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Stage level, volume, and time-frequency information content of Lake Tana using stochastic and wavelet analysis methods

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Abstract

Lake Tana is the largest fresh water body situated in the north western highlands of Ethiopia. It serves for local transport, electric power generation, fishing, ecological restoration, recreational purposes, and dry season irrigation supply. Evidence show, the lake has dried at least once at about 15 000–17 000 BP (before present) due to a combination of high evaporation and low precipitation events. Past attempts to observe historical fluctuation of Lake Tana based on simplistic water balance approach of inflow, out-flow and storage have failed to capture well known events of drawdown and rise of the lake that have happened in the last 44 years. This study is aimed at simulating the lake level, specifically extreme events of the lake variation using stochastic approaches. Fourty-four years of daily, monthly and mean annual lake level data has showed a Gaussian variation with goodness of fit at 0.01 significant levels of the Konglomorov-Simrnov test. Three stochastic methods were employed, namely perturbations approach, Monte-Carlo methods and wavelet analysis, and the results were compared with the stage level measurements. The stochastic simulations predicted the lake stage level of the 1972, 1984 and 2002/2003 historical droughts 99% of the time. The information content (frequency) of fluctuation of Lake Tana for various periods was resolved using Wigner's Time-Frequency Decomposition method. The wavelet analysis agreed with the perturbations and Monte Carlo simulations resolving the time (1970s, 1980s and 2000s) in which low frequency and high spectral power fluctuation has occurred. In summary, the Monte-Carlo and perturbations methods have shown their superiority for risk analysis over deterministic methods while wavelet analysis has met reconstructing stage level historical record at multiple time scales. A further study is recommended on dynamic forecasting of the Lake Tana stage level using a combined approach of the perturbation and wavelet analysis methods.

1 Introduction

Lake Tana is the largest fresh water body situated in the north western highlands of Ethiopia. It serves for local transport, electric power generation, fishing, ecological restoration, recreational purposes, and dry season irrigation supply. The lake is fed by four major rivers; Gumara, Megech, Rib and Gilgel Abay. The drainage basin receives an average annual rain fall of 1200 mm which helps to replenish the lake seasonally. The lake has one outlet, where the Blue Nile River begins (Chebud and Melesse, 2009).

Historically, the lake dried at least once at about 15 000–17 000 BP (before present) (Hautot et al., 2006). The same report indicated that the reason for desiccation was probably a combination of high evaporation and low precipitation events. Such evidences was obtained from sediment core samples of plant remnants that grew and deposited within the present lake area. The recent historical fluctuation of Lake Tana and shrinkage of wetlands is reported by Kebede et al. (2006b), Chebud and Melesse (2009), Wale (2008). All of the studies reported are based on simplistic water balance approach of inflow, out-flow and storage. The reports invariably indicated the stationarity of the mean lake level in the last 44 years. None of the studies explained how the lake stage level behaved in the 1972, 1984 and 2002/2003 historical droughts nor showed any extreme event. Whereas, the historical flow data showed a decline of the out flow below 500 million cubic meters (MCM) which was only 10% of its annual average 5 billion cubic meters (BCM).

The challenge of past studies that use deterministic water balance approaches is evident from the uncertainty of inputs. First, the major inflow variable (inflow from 4 major rivers) is gauged at a daily scale (a sample/24 h) interval, therefore most peak flows could have been missed any time of the day. Second, the considerable size of the contributing watershed is un-gauged (Wale, 2008) and is assumed. Third, sub-surface inflow and outflow is often assumed (Kebede et al., 2006b), or estimated using numerical models (Chebud and Melesse, 2009) that take into account geological inferences. In general, the propagation of error could limit the predictability of Lake Tana using deterministic water balance approaches.

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Considering the scarcity of data and absence of forecasting mechanisms for Lake Tana, simulating the lake level using stochastic methods was relevant. The stochastic methods hold the statistical variability of the historical lake level and also take into account the error term from the probabilistic distribution of the noise. This makes stochastic methods superior, in Lake Tana's context, over deterministic methods that suffer from poor sampling methods, crude assumptions as well as uncertainties of un-gauged watersheds. The stochastic method uses a historical time series of the daily lake level which precludes any propagated error from component measurements; rather it uses measurement of the lake level (net balance of storage).

In fact the stochastic methods could serve to simulate historical events as well as forecast the stage level dynamically. However, a complement using wavelet analysis is also necessary to resolve the return period and the spectral power of the events. So, the two objectives set for this study were to

1. simulate Lake Tana stage level and volumetric balance using stochastic approaches of Monte Carlo and perturbation methods, and
2. observe the time-frequency resolved changes of the spectral power and validate low frequency events of drought or flooding of Lake Tana.

The study hypothesis is that a combined approach of stochastic methods and wavelet analysis could improve predictability of Lake Tana water level compared to deterministic water balance models.

2 Study area

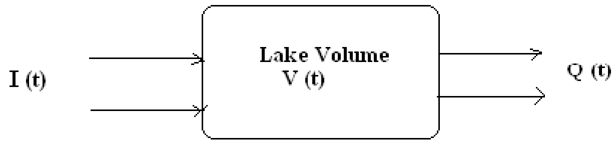
Lake Tana is located at 11°27′ N and 37°10′ E of Ethiopia at 1790 m a.s.l. The lake has a mean and maximum depth of 8 m and 14 m, respectively. The total drainage area of the Lake Tana basin is about 16 500 km^2. It receives about 90% of the inflow from the four rivers. The local and regional groundwater inflows contribute only 3%

and 7%, respectively (Chebud and Melesse, 2009). The lake is geologically dammed by quaternary and tertiary basalts in the south and western part where the out flowing Blue Nile River drains out (Kebede et al., 2005b). The damming has cutoff any seepage outflow which simplifies modeling of the lake stage (Hautot et al., 2006).

3 Background on stochastic approaches

3.1 Random nature of lake stage and the stochastic approach

The water balance approach is the basis for the stochastic simulation approach. Inflow and outflow determine lake stage (volume) variability which in retrospect is dependent on the amount of rainfall, evapotranspiration and, catchment characteristics that generates runoff. Rainfall influences the amount of inflow in two ways. First, it is a direct contributory over the lake area. Second, it drives the recharge and runoff and hence the inflow. Considering river inflow and precipitation as inputs $I(t)$ (L^3/T), river outflow at the outlet and evapotranspiration from the lake surface as outputs $Q(t)$ (L^3), the difference in the volume was formulated as the storage $V(t)$ (L^3) as shown in free diagram below.



The equation could be formulated by balancing the change in volume with time taking the difference of the inflow and outflow as indicated in Eq. (1), considering an analogous approach by Yevjevich (1972). Since the variables in Eq. (1) have their respective mean and noise, the perturbation approach takes the form shown in Eq. (2) where v, i, and q are noises from volume, inflow and outflow, respectively. Averaging Eq. (2) over several realizations, the noise terms will be eliminated giving only the equations

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in terms of the mean values. Detrending mean values from Eq. (2) results in a relationship of the noises only, Eq. (3). The formulation of the noises of inflow, outflow and lake volume as in Eq. (3) is the core of the perturbation approach which enables us to reformulate for the statistical behavior of the lake fluctuations in terms of the variances and covariance of inflow and outflow.

$$\frac{dV(t)}{dt} = I(t) - Q(t) \tag{1}$$

$$\frac{d(\bar{V}+v)}{dt} = (\bar{I}+i) - (\bar{Q}+q) \tag{2}$$

$$\frac{d(v)}{dt} = (i) - (q) \tag{3}$$

Multiplying Eq. (3) on both sides by the storage noise "v" and averaging over several realizations, a physically meaningful parameter $(\overline{v^2})$ or the storage variance (σ_v^2) and the other terms of Eq. (4) namely covariance of the inflow and outflow with the storage changes are derived. The equation relates rate of amplification or decay of storage variance (σ_v^2) with its covariance against inflow and outflow. Similarly, multiplying Eq. (3) on both sides by the inflow noise, averaging it over realizations and rearranging the terms gives $(\overline{i^2})$ which is the inflow variance (σ_i^2) that shows a relationship with its covariance against the outflow and the rate of change of its covariance with the storage volume. Using the same approach, the relationship of the variance of the outflow is found to be related with its covariance to the storage changes and inflow as shown in Eq. (6)

$$\frac{d(\overline{v^2})}{dt} - 2(\overline{iv}) - (\overline{qv}) \tag{4}$$

$$\frac{d(\overline{iv})}{dt} = (\overline{i^2}) - (\overline{iq}) \tag{5}$$

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$$\frac{d(\overline{qv})}{dt} = (\overline{iq}) - (\overline{q^2}) \quad (6)$$

The relationships in Eqs. (4), (5) and (6) reveal that the variance or covariance from any of the inflow, outflow or storage variables is propagated with time, suggesting the propagation of the error as part of the simulation. Though such error propagation is a limitation of the perturbation approach, it is powerful to inform all possible realizations. The limitations however could be constrained by prior knowledge of the historical variability and some boundary conditions. An alternative to the classical perturbation methods with less error propagation is the Monte-Carlo approach. The Monte Carlo approach (Christopher, 1997) is simplistic that relates probability of future realizations based on past events added to a noise. The relationship could be established discretizing Eq. (1),

$$V_{t+\Delta t} = V_t + \Delta t[I(t) - Q(t)] \quad (7)$$

From the definitions of the Monte Carlo approach, the probability of realizing the predicted Volume ($V_{t+\Delta t}$) is the sum of the current probabilities of the volume, inflow and outflow. This dependence of current and future probabilistic distributions is reported as a weakness of Monte Carlo approach (Gelhar, 1998). However it is merely inapplicable to non-stationary distributions. On the other hand, its simplicity and conformity to boundary conditions of realizations makes it appealing.

3.2 Wavelet analysis

Chebud and Melesse (2009), showed the two frequency ranges and the spectral power of Lake Tana but fall short of informing exactly when such changes of frequencies occur. In other words, the information content of the lake variation (the spectral power and return period) was not resolved in time. This study investigated the potential of the time-frequency resolution using wavelet analysis. Wavelet analysis could detect (1) any historical changes in frequency of lake level fluctuation which could help to

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relate the effect of the frequency with other ecological phenomenon (signatures) since the latter is recorded in the time domain and (2) the wavelet analysis could decompose the signal as local and global trends which could later be reconstructed and serve for lake level forecasting.

The wavelet transform can be considered as the convolution of the signal of interest (in this case the lake level fluctuation) with the conjugate of the Fourier transform of a mother wavelet. The mother wavelet has a magnifying as well as discrimination role over the signal. oreover the type of mother wavelet chosen usually depends on its pattern similarity with the signal type. For hydrological analysis, the Morelet wavelet (Fig. 1) is recommended by Tesche et al. (1998), Kang et al. (2007), and Torrence et al. (1997). The normalized Morelet wavelet's distribution, Eq. (8), has a mean value of zero and is known for its localization in time and frequency information content. The vertical axis, represented by $\Psi(t)$ is analogous to a density function for deterministic signals (Cohen, 1989) (a measure of amplitude in this case, though unit-less) while the horizontal distribution is time whose unit is dependent on the unit of signal with which it is convoluted .

$$\Psi_o(x) = \pi^{-1/4} e^{i\omega_0 x} e^{\frac{-x^2}{2}} \tag{8}$$

Ψ_o is the Morelet wavelet distribution, x constitutes the product of time and is a scale term which enables us to expand or shrink the wavelet in order to increase the window for analysis. Large scale factors expand mother wavelets (increase windows and suppress high frequencies) which results in higher time but lower frequency resolution. Whereas low scale values shrink the mother wavelets increasing the frequency content and decreasing time resolution. The wavelet characteristic at different scaling factors helps to decompose the lake level signal at localized as well as large scale trends. The practical implementation as described by Torrence et al. (1997) Eq. (9) involves the Fourier transforms of the lake level signal $\hat{x}_r$ and the complex conjugate of the Morelet

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wavelet;

$$W_n(s) = \frac{1}{n}\sum_{n=0}^{N-1} \hat{x}_r \widehat{\Psi^*}(s\varpi_r)e^{i\varpi_r n\Delta t} \tag{9}$$

Where ϖ_r is the angular frequency below the Niquist frequency $\varpi \leq 2\pi r/N\Delta t$, s is the wavelet scaling factor, and r 1,. . . , $N-1$ is the frequency index. For normalization purposes, the wavelet should be multiplied by $(2\pi s/\Delta t)^{1/2}$. The wavelet transfer, W_n in Eq. (9) has a physical meaning of similarity index (Torrence et al., 1997) between water level signal and pattern of the daughter wavelet (a reproduced signal). The square of the wavelet transform W_n^2 $(s, r\Delta t)$ at a given localized time and frequency content is known to be an indicator of the lake level fluctuation spectral power, shifted in time by $r\Delta t$ with a dilation of the mother wavelet by scale s (Torrence et al., 1997).

4 Methodology

For Monte Carlo and Perturbation methods, a mean annual stage level of Lake Tana was derived from 44 years of daily data and used to calculate initial input parameters such as variance, initial stage level and initial volume of the lake. For the wavelet analysis, the daily lake stage data collected from the Ethiopian Ministry of Water Resources was used.

The statistical characteristic of the lake level variation was tested fitting different probabilistic distributions on the historical data. Two probabilistic distributions were fitted using the annual averaged data namely the normal and lognormal. The goodness of fit was tested based on the Konglomorov-Simirnov tests (Chakravart et al., 1967). Appropriate the stationarity assumption of the annual lake level for Monte Carlo simulation was tested using autocorrelation at different lag times of the annual average lake level data.

Simulation using the perturbation approach was done discretizing Eqs. (4), (5) and (6) which takes the same format as Eq. (7) except that values discretized at different

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time steps are variance of the volume (σ_V^2), covariance of inflow and volume ($\overline{i\vartheta}$) and covariance of outflow and volume ($\overline{q\vartheta}$). On the other hand, Eq. (7) in its discretized format is used upon using realizations that were already generated for the perturbation approach. Practically, it would mean sampling all realizable values of the inflow $I(t)$, outflow $Q(t)$ and Volume V_t, based on the historical mean, variance and probabilistic distributions. The realizations were sampled from the normal probability distribution which was decided after the fitting in this paper.

The wavelet analysis involved two implementations, one intended to reconstruct the historical record after resampling and the other for time-frequency resolution of the lake level. Reconstructing the historical signal was done by decomposing existing signals at different sampling rates through a convolution of the mother wavelet at different scaling factors (related to the window width of the mother wavelet). The decomposed signal was first filtered at high and low frequency filters (to capture the local and global trends respectively), which was later denoised and added back, to reconstruct at the respective resolutions and test predictability of lake level.

Spectral density was resolved in time and frequency using Wigner methods (Cohen, 1989). The method uses Eq. (8) to create the mother wavelet distribution for a given time period and later calculates the complex conjugate of its Fourier transform. It subsequently convolves it with the Fourier transform of the observed lake level to obtain the Wigner intensity distribution in time and frequency Eq. (9). The output is used to derive instantaneous frequency for Wigner distribution as shown in Eq. (10) (Auguer et al., 1995–1996). W is the Wigner distribution from Eq. (8), t is time and θ is the frequency

$$\omega(t) = \frac{\int_{-\infty}^{+\infty} \theta W(t,\theta)d\theta}{\int_{-\infty}^{+\infty} W(t,\theta)d\theta} \tag{10}$$

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5 Results and discussion

5.1 Significance of perturbations, Monte Carlo and deterministic methods

With the intention to observe the fit of the data to probabilistic models, 44 years of lake elevation (1960–2003) was fitted to normal and lognormal distributions. The data is a monthly averaged from a daily measured value. Among the two probabilistic models the normal has shown a better goodness of fit at 0.01 significant levels of the Konglomorov-Simrnov test. The coefficient of determination (R^2) value was 0.58 and 0.47 for the normal and lognormal distributions respectively. Although, the graphical fit (Fig. 2a and b) is indistinguishable, which might be due to the size of annual data, the normal fit was used in this study for Monte Carlo implementation.

A test of the periodicity of the data was done using an autocorrelation analysis of monthly and annual data at all lag times. The analysis (using the integral of Fig. 3) showed 1.5 years of memory time and decay of the autocorrelation afterwards with no periodicity of the annual averaged data. As a large water body with an area of 3150 km^2 and mean annual volume of 29 BCM, 1.5 years of memory time for the input and output relationship show randomness of the lake level fluctuation data. So, annual resolution was used for the perturbation as well as Monte Carlo approaches.

The perturbation approach of the model used the annually disctetized form of Eqs. (3), (4), (5) and (6). Using statistical values of 5% coefficient of variation, 20 000 realizations of inflow (river inflow and precipitation added together), out flow (gauged values at outlet added with evapotranspiration), and their respective covariance were generated. The 1960 mean annual lake volume, derived from the historical lake level and bathymetric relationship of Lake Tana (Wale, 2008), was used as an initial value and updated for each realization of the inflow and outflow. The covariance of the flows with the lake volume was updated in parallel. The Monte Carlo approach of predictions was estimated using the same realizations substituting in Eq. (7).

ANOVA of the perturbation and Monte Carlo approaches showed nonexistence of significant differences between the two at 1% of significance. Whereas the lake level determined using deterministic water balance approach has shown significant differences with the perturbation as well as the Monte Carlo approach even at a 10% level. The graphical plot of the lake level as well as volumetric trends showed similar patterns (Figs. 4a and b). The stochastic approaches have explained the lowest lake levels around 1972, 1984 and 2002/2003 predicting extreme events of low lake levels. Each of these time periods were well documented drought years in Ethiopian history.

The lake level predicted by the perturbation as well as the Monte Carlo approach reaches the maximum lake level measured at 1987 (Fig. 3a). Adding twice the standard deviation on top of the mean, perturbation approach has shown a predicted level of 1789.8 m a.s.l., which was 3.2 m short of flooding the town adjacent to the lake called Bahir Dar. The recurrence period analysis using Monte Carlo and the perturbation approaches showed that a severe flooding could occur every 96 years. The deterministic approach has extended it to 290 years for flooding. The latter number does not seem to be realistic though historical evidence is needed from other sources. Due to lack of information on the position of the datum with respect to the zero out flow, no boundary condition was introduced in this study which would have contributed to the accuracy of the model. Another presumable source of error in this study could be the accuracy of the interpolation from the bathymetry as it was derived from others studies. The propagation of the error in this respect is obvious though maximum care is taken on the reading and resolution.

Lastly, it was observed that deterministically modeled lake level and the yearly minimum measured lake level fit at 1% significant test. The graphical plot of Fig. 4a showed constant lake level depicting a smoothing effect which failed to predict any of the drought years, underestimating the lake level at all times.

5.2 Time-frequency decomposition of Lake Tana fluctuation

The choice of scale for the wavelet analysis was made at a range of 1–5 for testing. The result showed that Lake Tana has not encountered a change in frequency of its global patterns over the last 44 years as indicated by decomposed and reconstructed signals of Fig. 5a. Second, decomposition at several scales (d_1–d_5) and the reconstructed signal as shown by the simulated signals in Fig. 5a, (approximated by, $a_1 \ldots a_5$) at each scale suggested that Lake Tana's stage level sampling period interval between daily and monthly level ($d_1 \ldots d_5$ in Fig. 5c) had no differences in simulation and prediction at least for the wavelet analysis. Third, the exact fit of the original signal (s) and reconstructed signal (cs) in Fig. 5b) showed that Lake Tana stage level could be forecasted at daily and monthly scale without being affected by the sampling rate. This is a challenge for deterministic approaches that are dependent on component measurements as mentioned elsewhere. Last, early 1970s, mid 1980s and early 2000 are characterized by rapid fluctuations as shown in decomposed signals at all scales ($d_{1 \ldots} d_5$) (Fig. 5c). The effects of such perturbation and possible ecological signatures are important research areas that need further analysis.

The spectral power was resolved very well using Wigner time-frequency decomposition. The negative spectral values (Fig. 6, 3-D) that are cross product of the mother wavelet (especially Morelet) with the signal need to be interpreted in their absolute value as suggested by Boreghetti et al. (2009). The early 1970s, mid 1980s and early 2000, show low frequency (high return period) events of high spectral power confirming the previous scaled signal decomposition analysis. The intensity of the change in the amplitude could be obtained upon integrating the spectral power over the specific time-frequency localization. The spectral power distribution viewed from the 2-D shows the peak occurrence at the 1984 drought year. It is also shown that time and frequency (return period) for the case of Lake Tana are related by a polynomial distribution mainly by the second power 2 (parabolic) as shown in the 2-D plot (Fig. 6) which warns us against the use of direct ranking technique for return period estimation of lake level extremes. The spectral power distribution could help to sharpen questions about whether such distributions of spectral power (in Fig. 6) and noise dominated fluctuation could cause any impact on fisheries and other aquatic life which would be an interest for other studies.

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6 Conclusion and recommendation

At the outset, this study aimed simulate the stage level and volume of Lake Tana using stochastic approaches. It also intended to simulate a time-frequency break down of the spectral power using wavelet analysis. Probabilistically, the result showed that the two stochastic approaches (Monte Carlo and perturbations approach) were significantly different from deterministic water balance simulations 99% of the time. Whereas the perturbation and Monte Carlo approaches have shown no significant differences at 99% of the time.

For the perturbation and Monte Carlo approaches the mean annual lake level prediction is relevant (for its stationrity) and hence the method serves to predict extreme events inclusive of the timing. The wavelet analysis added key knowledge on three issues. First, daily and monthly stage level of Lake Tana could be simulated easily using Morelet mother wavelet as it was shown on the fit between the observed and constructed signal. It has reproduced the observed data as exact fit and hence it is recommended for further employment on forecasting of the lake level at monthly or daily level where neither perturbations nor Monte Carlo method could work. Second, the wavelet analysis predicts at flexible sampling rate which outweighs both deterministic as well as the other stochastic methods for prediction purposes which reduces error propagation. So, complementarity of the wavelet analysis with the perturbation or Monte Carlo approach makes it appealing for a combined forecast of Lake Tana. The daily and monthly forecast is best tackled by wavelet analysis while annual lake level, especially extreme events, are well captured by perturbations or Monte Carlo methods. In summary, the combined use of stochastic wavelet analysis has shown the potential for employment in operational forecasting. And a further study is recommended to develop a dynamic approach combining perturbations and wavelet methods for operational forecasting of mean annual and daily stage level of Lake Tana. Last, the Wigner time-frequency resolution had an added value on the spectral power content (integrated locally in time and return period) which helps for interdisciplinary analysis cross correlating it with other ecological events around the lake.

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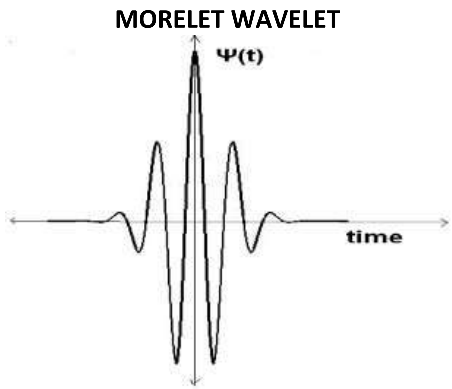

Fig. 1. A Morelet mother wavelet density function distribution used for decomposing the water level signal.

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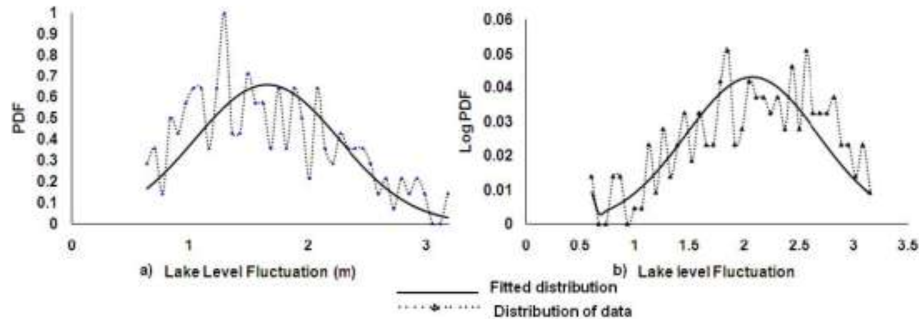

Fig. 2. **(a)** Normal fit to lake level probabilistic distribution, **(b)** lognormal distribution fit.

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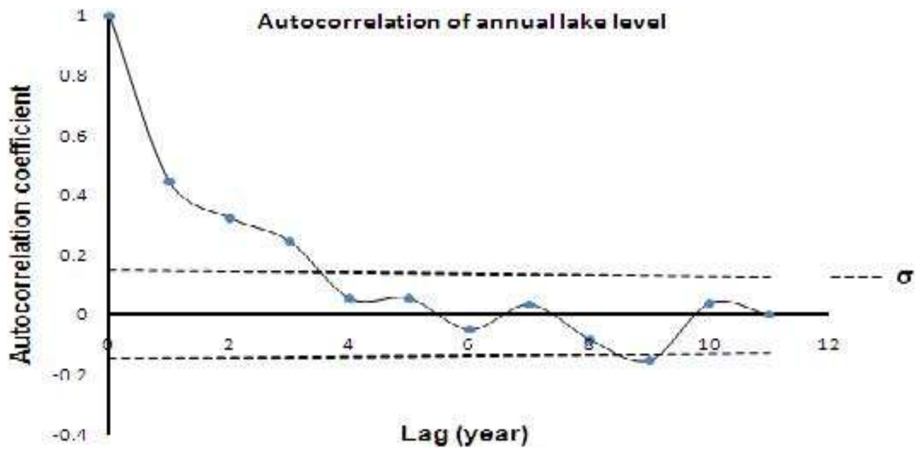

Fig. 3. Autocorrelation of annual averaged lake level for a lag time of 1 to 12 years.

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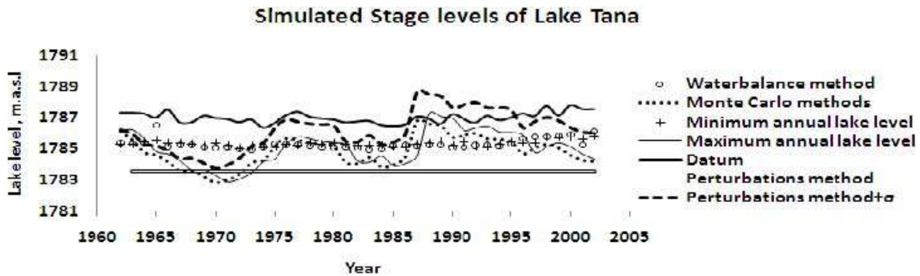

Fig. 4a. Lake level based on perturbations, Monte-Carlo and deterministic methods.

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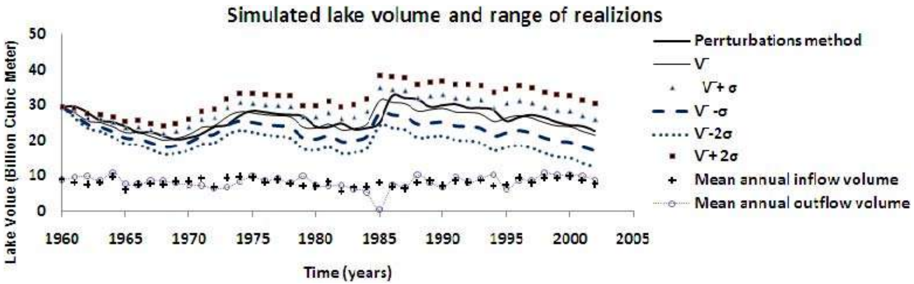

Fig. 4b. Simulated volume based on Monte-Carlo and perturbations methods.

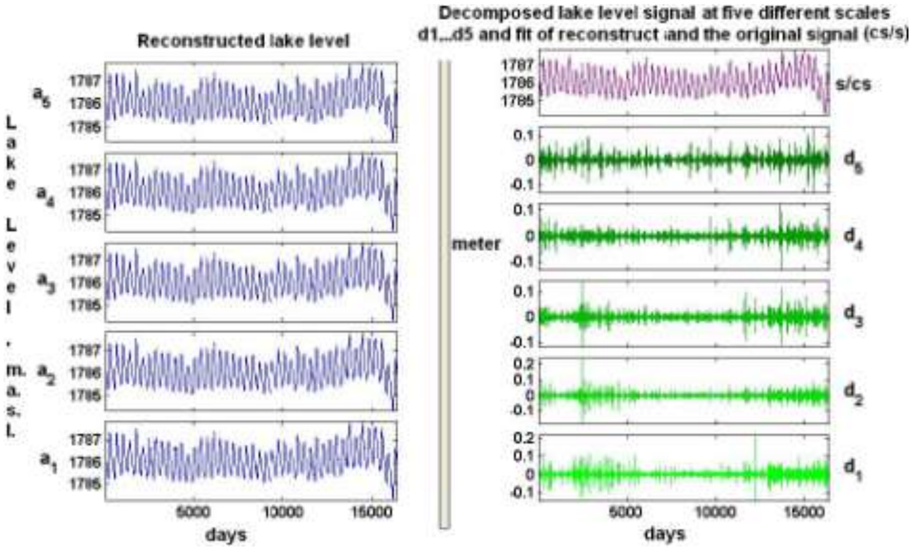

Fig. 5. **(a)** Reconstructed signals of lake level $a_1 \ldots a_5$ based on decompositions at scales of one up to 5. **(b)** Decomposed signals fluctuation for the scales of 1 to 5. **(c)** Fit of the reconstructed and original lake level signal (Cs/s).

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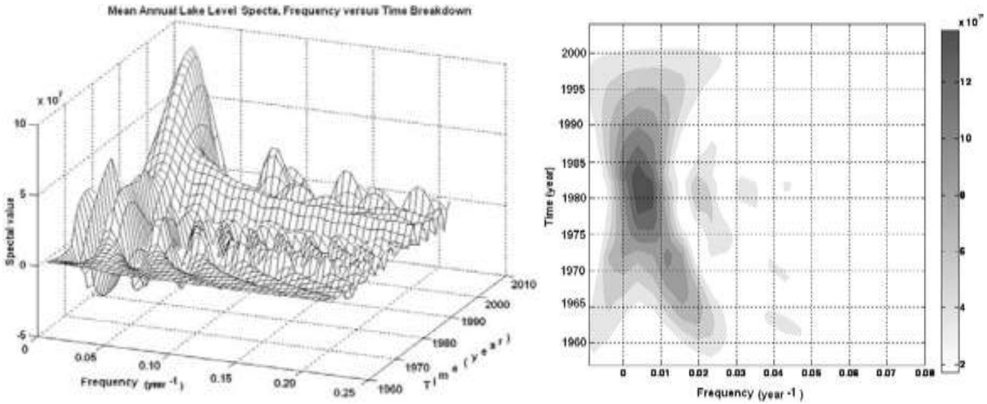

Fig. 6. Spectral power under frequency-time resolution of annual lake level spectra, 3-D and 2-D.

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