} \psi dV. \quad (7c)$$

At the pore-scale, where the distributions of the solid, liquid, and gas phases are known, integration of ψ over V_l in Eqs. (7b and 7c) can be achieved by integrating over V while including in the integrand an indicator function that assumes the value of one in locations within V_l and zero elsewhere. The volume V_l in Eq. (7c) equals the volume integral of that indicator function over V . At the field and catchment scales, the continuum approach (Bear and Bachmat, 1991) is invoked. In that case, to arrive at the phase average, the local value of ψ needs to be multiplied by the local volume fraction occupied by its phase, which is the volumetric water content θ for the liquid phase. The resulting expression for the phase average is:

$$\langle \psi \rangle = \frac{1}{V} \int_V \theta \psi dV. \quad (7d)$$

The intrinsic phase average then becomes:

$$\langle \psi \rangle_l = \frac{\int_{V_l} \theta \psi dV}{\int_{V_l} \theta dV}. \quad (7e)$$

Equation (7a) suffers from undefined values of ψ outside V_l when applied to the pore scale (for which Zehe et al., 2006, did not intend it to be used). Only when ψ is set to zero outside V_l does its value there not influence the value of the volume integral. Equation (7a) then becomes equivalent to the phase average (Eq. 7b), because the indicator function has the same effect. When applied to the field and catchment scale (its intended application), Eq. (7a) weighs every local value of the potential in the

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solid-liquid-gas continuum equally, irrespective of the local water content. As a consequence, Eq. (7a) does not satisfy Gray's (2002) additivity property that ensures that the potential energy is conserved during the volume averaging, and is therefore not recommended. The inability of Eq. (7a) to conserve potential energy can be illustrated by

5 volume-averaging the gravitational potential energy of the water in a vertical soil column of area A extending between heights $-\Delta x_3$ to 0. Irrespective of the water distribution in the column, its average gravitational potential will be calculated as $-\rho g \Delta x_3 / 2 \text{ J m}^{-3}$, which obviously is only true if the vertical soil water distribution is symmetrical around $-\Delta x_3 / 2$.

10 The expressions for the phase average (Eqs. 7b and 7d) and intrinsic phase average (Eqs. 7c and 7e) both satisfy the conservation property, even though the numerical values of the spatial averages will differ. The phase average expresses the potential energy as Joule per volume for the entire averaging volume V , while the intrinsic phase average gives the potential energy per volume over the smaller volume V_l , with its value

15 correspondingly higher. When the respective averages are multiplied by the volumes for which they hold, they yield the same numerical value of potential energy in Joules. When ψ is uniform over V_l , the intrinsic phase average $\langle \psi \rangle_l$ will be equal to the local value ψ , which is an attractive property that makes the intrinsic phase average the most representative of the three averages of the conditions in the phase of interest

20 (Whitaker, 1986).

The above volume integrations are all carried out for potentials expressed as potential energy per volume (Pa), and therefore weighting by volume is appropriate. For potentials expressed by mass (J kg^{-1}) or weight (potential head, m), the weighting needs to be adjusted accordingly. The corresponding expressions for the intrinsic phase averages are given below (compare e.g., Gray, 2002, Eq. 5e). Expressions for the phase average are analogous. Note that the various expressions are equivalent if the phase density and the gravitational acceleration are uniform within V .

$$\langle \mu \rangle_l = \frac{\int_V \theta \rho \mu dV}{\int_V \theta \rho dV}. \quad (8a)$$

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Here, μ (J kg^{-1}) denotes the potential expressed per unit mass. When the potential head is used instead, the correct expression is:

$$\langle H \rangle_l = \frac{\int_V \theta \rho g H dV}{\int_V \theta \rho g dV}, \quad (8b)$$

where H denotes the hydraulic head (m) and can be replaced by the matric head h or the gravitational head x_3 .

3 Illustrative cases and large-scale consequences

3.1 Rapid rise of groundwater levels during rainfall

In fine-textured soils, a fully saturated capillary fringe can extend over several decimeters above x_p . Above the capillary fringe, the largest pores are filled with air, but most of the pores space is still filled with water. This is the equivalent of the water vessel with the narrow tube on top (A_{lg} small, curved liquid-gas interface, Fig. 1b) discussed in the Theory section. A small amount of rainfall suffices to saturate the pore space over a much larger vertical extent than the equivalent water layer of the rainfall, and x_p as well as the capillary fringe shoot upward (e.g., Seibert et al., 2003). For soils with x_p only a few decimeters deeper than the height of the capillary fringe, most of the pore space below the soil surface is water-saturated. As a consequence, the rising capillary fringe may extend to the soil surface, causing saturation-excess overland flow (see observations by van der Velde et al., 2010, among others). The menisci at the soil surface can no longer go up and can only flatten (Fig. 3). The pressure increase by adding a water layer then combines with the reduction of the curvature of the menisci to make x_p rapidly rise to the soil surface, even though the phreatic level prior to rainfall may have been decimeters below the soil surface.

Table 1 illustrates the magnitude of this effect for hypothetical hydrophilic soils with capillaries of uniform radius. The table gives the amount of water needed to let the

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matric potential at the soil surface go from the air entry value to zero, thereby bringing the phreatic level to the soil surface. The air-entry value is calculated as $-\Delta P_{lg}$ from Eq. (1) for a zero contact angle between solid and liquid. Under hydrostatic equilibrium, with the soil surface being at the air-entry value, the phreatic level x_p would be at $\Delta P_{lg}/\rho g$ m below the soil surface. For fine-textured soils, the minute amount of water needed to bring the phreatic level to the soil surface is even smaller than for coarse soils (3rd column of Table 1), yet the matric head jump $\Delta P_{lg}/\rho g$ that it causes (2nd column) is much larger and occurs much faster (4th column). Since fine-textured soils have the most extensive capillary fringes (2nd column), their ratio of the rise of the phreatic level and the thickness of the added water layer that causes the rise is enormous (last column).

3.2 Shallow infiltration of small amounts of rainfall/irrigation in dry soil

Small amounts of rainfall or irrigation water on well-sorted, dry soils not always percolate to the subsoil but instead only wet the top soil (Youngs, 1958). Raats (1973) explained this by the difference between the water-entry pressure at the wetting front and the air-entry pressure at the soil surface. His explanation can be cast within the framework outlined above. Incomplete wetting at the wetting front leads to non-zero contact angles and meniscus radii larger than the pore radii. The pressure jump ΔP_{lg} across the liquid-gas interface at wetting front depth D (m) below the soil surface is therefore limited (Eq. 1), and the water pressure behind the front only slightly subatmospheric. For the soil surface to dry, water-filled pores must empty, and these therefore will be completely wetted. The pressure jump across the interface is quite large, and the water behind the drying front will be at a considerably lower pressure than the water at the wetting front. For slow-moving fronts, the water pressure profile within the wetted layer will be nearly hydrostatic, and Eq. (5) can be applied to the problem. The pressure difference from the soil water to the atmosphere at the soil surface will be $\Delta P_{lg} + \rho g D$. According to Eq. (1) this pressure difference must be larger than the air entry pressure difference $2\gamma_{lg}/r$ required for air to enter the soil, with r the pore radius.

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This will only occur if $D > (2\gamma_{lg}/r - \Delta P_{lg})/\rho g$: the thickness of the wetted layer D must be larger than the difference between the matric potential heads at water entry and air entry.

3.3 Relation between large-scale average matric potential and the average curvature of the menisci

The total volume-averaged hydraulic potential of a body of water at equilibrium locked in the porous medium behind gas-liquid interfaces can be calculated from Eq. (4) and the expression for the pressure equivalent of its intrinsic phase average gravitational potential $\langle \psi_g \rangle_l$ (Pa) (compare de Rooij, 2009):

$$\langle \psi_g \rangle_l = \int_{-\infty}^{\infty} A(x_3) \rho g x_3 dx_3 \left(\int_{-\infty}^{\infty} A(x_3) dx_3 \right)^{-1} \quad (9)$$

This equation is more general if $A(x_3)$ is interpreted as the water-filled portion of the horizontal cross-section at x_3 : it then applies to any body of subsurface water, stagnant or mobile. But since Eq. (4) is valid for a single connected body of stagnant water, the total intrinsic phase averaged hydraulic potential $\langle \psi_H \rangle_l$ (Pa) is subject to the more limiting constraints of Eq. (4). Its expression is:

$$\begin{aligned} \langle \psi_H \rangle_l = \langle \psi_m \rangle_l + \langle \psi_g \rangle_l = & P_g - P_{atm} \pm \Delta P_{lg} \\ & + \int_{-\infty}^{\infty} A(x_3) \rho g (x_3^* - x_3) dx_3 \left(\int_{-\infty}^{\infty} A(x_3) dx_3 \right)^{-1} \\ & + \int_{-\infty}^{\infty} A(x_3) \rho g x_3 dx_3 \left(\int_{-\infty}^{\infty} A(x_3) dx_3 \right)^{-1} \end{aligned}$$

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$$\begin{aligned}
 &= P_g - P_{\text{atm}} \pm \Delta P_{\text{lg}} \int_{-\infty}^{\infty} A(x_3) \rho g x_3^* dx_3 \left(\int_{-\infty}^{\infty} A(x_3) dx_3 \right)^{-1} \\
 &= P_g - P_{\text{atm}} \pm \Delta P_{\text{lg}} + \rho g x_3^*
 \end{aligned}
 \tag{10}$$

with $+\Delta P_{\text{lg}}$ for hydrophobic soils and $-\Delta P_{\text{lg}}$ for hydrophilic soils.

The inclusion of the water-filled cross-section A in the equations above limits the volume integration to the liquid phase only. This is a refinement of the volume integration of Zehe et al. (2006) who integrated over the entire subsurface volume, i.e., over the gas, solid, and liquid phases. As discussed above and in line with Gray (2002), integrating only over the liquid phase ensures that the upscaling operation conserves potential energy.

Since hydrostatic equilibrium requires the hydraulic potential to be the same everywhere, the simple expression for the volume-averaged hydraulic head expressed in the final result of Eq. (10) is not surprising. But it does point out that under equilibrium conditions, the gas pressure and interface curvature at any gas-liquid interface, and its vertical position, suffice to describe the total volume-averaged hydraulic potential in a body of subsurface water at equilibrium, but not its volume-averaged matric/pressure potential $\langle \psi_m \rangle_l$, not even for a water pocket behind a single pore. This is contrary to Zehe et al.'s (2006) hypothesis that the volume-averaged capillary pressure should reflect the average pressure drop across the liquid-gas interfaces in the pores at equilibrium. For multiple-pore systems, the situation becomes even more troublesome, since the average pressure drop over the menisci will not only be governed by the matric potential field (see Eqs. 5 and 6), but also by the distribution of menisci. Below the top of the capillary fringe, for instance, the number of menisci drops to zero while a significant portion of the soil water is likely to reside in the capillary fringe; the other extreme is the dry top layer of coarse soils, where a meniscus is present at each pendular ring of water that retracted around a contact point of solid grains. A large population of menisci thus represents only a small quantity of water. The relation between average

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matric potential and average meniscus curvature is analyzed in detail for a bundle of capillary tubes reflecting a soil water characteristic in the Appendix.

Zehe et al.'s (2006) hypothesis gains credibility under unit gradient flow conditions. Then, the flow is driven by gravity only, and the matric potential is uniform. (Such conditions are unlikely in saturated conditions, and we therefore disregard the pressure potential here.) In this case, the difference between the matric potential terms in Eq. (6) is zero. The matric potential determined behind any individual interface is representative for the entire body of water, and the average meniscus curvature is obviously equal to each of the individual curvatures. This warrants the conclusion that Zehe et al.'s (2006) hypothesis is correct for unit gradient flow conditions, but not for the equilibrium conditions as stipulated by the authors.

Zehe et al. (2006) formulated their assertion for a population of interfaces and large bodies of water, and for the case of unit gradient flow that is appropriate: at the pore scale, and even at the Darcy-scale, unit gradient flow cannot exist in heterogeneous media because it precludes lateral flows by assuming horizontal matric potential gradients to be zero. Thus, the redistribution of water needed to keep the local vertical flux densities equal to the local hydraulic conductivities (a necessity if the vertical matric potential gradient is to vanish) is not allowed. Nevertheless, unit gradient conditions have been observed in the field at depths sufficient to dampen out fluctuations in the boundary conditions at the soil surface. At scales slightly larger than the Darcy scale (1 m and beyond) unit gradient flow can safely be assumed to occur if conditions are favorable (e.g., Davidson et al., 1969; Reichardt et al., 1998): the lateral matric gradients inevitably caused by soil heterogeneity often turn out to be relatively small owing to the dissipative nature of matric potential gradients. Consequently, the matric potential corresponding to the mean curvature of the menisci should be a rather good approximation of the intrinsic phase average of the matric potential during unit gradient flow. However, it seems unlikely that the unsaturated flow in an entire catchment is strictly gravity-driven or at hydrostatic equilibrium.

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For more dynamical conditions and variations of flow directions within the water body (upward, downward, lateral, water removal by plant roots, etc.), the relationship between the matric potential field within the body of water and the matric potentials just behind its liquid-gas interfaces seems to be approximate at best. Therefore, caution should be exercised when relating the average matric potential of large unsaturated water bodies to the curvatures of their menisci.

4 Summary and conclusions

The various roles of liquid-gas interfaces in the pores of soils were highlighted in view of their effect on phenomena observable and relevant at the field and the catchment scales, i.e., much larger than the pore scale. Their effect on water pressure and soil matric potential was analyzed in some detail, including the role of menisci in highly dynamic matric/pressure potential responses of fine-textured soils to rainfall: one well-known part of this response is the result of limited available storage in the pore space, another part is caused by extremely rapid water pressure changes when curved interfaces at the soil surface are flattened. The view of menisci as one-way pressure ports with an offset proportional to their curvature was proposed in this context. A detailed analysis of the relationship between the curvature of a single meniscus or a population of menisci and the volume-averaged matric potential of a body of subsurface water did not offer theoretical support for the direct relationship between average matric potential and average meniscus curvature at equilibrium that was hypothesized in the literature. However, for strictly gravity-driven flow, a direct proportionality between average meniscus curvature and average matric potential is plausible.

The analysis relied on the conventional tools in use at the Darcy scale: Darcy's Law and the Laplace-Young Law, and often assumed conditions to be close to hydrostatic equilibrium. Nevertheless, considerable insight was gained in the interactions between pore scale processes and phenomena at scales that are several orders of magnitude larger and surpass the Darcy scale, which justifies the simplifications made. For truly

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dynamic processes, this approach may be less attractive, but a workable alternative is currently not available.

Appendix A

5 Average meniscus curvature and average matric potential in a bundle of capillary tubes: a direct test of Zehe et al.'s (2006) hypothesis

The soil water characteristic of a soil can be converted to a distribution of pore radii through Eq. (1). The pressure jump can be converted to a matric head h (m) : $P_{lg}/\rho g = -h$. For cylindrical pores the principle radii are equal: $r_1=r_2=r$. Note that the minus sign in the conversion implies that the soil is hydrophilic. Thus we have:

$$10 \quad h = -\frac{2\gamma}{\rho g r} = -\frac{a}{r}, \quad (A1)$$

with a (m²) defined by the equation. A convenient closed-form expression of the soil water characteristic is that of Brooks and Corey (1964):

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \left(\frac{h_{ae}}{h}\right)^\lambda \quad \text{for } h \leq h_{ae}, \quad (A2)$$

15 where θ is the volumetric water content, θ_r and θ_s are its residual and saturated values, respectively, h_{ae} (m) is the soil-specific air entry value, and λ is a soil-specific constant. Note that the potentials here are expressed as energy per weight, giving the dimension length. This clarifies the derivation below.

If h_{ae} and λ are given for a particular soil, we can conceive of a bundle of vertical capillary tubes that has the same water characteristic as the soil. Such a schematization allows to calculate analytically the average matric potential and the average curvature of the menisci, which permits a direct test of Zehe et al.'s (2006) hypothesis that the average curvature of the menisci reflects the average matric potential.

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The widest tube of the population would have a radius of $-ah_{ae}^{-1}$. Equation (A2) gives the water remaining in the soil at a given depth after h at that depth has been lowered from zero to its current value, i.e., all pores with a h_{ae} -value larger than h have already emptied. By inserting Eq. (A1) into Eq. (A2) we can express the water content as a function of the pore radius that is about to empty:

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \left(-\frac{h_{ae}}{a} r \right)^\lambda \quad (\text{A3})$$

By calculating the water content for two values of r we can find the fraction of the volume occupied by water residing in tubes with radii within the range defined by the two values. Letting this range go to zero gives the derivative:

$$\frac{d\theta}{dr} = -(\theta_s - \theta_r) \frac{\lambda h_{ae}}{a} \left(-\frac{h_{ae}}{a} r \right)^{\lambda-1} \quad (\text{A4})$$

If N (m^{-2}) denotes the number of water-filled tubes per square meter and n (m^{-3}) its derivative dN/dr , Eq. (A4) provides the number of capillary tubes per square meter with radius r by noting that $d\theta/dr$ is the fraction of the total tube area occupied by the combined inner area of these tubes, i.e., $n \cdot \pi r^2$:

$$n(r) = -(\theta_s - \theta_r) \frac{\lambda h_{ae}}{\pi a r^2} \left(-\frac{h_{ae}}{a} r \right)^{\lambda-1} = -(\theta_s - \theta_r) \frac{\rho g \lambda h_{ae}}{2\gamma \pi r^2} \left(-\frac{\rho g h_{ae}}{2\gamma} r \right)^{\lambda-1} \quad (\text{A5})$$

In a bundle of capillary tubes with its lower end submerged in a water reservoir (mimicking the groundwater), all tubes will contain water, with the level rising to a/r above the water level (Eq. A1). Equation (A5) then gives the number of menisci per square meter with curvature $2/r$ (m^{-1}). The average curvature of the menisci in the entire bundle of tubes can then be found by deriving the distribution function of $2/r$ from Eq. (A5). By changing variables to make $y=2/r$ (thereby letting y (m^{-1}) denote the meniscus

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curvature), Eq. (A5) can be written as:

$$n(y) = -(\theta_s - \theta_r) \frac{\rho g \lambda h_{ae} y^2}{8 \gamma \pi} \left(-\frac{\rho g h_{ae}}{\gamma y} \right)^{\lambda-1} \quad (A6)$$

Note that the minimum curvature is $-\rho g h_{ae}/\gamma$ and belongs to the largest capillary, emptying at h_{ae} . The curvatures of tubes that are water filled over their entire length (where $h = -x_p$ if we set x_3 equal to zero at the soil surface) are ignored, since they represent water pockets in fine pores behind larger pores with menisci represented by the larger capillaries. The arithmetic mean of the curvature $\langle y \rangle_a$ (m^{-1}) then is:

$$\langle y \rangle_a = \frac{-(\theta_s - \theta_r) \frac{\rho g \lambda h_{ae}}{8 \gamma \pi} \int_{-\rho g h_{ae}/\gamma}^{-\rho g x_p/\gamma} y^3 \left(-\frac{\rho g h_{ae}}{\gamma} y^{-1} \right)^{\lambda-1} dy}{-(\theta_s - \theta_r) \frac{\rho g \lambda h_{ae}}{8 \gamma \pi} \int_{-\rho g h_{ae}/\gamma}^{-\rho g x_p/\gamma} y^2 \left(-\frac{\rho g h_{ae}}{\gamma} y^{-1} \right)^{\lambda-1} dy}, \quad (A7)$$

which reduces to:

$$\langle y \rangle_a = \frac{\int_{-\rho g h_{ae}/\gamma}^{-\rho g x_p/\gamma} y^3 \left(-\frac{\rho g h_{ae}}{\gamma} y^{-1} \right)^{\lambda-1} dy}{\int_{-\rho g h_{ae}/\gamma}^{-\rho g x_p/\gamma} y^2 \left(-\frac{\rho g h_{ae}}{\gamma} y^{-1} \right)^{\lambda-1} dy} = \frac{\left[\frac{y^4}{3-\lambda} \left(-\frac{\rho g h_{ae}}{\gamma} y^{-1} \right)^{\lambda-1} \right]_{-\rho g h_{ae}/\gamma}^{-\rho g x_p/\gamma}}{\left[\frac{y^3}{2-\lambda} \left(-\frac{\rho g h_{ae}}{\gamma} y^{-1} \right)^{\lambda-1} \right]_{-\rho g h_{ae}/\gamma}^{-\rho g x_p/\gamma}} \quad (A8)$$

to give

$$\langle y \rangle_a = -\left(\frac{\rho g}{\gamma} \right) \left(\frac{2-\lambda}{3-\lambda} \right) \left(\frac{x_p^{5-\lambda} - h_{ae}^{5-\lambda}}{x_p^{4-\lambda} - h_{ae}^{4-\lambda}} \right). \quad (A9)$$

The radius of a meniscus in a fully wetted cylindrical capillary corresponding to this curvature would be $2/\langle y \rangle_a$. The matrix head of water just behind this meniscus is:

$$h(\langle y \rangle_a) = -\left(\frac{2-\lambda}{3-\lambda} \right) \left(\frac{x_p^{5-\lambda} - h_{ae}^{5-\lambda}}{x_p^{4-\lambda} - h_{ae}^{4-\lambda}} \right). \quad (A10)$$

We can also calculate the average matric head of all the water in the population of tubes. For a tube of radius r , the water rises to $2\gamma/\rho g r$ m above x_p , and the tube therefore contains $2\pi r\gamma/\rho g$ m³ of water. The total volume of water per square meter that is stored in tubes in which the water rises to $2\gamma/\rho g r$ m above x_p is $n(r) \cdot 2\pi r\gamma/\rho g$, and that volume of water has an average matric head of $-\gamma/\rho g r$ m. The overall average matric head equals the weighted average of these average matric heads for all r , with the weighting factor being the amount of water residing in tubes of radius r . The tubes that are so narrow that they are filled with water all the way to the soil surface all have an average matric head of $-x_p/2$. The largest of these tubes has radius $-2\gamma/\rho g x_p$.

The average matric head of the population of tubes larger than this is:

$$\langle h_1 \rangle_a = \frac{\int_{-2\gamma/\rho g h_{ae}}^{-2\gamma/\rho g x_p} n(r) \frac{2\pi r\gamma}{\rho g} \frac{-\gamma}{\rho g r} dr}{\int_{-2\gamma/\rho g h_{ae}}^{-2\gamma/\rho g x_p} n(r) \frac{2\pi r\gamma}{\rho g} dr} \quad (\text{A11})$$

With Eq. (A.5) this results in:

$$\begin{aligned} \langle h_1 \rangle_a &= -\frac{\gamma}{\rho g} \frac{\int_{-2\gamma/\rho g h_{ae}}^{-2\gamma/\rho g x_p} r^{-2} \left(-\frac{\rho g h_{ae}}{2\gamma} r\right)^{\lambda-1} dr}{\int_{-2\gamma/\rho g h_{ae}}^{-2\gamma/\rho g x_p} r^{-1} \left(-\frac{\rho g h_{ae}}{2\gamma} r\right)^{\lambda-1} dr} \\ &= -\frac{\gamma}{\rho g} \frac{\lambda-1}{\lambda-2} \frac{\left[\left(-\frac{\rho g h_{ae}}{2\gamma} r\right)^{\lambda-1} r^{-1} \right]_{-2\gamma/\rho g h_{ae}}^{-2\gamma/\rho g x_p}}{\left[\left(-\frac{\rho g h_{ae}}{2\gamma} r\right)^{\lambda-1} \right]_{-2\gamma/\rho g h_{ae}}^{-2\gamma/\rho g x_p}}, \end{aligned} \quad (\text{A12})$$

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which gives:

$$\langle h_1 \rangle_a = \left[\frac{x_p(\lambda - 1)}{2(\lambda - 2)} \right] \frac{1 - \left(\frac{h_{ae}}{x_p} \right)^{2-\lambda}}{1 - \left(\frac{h_{ae}}{x_p} \right)^{1-\lambda}}. \quad (A13)$$

This average matric head relates to a volume of water present in those tubes in a 1 m^2 area that are not entirely filled over their full length $-x_p$. This water volume is given by the denominator of Eq. (A11). The remaining portion of the water resides in the capillaries that are filled with water up to the soil surface. The volume of water per square meter of soil stored in these capillaries with radii between 0 and $-2\gamma/\rho g x_p$ is:

$$\begin{aligned} \int_0^{-2\gamma/\rho g x_p} -x_p \pi r^2 n(r) dr &= \frac{\rho g \lambda h_{ae} (\theta_s - \theta_r) x_p}{2\gamma} \int_0^{-2\gamma/\rho g x_p} \left(-\frac{\rho g h_{ae}}{2\gamma} r \right)^{\lambda-1} dr \\ &= -(\theta_s - \theta_r) x_p \left(\frac{h_{ae}}{x_p} \right)^\lambda. \end{aligned} \quad (A14)$$

To arrive at the overall average matric head $\langle h \rangle_a$ (m), the average matric head for the partially filled tubes $\langle h_1 \rangle_a$ and that for the full tubes $(-x_p/2)$ have to be averaged, weighted by their respective portions of the total water they represent:

$$\begin{aligned} \langle h \rangle_a &= \left[-\frac{x_p(\lambda - 1)}{2(\lambda - 2)} \right] \frac{1 - \left(\frac{h_{ae}}{x_p} \right)^{2-\lambda}}{1 - \left(\frac{h_{ae}}{x_p} \right)^{1-\lambda}} (\theta_s - \theta_r) \left(\frac{\lambda h_{ae}}{\lambda - 1} \right) \left[\left(\frac{h_{ae}}{x_p} \right)^{\lambda-1} - 1 \right] \\ &\quad + \frac{x_p^2}{2} (\theta_s - \theta_r) \left(\frac{h_{ae}}{x_p} \right)^\lambda, \end{aligned} \quad (A15)$$

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which can be rearranged to give:

$$\langle h \rangle_a = \frac{x_p(\theta_s - \theta_r)}{2} \left\{ \left(\frac{\lambda h_{ae}}{\lambda - 2} \right) \left(\frac{h_{ae}}{x_p} \right)^{\lambda-1} \left[\left(\frac{h_{ae}}{x_p} \right)^{2-\lambda} - 1 \right] + x_p \left(\frac{h_{ae}}{x_p} \right)^\lambda \right\}. \quad (\text{A16})$$

Clearly, Eqs. (A10) and (A16) are unequal, and for the case of a soil simplified to a bundle of capillary tubes, the matric potential derived from the mean curvature of the menisci does not represent the volume-averaged matric potential of the water above the groundwater table if the water is at hydrostatic equilibrium with a fixed groundwater table.

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Table 1. Amount of water needed to bring the phreatic level up to the soil surface when initially the capillary fringe extends to the soil surface, and the time needed to deliver this water during rainfall of moderate intensity. The soils are assumed to have cylindrical pores of uniform radius. The soil water is 15 °C (with corresponding density and air-water surface tension), and the gravitational acceleration is 9.812 m s⁻². For the third column, a porosity of 0.60 is assumed for pores ≤0.1 mm, and of 0.40 for larger pores. Note that the last row gives the limiting radius where the capillary rise equals the pore radius.

Pore radius (mm)	Height of capillary fringe (equals the initial depth of the phreatic level) (m)	Equivalent water layer needed to raise the phreatic level to the soil surface (mm)	Time needed during 10 mm h ⁻¹ rainfall (s)	Matric head increase divided by the added water layer
1.00×10 ⁻²	1.499	4.00×10 ⁻³	1.44	3.75×10 ⁵
3.00×10 ⁻²	0.500	1.20×10 ⁻²	4.32	4.16×10 ⁴
5.00×10 ⁻²	0.300	2.00×10 ⁻²	7.20	1.50×10 ⁴
0.100	0.150	4.00×10 ⁻²	14.4	3.75×10 ³
1.00	1.50×10 ⁻²	0.267	96.0	56.2
3.872	3.872×10 ⁻³	1.03	371	3.75

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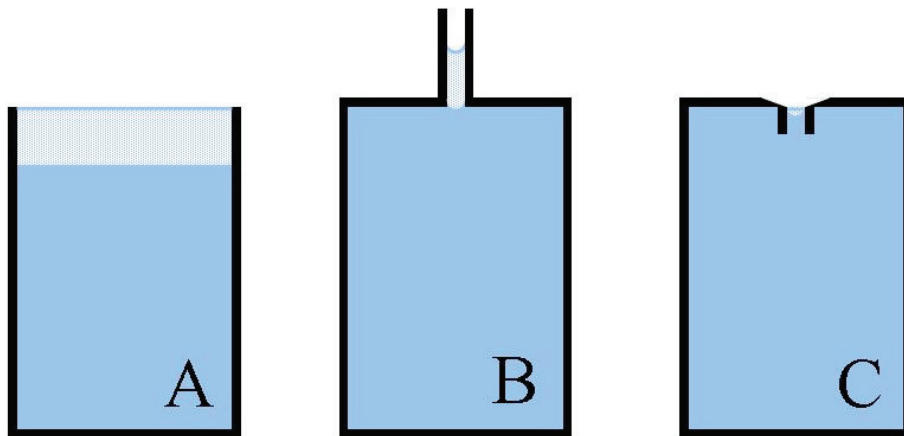


Fig. 1. Various liquid-filled vessels, with blue indicating the liquid. **(A)** Open at the top **(B)** Closed top with a capillary tube extending upward. **(C)** Closed top with a capillary tube extending inward. The dashed regions indicate the amounts of liquid that need to be added to cause equal pressure increases in the bulk of the liquid.

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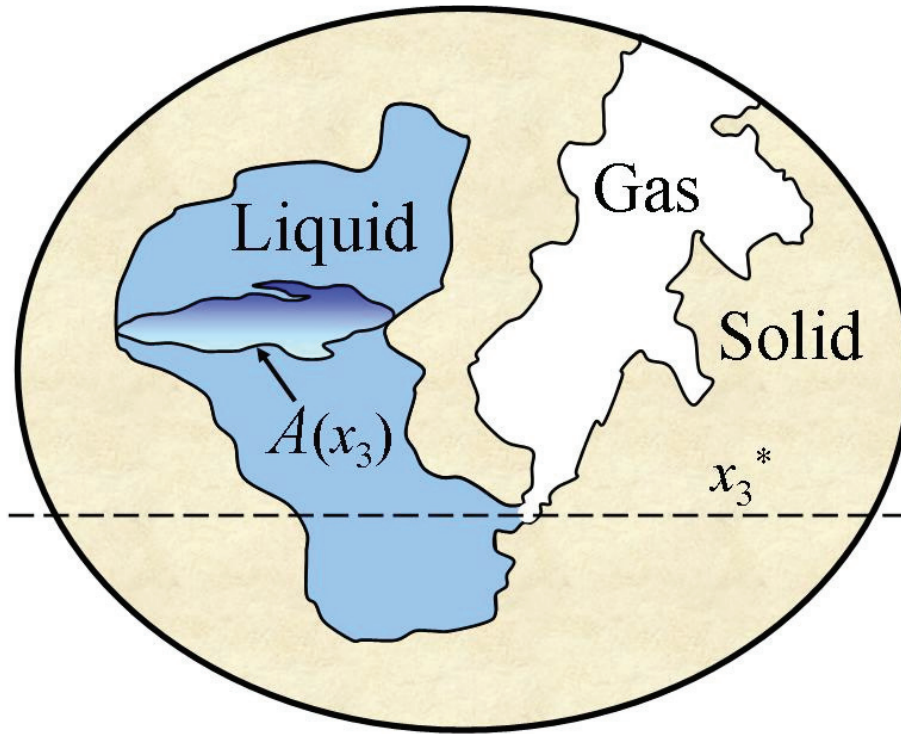


Fig. 2. A hypothetical liquid-filled pore with a single exit in which the liquid-gas interface is located at vertical position x_3^* . The horizontal liquid-filled cross-section A at arbitrary elevation x_3 is also indicated.

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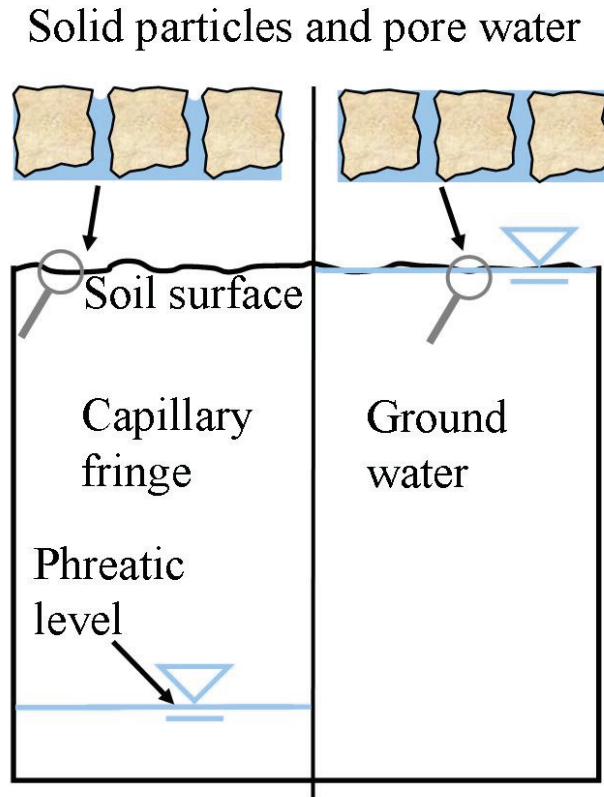


Fig. 3. A fine-textured soil at hydrostatic equilibrium with different phreatic levels. In the left pane the phreatic level is at such a depth that the capillary fringe exactly reaches the soil surface. The menisci at the soil surface are curved, but all pores are saturated. In the right pane, just enough water was added to flatten these menisci. As a result the phreatic level (where the water is at atmospheric pressure) moved up to the soil surface.