Hydrol. Earth Syst. Sci. Discuss., 7, 4473–4499, 2010
www.hydrol-earth-syst-sci-discuss.net/7/4473/2010/
doi:10.5194/hessd-7-4473-2010

Hydrology and Earth System Sciences Discussions

Groundwater fluctuations in heterogeneous coastal leaky aquifer systems

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Received: 4 June 2010 – Accepted: 9 June 2010 – Published: 9 July 2010

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Published by Copernicus Publications on behalf of the European Geosciences Union.

Abstract

In the past, the coastal leaky aquifer system, including an unconfined aquifer on the top, a confined aquifer at the bottom, and an aquitard in between, was commonly assumed to be homogeneous and of an infinite extent in the horizontal direction. The leaky aquifer system may however be nonhomogeneous and of a finite extent due to variations in depositional and post depositional processes. Thus, in the paper, the leaky aquifer system is divided into several horizontal regions for simulating the heterogeneity involved in both the confined aquifer and aquitard. A one-dimensional analytical model is developed for describing the head fluctuation in such a heterogeneous leaky aquifer system. The head of the upper unconfined aquifer is assumed to remain constant. It is found that both the length and location of the discontinuous aquitards presented in the coastal area have significant effects on the amplitude and phase shift of the head fluctuation in the confined aquifer. In addition, the influences of both the heterogeneous aquifer and aquitard on the spatial head distribution are investigated.

1 Introduction

The groundwater near the coast fluctuating with periodical tides is interesting and practical topics for hydrogeologists. These topics include coastal aquifer parameter estimation (e.g., Li and Chen, 1991; Pandit et al., 1991), marine environment (e.g., Uchiyama et al., 2000), marine retaining structures (e.g., Farrell, 1994) and numerical as well as analytical simulations (e.g., Sun, 1997; Oki et al., 1998; Li et al., 2000; Jeng et al., 2002; Jiao and Li, 2004; Li et al., 2007; Song et al., 2007; Xia et al., 2007; Chuang and Yeh, 2008; Sun et al., 2008; Chang et al., 2010).

The coastal leaky aquifer system usually consists of an upper unconfined aquifer bounded from below by one or more confined aquifers (Chen and Jiao, 1999). Many researches reveal that the head fluctuation in the unconfined aquifers due to tides is significantly damped by the storage of the unconfined aquifer (e.g., Millham and Howes,

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1995; Chen and Jiao, 1999). White and Roberts (1994) indicated that the farthest distance that tidal propagation can reach is usually not over 30 m if the unconfined aquifer is heavily damped. The water table is therefore assumed to maintain constant.

Several previous researches indicated that the effect of the leakage on the groundwater in coastal leaky aquifer systems is significant. Jiao and Tang (1999) developed an analytical solution for describing the head fluctuations in coastal confined aquifers, where the aquitard infinitely extends from the coastal line to the inland. They indicated that the leakage diminishes both the amplitude of the head fluctuation and the farthest distance that tidal propagation can reach. Li and Jiao (2001) presented an analytical solution for describing leaky aquifer systems with considering the leakage and the effect of aquitard storage. They found that both the leakage and the effect of aquitard storage on the head fluctuation in the confined aquifer are negligible when the aquitard storage is small and the storage ratio of the aquitard to confined aquifer is less than 0.5. Guo et al. (2010) developed an analytical solution for describing the groundwater head response to tidal fluctuation in a coastal aquifer consisting of two zones of different hydraulic properties. They introduced a parameter r as the ratio between the product of transmissivity and storativity of the inland zone to that of the costal zone. The solutions were used to investigate the behaviors of amplitude and phase shift of the head fluctuation. In addition, the solutions were also used to estimate the aquifer parameters in a real coastal two zone aquifer.

The long-time variations in depositional and post depositional processes may result in heterogeneous hydraulic conductivity and nonuniform thickness of the aquitard (Cherry et al., 2006) and thus produce variations in the leakage to the lower confined aquifer. The objective of this paper is to develop a mathematical model for describing the head fluctuation in the confined aquifer due to the tidal effect in a coastal leaky aquifer system. This system is divided into several horizontal regions to simulate the heterogeneous conductivities of the aquitard and underlying confined aquifer. The head of the upper unconfined aquifer is assumed constant. The analytical solution of the model is then developed by the direct Fourier method. The effects of the length

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and location for a discontinuous aquitard on the amplitude and phase shift of the head fluctuation in the confined aquifer are investigated. The present solution is applied to simulate the head distribution for the coastal aquifer system in the Chek Lap Kok (CLK) airport, Hong Kong (Jiao and Tang, 1999) and the results are compared with the observed data taken at this airport. In addition, the influences of heterogeneities of the aquitard and confined aquifer on the spatial head distribution in the underlying confined aquifer are also examined.

2 Methods

2.1 Conceptual model

Figure 1 shows a coastal leaky aquifer system in which the aquitard and underlying confined aquifer are heterogeneous in the horizontal direction. The system is divided to N different horizontal regions and the formation materials of the aquitard and confined aquifer in each region are homogeneous. The origin of the coordinates in x direction is located at the coastal line. The distance measured from the origin to the interface between regions 1 and 2 is d_1. Similarly, the distance measured from the origin to the interface between regions $n-1$ and n is d_{n-1}. The mean sea level (MSL) is chosen as reference datum. Consider that the unconfined aquifer may store large quantities of water which can damp the tidal effect so that the water table fluctuation in the unconfined aquifer is negligible in comparison with that in the confined one. Therefore, the water table is assumed to be the same level as the MSL.

The governing equation describing the head distributions in these regions is expressed as

$$S_n \frac{\partial h_n}{\partial t} = T_n \frac{\partial^2 h_n}{\partial x^2} + L_n h_n \tag{1}$$

where n is an integer from 1, 2, 3, ..., N; h_n, T_n and S_n are the hydraulic head,

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transmissivity, and storativity for the confined aquifer in nth region, respectively; and L_n is the leakage for the aquitard in nth region. The tidal boundary is expressed as

$$h_1(0,t) = A\cos(\omega t) \tag{2}$$

where A and ω are the amplitude and frequency of the tide, respectively. The remote boundary condition is

$$\lim_{x\to\infty} \frac{\partial h_N(x,t)}{\partial x} = 0 \tag{3}$$

where h_N is the hydraulic head for the farthest confined aquifer from the coastal line. The continuities of the head and flux required at the interfaces d_n are, respectively

$$h_n(x,t) = h_{n+1}(x,t) \quad \text{at} \quad x = d_n \tag{4}$$

and

$$T_n \frac{\partial h_n(x,t)}{\partial x} = T_{n+1} \frac{\partial h_{n+1}(x,t)}{\partial x} \quad \text{at} \quad x = d_n \,. \tag{5}$$

2.2 Solution for heterogeneous aquifer systems

Based on the direct Fourier method, the solutions for describing the head distributions in the regions demonstrated in Fig. 1 can be expressed as

$$h_n(x,t) = \mathrm{Re}[A \cdot X_n(x) \cdot e^{-i\omega t}] \tag{6}$$

where Re means the real part of the complex expression and i is equal to $\sqrt{-1}$.

Substituting Eq. (6) into Eq. (1) and eliminating the exponential terms can obtain ordinary differential equations in terms of $X_n(x)$. Substituting Eq. (6) into Eq. (2) with n=1 and eliminating the exponential terms acquire the boundary condition expressed as $X_1(0)$=1. Similarly, the equation $\lim\limits_{x\to\infty} \partial X_N/\partial x$=0 can be obtained from Eqs. (3) and (6). The continuity requirements (i.e., X_n=X_{n+1} and $T_n \partial X_n/\partial x$=$T_{n+1}\partial X_{n+1}/\partial x$) at

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$x = d_n$ can also be acquired from Eqs. (4)–(6) in a similar manner. The general solutions for $X_n(x)$ can then be solved as

$$X_n(x) = c1_n e^{\lambda_n x} + c2_n e^{-\lambda_n x} \tag{7}$$

with

$$\lambda_n = a_n \sqrt{2(u_n - i)} \tag{8}$$

where the parameter $a_n = \sqrt{\omega S_n / 2T_n}$ is a reciprocal of the decay length for the nth confined aquifer and $u_n = L_n / \omega S_n$ is the dimensionless leakage. The coefficients, $c1_n$ and $c2_n$, for nth confined aquifer can be obtained by the boundary conditions $X_1(0) = 1$ and $\lim_{x \to \infty} \partial X_N / \partial x = 0$ as well as the continuity requirements $X_n = X_{n+1}$ and $T_n \partial X_n / \partial x = T_{n+1} \partial X_{n+1} / \partial x$.

Based on Cramer's rule, the results for $c1_n$ and $c2_n$ can be expressed as

$$c1_n = \frac{\det \mathbf{D}_n}{\det \mathbf{D}} \tag{9}$$

and

$$c2_n = \frac{\det \mathbf{D}_{N+n}}{\det \mathbf{D}} \tag{10}$$

with

$$\mathbf{D} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{I} \\ \mathbf{B} & \mathbf{C} \\ \mathbf{E} & \mathbf{F} \\ \boldsymbol{G} & \boldsymbol{H} \end{bmatrix}_{2N \times 2N}, \tag{11}$$

$$\boldsymbol{I} = \begin{bmatrix} 1 & 0 & \ldots & 0 & 0 \end{bmatrix}_{1 \times N}, \tag{12}$$

$$\boldsymbol{G} = \begin{bmatrix} 0 & 0 & \ldots & 0 & 1 \end{bmatrix}_{1 \times N}, \tag{13}$$

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$$H = [0\,0\,\ldots\,0\,0]_{1\times N}, \tag{14}$$

$$\mathbf{B} = \begin{bmatrix} e^{\lambda_1 d_1} & -e^{\lambda_2 d_1} & 0 & \ldots & 0 & 0 \\ 0 & e^{\lambda_2 d_2} & -e^{\lambda_3 d_2} & 0 & \ldots & 0 \\ 0 & 0 & & & & \\ \vdots & \vdots & & \ddots & \vdots & \vdots \\ & & & & 0 & \\ 0 & \ldots & 0 & e^{\lambda_{N-2} d_{N-2}} & -e^{\lambda_{N-1} d_{N-2}} & 0 \\ 0 & 0 & \ldots & 0 & e^{\lambda_{N-1} d_{N-1}} & -e^{\lambda_N d_{N-1}} \end{bmatrix}_{(N-1)\times N} \tag{15}$$

and

$$\mathbf{E} = \begin{bmatrix} T_1\lambda_1 e^{\lambda_1 d_1} & -T_2\lambda_2 e^{\lambda_2 d_1} & 0 & \ldots & 0 & 0 \\ 0 & T_2\lambda_2 e^{\lambda_2 d_2} & -T_3\lambda_3 e^{\lambda_3 d_2} & 0 & \ldots & 0 \\ 0 & 0 & & & & \\ \vdots & \vdots & & \ddots & \vdots & \vdots \\ & & & & 0 & \\ 0 & \ldots & 0 & T_{N-2}\lambda_{N-2} e^{\lambda_{N-2} d_{N-2}} & -T_{N-1}\lambda_{N-1} e^{\lambda_{N-1} d_{N-2}} & 0 \\ 0 & 0 & \ldots & 0 & T_{N-1}\lambda_{N-1} e^{\lambda_{N-1} d_{N-1}} & -T_N\lambda_N e^{\lambda_N d_{N-1}} \end{bmatrix}_{(N-1)\times N} \tag{16}$$

where det represents a determinant of the matrix; **C** and **F** are $(N-1)\times N$ matrixes and identical to matrixes **B** and **E**, respectively, except that minus sign of the exponent of exponential terms; $\mathbf{D}_n$ and $\mathbf{D}_{N+n}$ can be obtained by replacing the n and $(N+n)$th column of matrix **D**, respectively, with a column matrix that the top element is one and the others are zero.

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Define the variable a_{X_n} and b_{X_n} representing the real and imaginary parts of the result calculated from Eq. (7), respectively, and thus one can have $X_n(x)=a_{X_n}+ib_{X_n}$. Substituting $X_n(x)=a_{X_n}+ib_{X_n}$ and $e^{-i\varpi t}=\cos(\varpi t)+i\sin(\varpi t)$ into Eq. (6) and taking the real part of Eq. (6) lead to the result in terms of cosine function as

$$h_n(x,t) = A \cdot c_n \cdot \cos(\varpi t + \phi_n) \tag{17}$$

with

$$c_n = \sqrt{a_{X_n}^2 + b_{X_n}^2} \tag{18}$$

and

$$\phi_n = \arccos\left(\frac{a_{X_n}}{c_n}\right) \tag{19}$$

where ϕ_n is the phase shift and c_n is the amplitude coefficient representing the decreasing amplitude of head fluctuations with increasing inland distance.

2.3 Special case

Jiao and Tang (1999) presented an analytical solution for describing the head fluctuations in a coastal leaky aquifer system. Both the aquifer and aquitard in their system are homogeneous and of an infinite extent from the coastal line. The present solution for N=1 can be reduced to their solution (1999, Eq. 4) and written, in our notation, as

$$h_1(x,t) = A \cdot c_1 \cdot \cos(\varpi t + \phi_1) \tag{20}$$

where

$$c_1 = -\exp\left\{\sqrt{a_1\left[\left(u_1^2+1\right)^{0.5}+u_1\right]}\right\}x \tag{21}$$

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and

$$\phi_1 = -\frac{a_1^2}{\sqrt{a_1\left[\left(u_1^2+1\right)^{0.5}+u_1\right]}}x. \quad (22)$$

Accordingly, the solution derived by Jiao and Tang (1999) for homogeneous coastal leaky aquifer systems can be considered as our special case.

3 Results and discussion

3.1 The effect of aquitard length

Consider a case that the leaky aquifer system with a homogeneous confined aquifer overlain by a heterogeneous aquitard can be divided into two regions (i.e., regions 1 and 2). The aquitard in region 1 is semi-permeable with leakage u_1=5 and a finite length d_1 while the aquitard in region 2 is impermeable (u_2=0) and of an infinite extent in x direction. Figure 2 shows the normalized amplitude of the head fluctuations in the confined aquifer versus the inland distance for $a_1=a_2=1\times10^{-2}\,\text{m}^{-1}$ and aquitard length d_1 of 0, 30, 50, 850 m. The normalized amplitude is defined as the ratio of the amplitude of head fluctuations to that of the tide. The data represented by the open circle is obtained by the present solution with no leakages (i.e., $u_1=u_2=0$). The curves shown in Fig. 2 indicate that a larger d_1 has a smaller normalized amplitude and shorter tidal intrusion distance, defined as the farthest landward distance from the coastline to the location where the normalized amplitude of head fluctuations is less than 10^{-2}. In addition, the present solution approaches Jiao and Tang's solution (1999) for u_1=5 and $a_1=1\times10^{-2}\,\text{m}^{-1}$ when d_1 goes large (say d_1=850m). This is because the aquitard length is larger than the tidal intrusion distance.

Figure 3 shows the phase shift of the head fluctuations versus the inland distance for u_1=5, u_2=0, and $a_1=a_2=1\times10^{-2}\,\text{m}^{-1}$ at d_1=50, 150, 850 m. The phase shift increases

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with the inland distance for various values of d_1, indicating that the response of groundwater in the confined aquifer to the tide becomes slow for a larger inland distance. The phase shift for the case of no leakage represented by the open circle is significantly larger than that of Jiao and Tang's solution (1999) represented by the solid circle, indicating that the leakage can diminish the phase shift of head fluctuation in confined aquifers. The slopes of the curves with d_1=50 m and d_1=150 m begin to close to that of Jiao and Tang's solution (1999) near x=35 m and x=135 m, respectively, which are in fact fairly close to the end of the aquitards. On the other hand, these two curves match with the solid circle near the coastal line, reflecting the fact that they are all influenced by the leakage.

3.2 Effect of aquitard location

In contrast to the previous case, the aquitard in region 1 with a length d_1 is now treated as impermeable, i.e., u_1=0, while the aquitard in region 2 is semi-pervious with u_2=5 and a length denoted as $l_2=d_2-d_1$. Note that d_1 can be considered as the location of the beginning of the aquitard in region 2 and the aquitard in region 2 may be considered to be of an infinite extent when l_2 exceeds the tidal intrusion distance. Figure 4 shows that the normalized amplitude of head fluctuations in the homogeneous confined aquifer versus the inland distance for l_2=1000 m, $a_1=a_2=1\times10^{-2}\,\mathrm{m}^{-1}$ and d_1=0, 50, 100, or 350 m. The figure shows that the curve with d_1=350 m matches with the case of no leakage, indicating that the aquitard located far away from the costal line does not affect the head distribution in the coastal confined aquifer. In addition, the figure also shows that those dashed lines approach the open circle for a large d_1, indicating that the effect of the aquitard on head fluctuations decreases with an increasing d_1.

Figure 5 shows the phase shift of the head fluctuations in confined aquifers versus the inland distance for u_2=5, l_2=1000 m, $a_1=a_2=1\times10^{-2}\,\mathrm{m}^{-1}$ and d_1=0, 100, 200, or 350 m. The figure indicates that the curve with d_1=350 m matches with the case of no leakage, indicating that the aquitard located far away from the costal line has no influence on the phase shift of the head fluctuation in coastal confined aquifers. In

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addition, those dashed lines shown in the figure approach the open circle at large d_1, indicating that the influence of the aquitard on the phase shift also decreases with an increasing d_1. Once the d_1 is larger than the tidal intrusion distance, the effect of the leakage on the phase shift becomes negligible.

3.3 Effect of heterogeneity in aquitard or aquifer

A leaky aquifer system with a heterogeneous aquitard is divided into three regions and the hydraulic conductivity of the aquitard in each region is homogeneous. The underlying confined aquifer is considered to be homogeneous for the investigation of the effect of aquitard heterogeneity. Figure 6 shows the spatial head distribution of the confined aquifer for $a_1=a_2=a_3=5\times10^{-3}$ m^{-1} (homogeneous) for four cases of aquitard conditions. Two cases are designed to have heterogeneous aquitards while the other two cases have homogeneous aquitards but different values of hydraulic conductivity. The normalized hydraulic head is defined as the ratio of the hydraulic head of the confined aquifer to the amplitude of the tide. For case 1, the leakages in regions 1, 2 and 3 are chosen as $u_1=10$, $u_2=5$ and $u_3=1$, respectively. In contrast, the leakages in these three regions are selected as $u_1=1$, $u_2=5$ and $u_3=10$ for case 2. In addition, the leakages are chosen as $u_1=u_2=u_3=1$ for case 3 and $u_1=u_2=u_3=10$ for case 4. The head distribution curve of case 1 is located below that of case 3 and close to that of case 4, indicating that the larger leakage near coastal line (region 1) significantly diminishes the hydraulic head distribution. On the other hand, the curve of case 2 in region 1 is located below that of case 3 even when the leakage in region 1 of case 2 is the same as that of case 3. This is because the large leakages in regions 2 and 3 will influence the hydraulic head distributions in regions 1 and 2. Accordingly, the aquitard heterogeneity has significant effect on the hydraulic head distribution.

Alluvial fans formed at the base of mountains usually have coarser sediment at the upper part of the fan and finer sediment near the coastal area. The formation of the coastal leaky aquifer may therefore exhibit the phenomenon of trending heterogeneity (Freeze and Cherry, 1979). The leaky aquifer system is divided into three regions

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for simulating trending heterogeneity of the aquifer (i.e., T_1=10 m^2/d, T_2=50 m^2/d and T_3=100 m^2/d). The leakages in these three regions are chosen the same for assessing the effect of aquifer heterogeneity. Figure 7 shows the spatial head distributions for both the homogeneous aquifer and heterogeneous trending aquifer at high-tide and middle-tide conditions. The following parameters are used: $u_1=u_2=u_3=5$, $S_1=S_2=S_3=10^{-4}$, d_1=100 m, d_2=200 m, $\omega=2\pi/0.5$ d^{-1} and $T_1=T_2=T_3$=50 m^2/d for the homogeneous aquifer. The figure indicates that the heterogeneous trending aquifer has a smaller tidal intrusion distance than the homogeneous one. This is because the region 1 has a smaller hydraulic conductivity. In addition, the slopes of the normalized head distribution change at x=100 m and x=200 m due to the changes in hydraulic conductivity. In order to satisfy the continuity of the flux at the interface, region 1 has a smaller T_1 and larger hydraulic gradient (slope). Obviously, the aquifer heterogeneity has an impact on the hydraulic head distribution.

3.4 Comparison of present solution with observed data

Figure 8 shows the geological section of the leaky aquifer system in the CLK airport, Hong Kong (Jiao and Tang, 1999). The lower aquifer lies on the bedrock composed of granite as the impermeable boundary and the upper aquifer of fill materials such as gravel and sand is an unconfined aquifer. The aquitard in between mainly consists of clay. The vertical dashed line shown in the figure divides this aquifer system into two regions, where the thickness of the aquitard is spatial-variant. The thickness of the aquitard in the region 1 is about a half of that in the region 2; therefore, the leakage in the region 1 is about two times that in the region 2. The lower confined aquifer is assumed homogeneous.

Jiao and Tang (1999) applied their solution to simulate the head fluctuations for the coastal aquifer system in the CLK airport. The observed well was located at 271 m distance from the coastal line. The confined aquifer was considered to be homogeneous ($a_1=7.65\times10^{-3}$ m^{-1}). The thickness of the aquitard was assumed constant in

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the horizontal direction and the leakage is chosen as u_1=9.38×10^{-3}. The amplitude and frequency of the tide were chosen as A=0.8 m and $\omega=2\pi/0.5\,d^{-1}$, respectively.

Figure 9 shows the observed data taken from the aquifer system in the CLK airport, Hong Kong and the present solution computed for d_1=300 m, $a_1=a_2=7.65\times10^{-3}\,m^{-1}$, $\omega=2\pi/0.5\,d^{-1}$, A=0.8 m, $u_1=9.38\times10^{-3}$ and $u_2=0.5u_1$. This figure shows that the present solution is identical to Jiao and Tang's solution (1999) and has a good agreement with the observed data. Such a result can be attributed to the fact that the leakages are very small and the length of the thin aquitard exceeds the tidal intrusion distance (150 m), which is estimated based on Eq. (18).

3.5 A hypothetical case for aquifer with high transmissivity

Jiao and Tang (1999) estimated the hydraulic diffusivity T/S of $10.6\times10^4\,m^2$/d for the leaky aquifer system in the CLK airport based on the measured data and their analytical solution. Accordingly, T will be 10.6 m^2/d if S is 1×10^{-4}. The coastal aquifer usually consists of sandy materials which may have the transmissivity T ranges from 0.1 to $1.0\times10^4\,m^2$/d (Freeze and Cherry, 1979) if the thickness of aquifers is 10 m. Assume that the aquifer material of the aquifer in the CLK airport is mainly the coarse sand with T=250 m^2/d. Under this circumstance, the tidal intrusion distance in the aquifer in the CLK airport will be 2000 m which is much larger than the thin aquitard length (300 m). Figure 10 shows the predicted hydraulic head fluctuations by Jiao and Tang's solution (1999) for T_1=250 m^2/d, $S_1=10^{-4}$, as well as u_1=5 and the present solution for $T_1=T_2$=250 m^2/d, u_1=5, u_2=2.5 and $S_1=S_2=10^{-4}$ while the other parameters are kept the same as those used in plotting Fig. 9. The difference between Jiao and Tang's solution (1999) and present solution reflects the effect of the change in aquitard leakage. Define the relative difference as

$$R = \left| \frac{h_J - h_P}{h_P} \right| \times 100\% \tag{23}$$

where h_J and h_P represent Jiao and Tang's solution (1999) and present solution, respectively. The maximum relative difference is about $R \cong 8\%$ in the hydraulic head. Therefore, their solution, without considering the change in aquitard leakage, gives good results only when the leakages are very small or the length of the aquitard is larger than the tidal intrusion distance.

4 Concluding remarks

Based on the direct Fourier method, a one-dimensional analytical model is developed for describing head fluctuations due to the tide in a coastal leaky aquifer system. The underlying confined aquifer and aquitard are considered to be heterogeneous and divided into several regions. Each region has a homogenous hydraulic conductivity in the confined aquifer and aquitard. The water table of the upper unconfined aquifer is assumed constant while the hydraulic head of the lower confined aquifer is subject to the effect of tidal fluctuation. The solution developed by Jiao and Tang (1999) can be considered as a special case of the present solution when $N=1$ or the length of the first aquifer is larger than the tidal intrusion distance. With considering the change in aquitard thickness, the present solution has a good agreement with the observed data from the aquifer system in the CLK Airport, Hong Kong (Jiao and Tang, 1999). Based on the present solutions, the effects of formation heterogeneity as well as the length and location of the aquitard on head fluctuations of the confined aquifer can be observed as follows:

1. The aquitard with a large leakage and/or long length near the coastal line will diminish the amplitude of the head fluctuation in the confined aquifer within the tidal intrusion distance.
2. The phase shift of the head fluctuation in the underlying confined aquifer increases slowly with the inland distance for the case of an aquitard of large leakages and dramatically for the case of no leakage.

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3. A heterogeneous trending confined aquifer may possibly produce a smaller tidal intrusion distance and lower head distribution in a coastal leaky aquifer system.

Acknowledgements. This study was supported by "Aim for the Top University Plan" of the National Chiao Tung University and Ministry of Education, Taiwan and the Taiwan National Science Council under the contract numbers NSC96–2221–E–238–009, NSC97-2221-E-130-011 and NSC98-3114-E-007-015.

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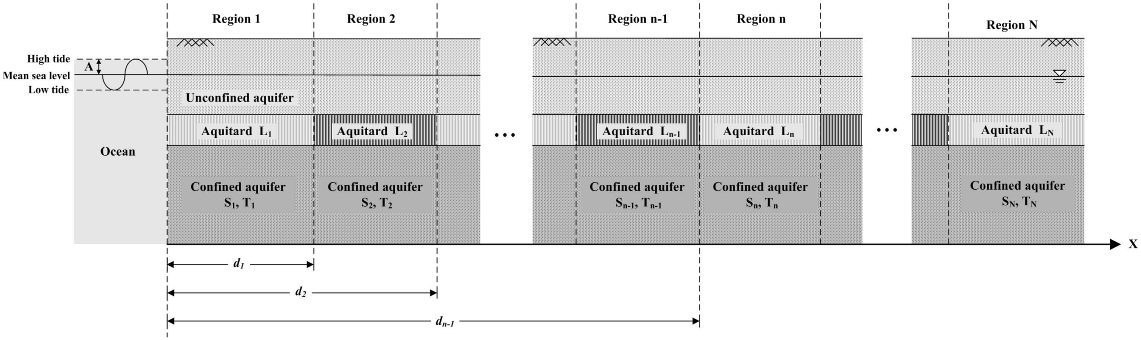

Fig. 1. Schematic diagram of the heterogeneous coastal leaky aquifer system.

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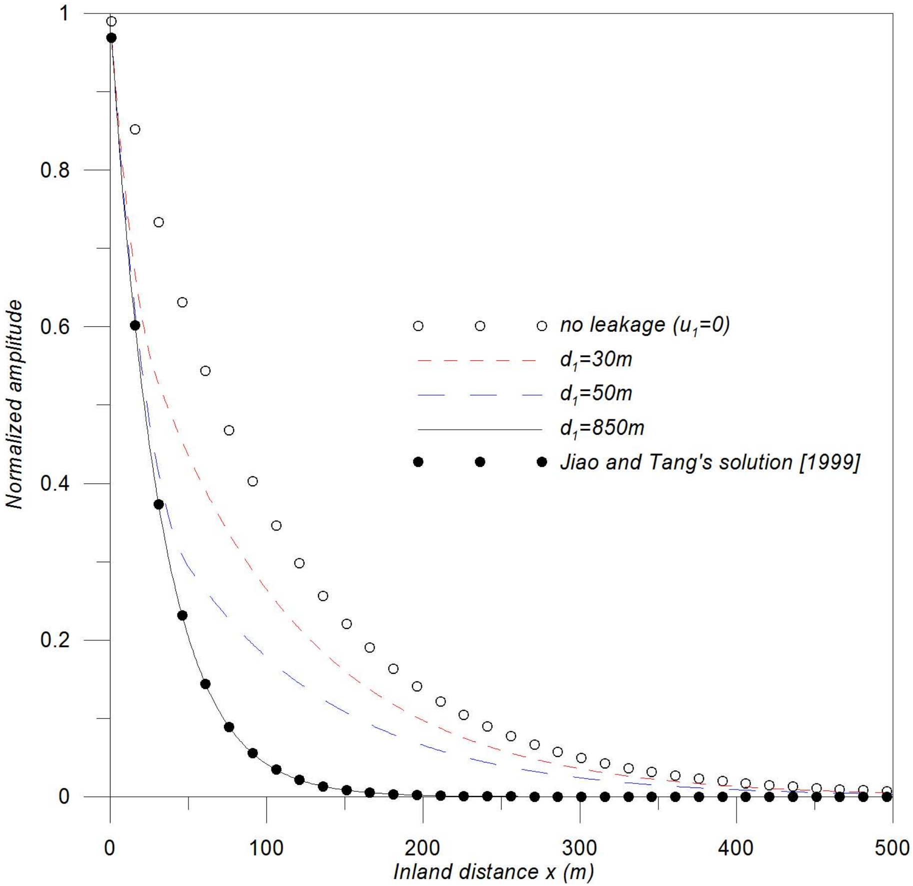

Fig. 2. The normalized amplitude of the head fluctuations versus the inland distance for $u_1=5$, $u_2=0$, $a_1=a_2=1\times10^{-2}\,\mathrm{m}^{-1}$, and various aquitard lengths.

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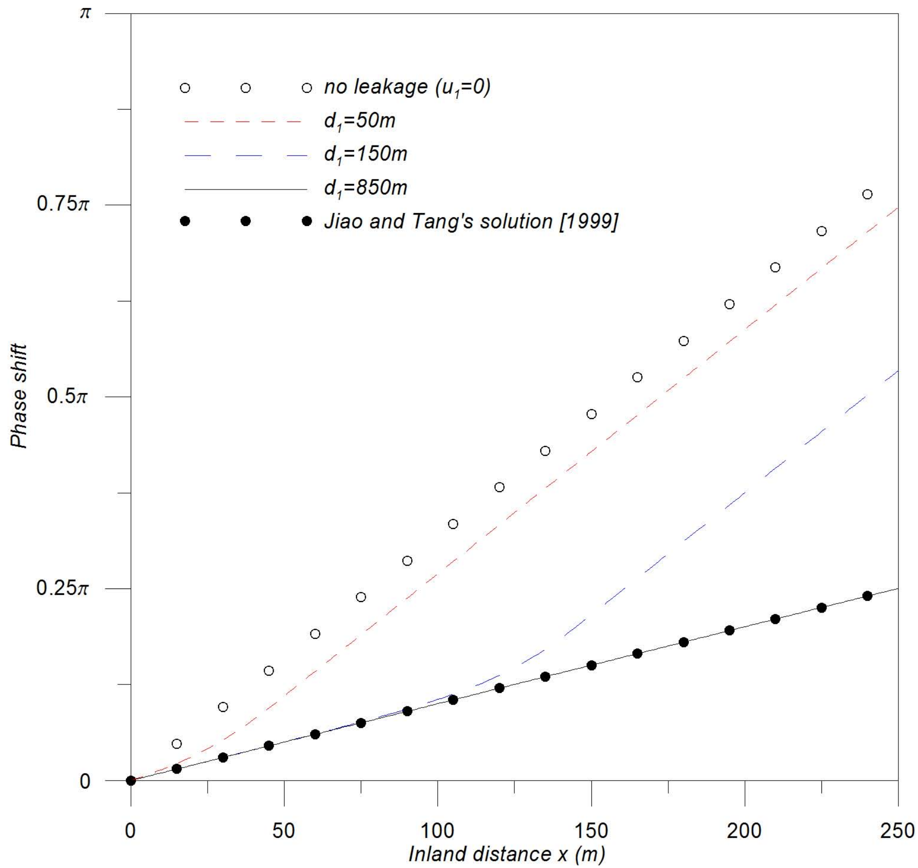

Fig. 3. The phase shift of the head fluctuations versus the inland distance for $u_1=5$, $u_2=0$, and $a_1=a_2=1\times10^{-2}\ \mathrm{m}^{-1}$ and various aquitard lengths.

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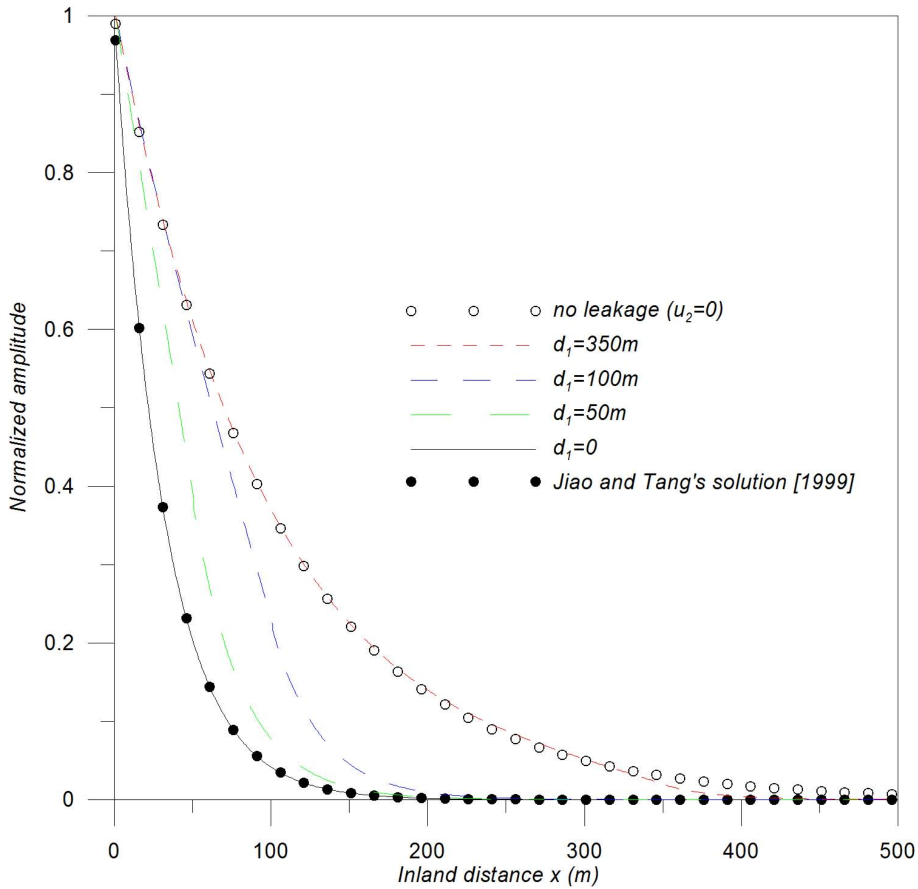

Fig. 4. The normalized amplitude of the head fluctuations versus the inland distance for $u_2=5$, $d_2-d_1=1000$ m, $a_1=a_2=1\times10^{-2}$ m^{-1} and various values of d_1.

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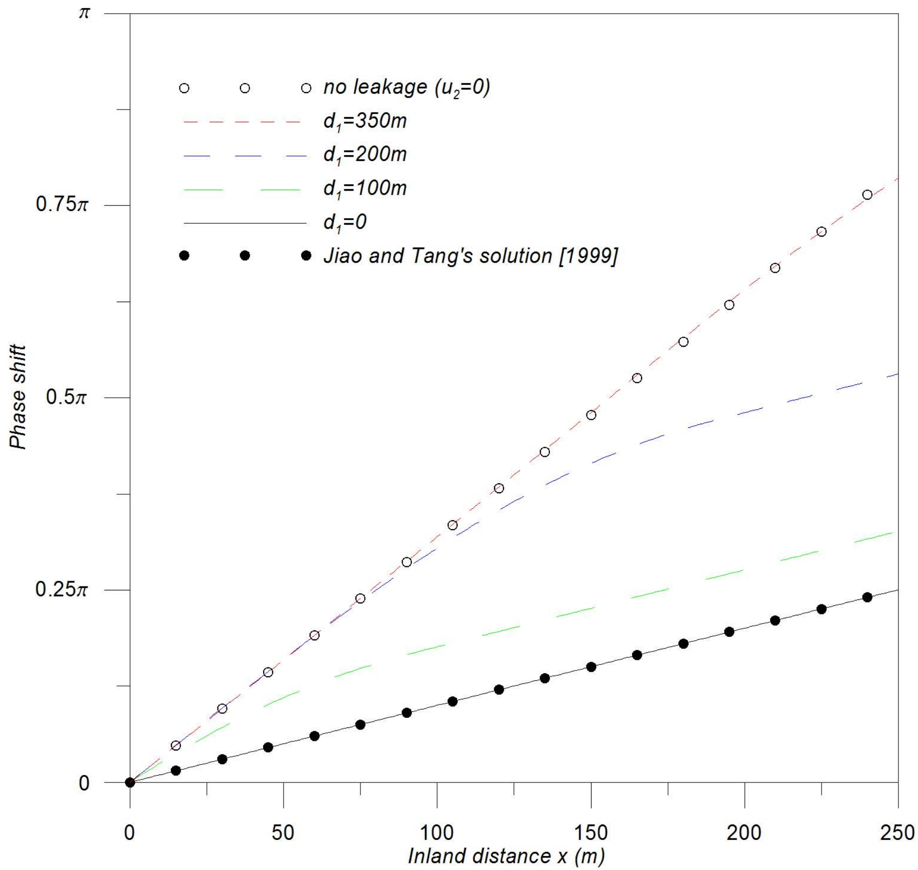

Fig. 5. The phase shift of the head fluctuations versus the inland distance for u_2=5, d_2-d_1=1000 m, $a_1=a_2=1\times10^{-2}$ m^{-1} and various values of d_1.

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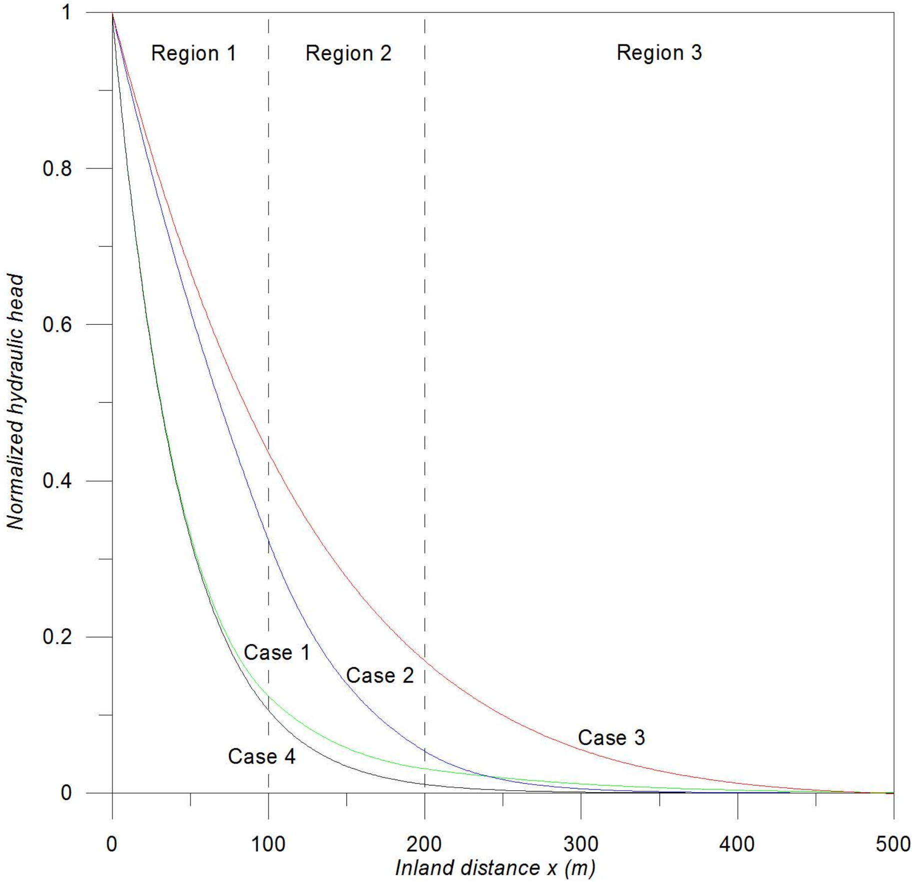

Fig. 6. The spatial head distribution in the homogeneous confined aquifer for cases 1 and 2 of heterogeneous trending aquitards and cases 3 and 4 of homogeneous aquitards but different conductivity values at high tide condition.

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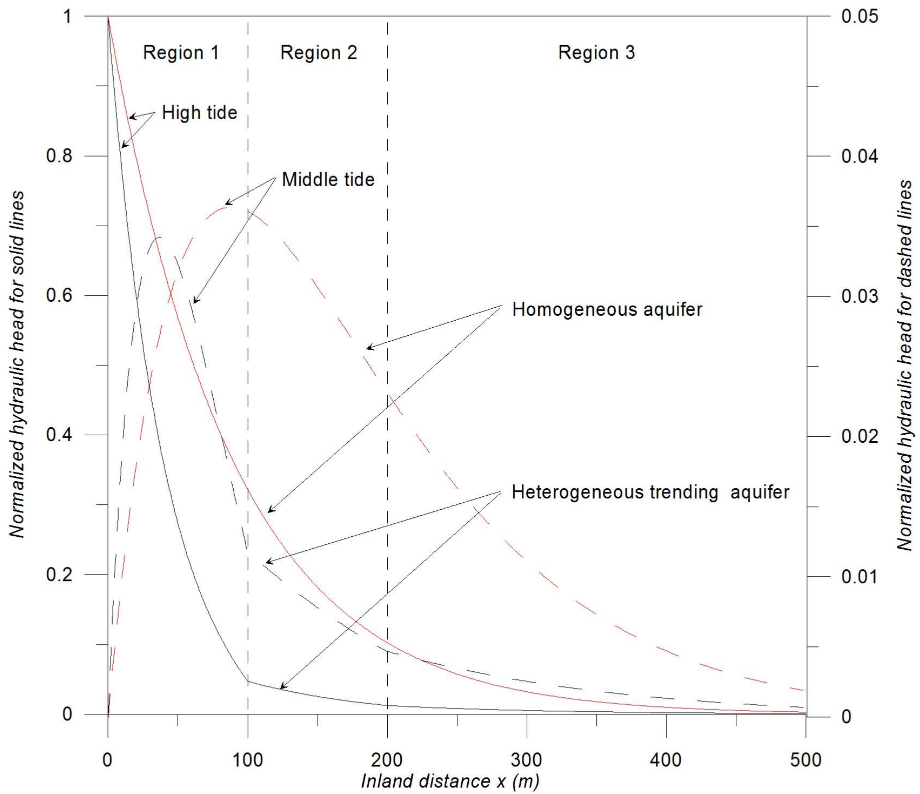

Fig. 7. The spatial head distribution for the homogeneous aquifer and heterogeneous trending aquifer at high and middle tide period.

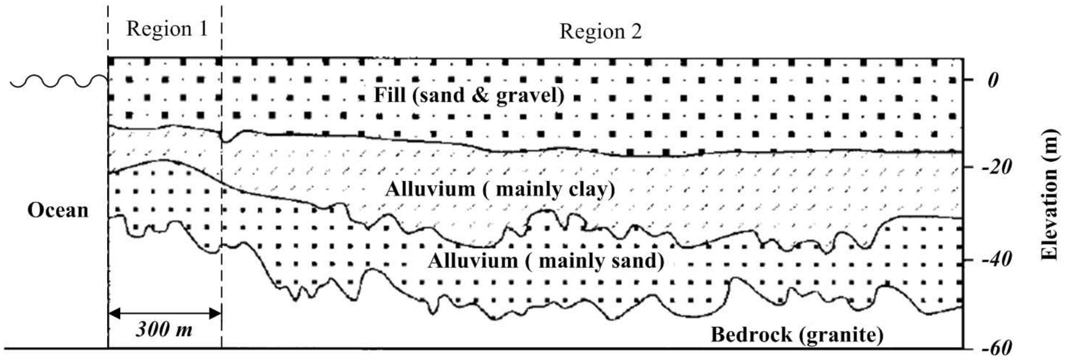

Fig. 8. Geological sections in the CLK Airport, Hong Kong (modified from Jiao and Tang, 1999).

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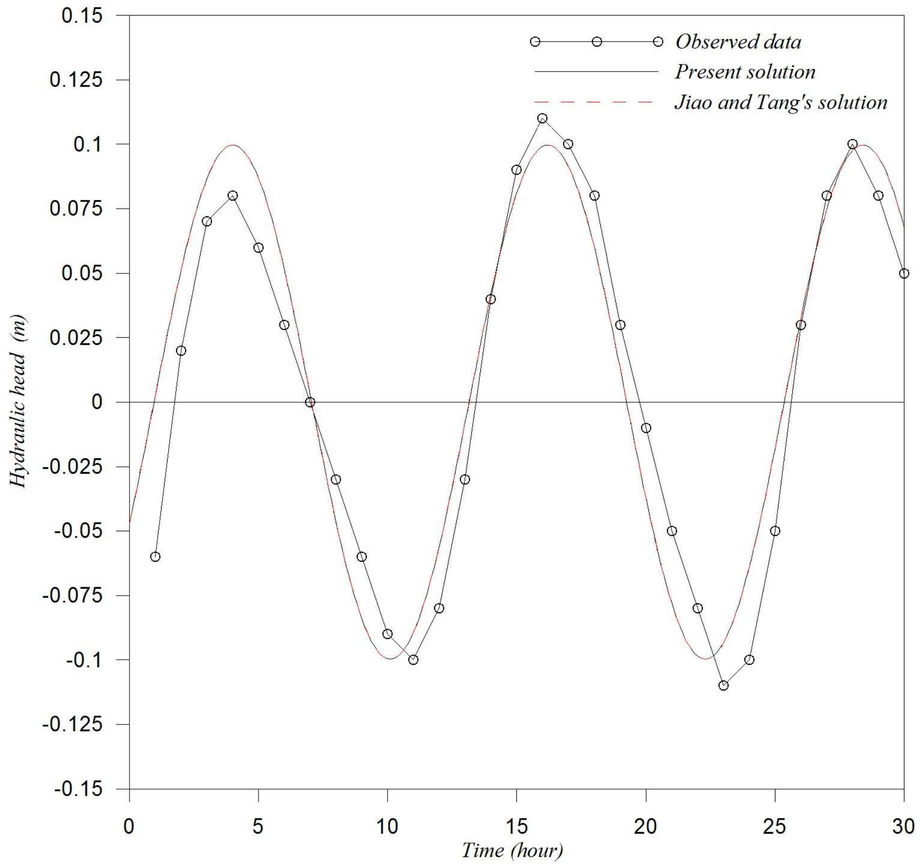

Fig. 9. Predicted hydraulic heads by the present solution and Jiao and Tang's solution (1999) and the observed data measured in the CLK airport, Hong Kong.

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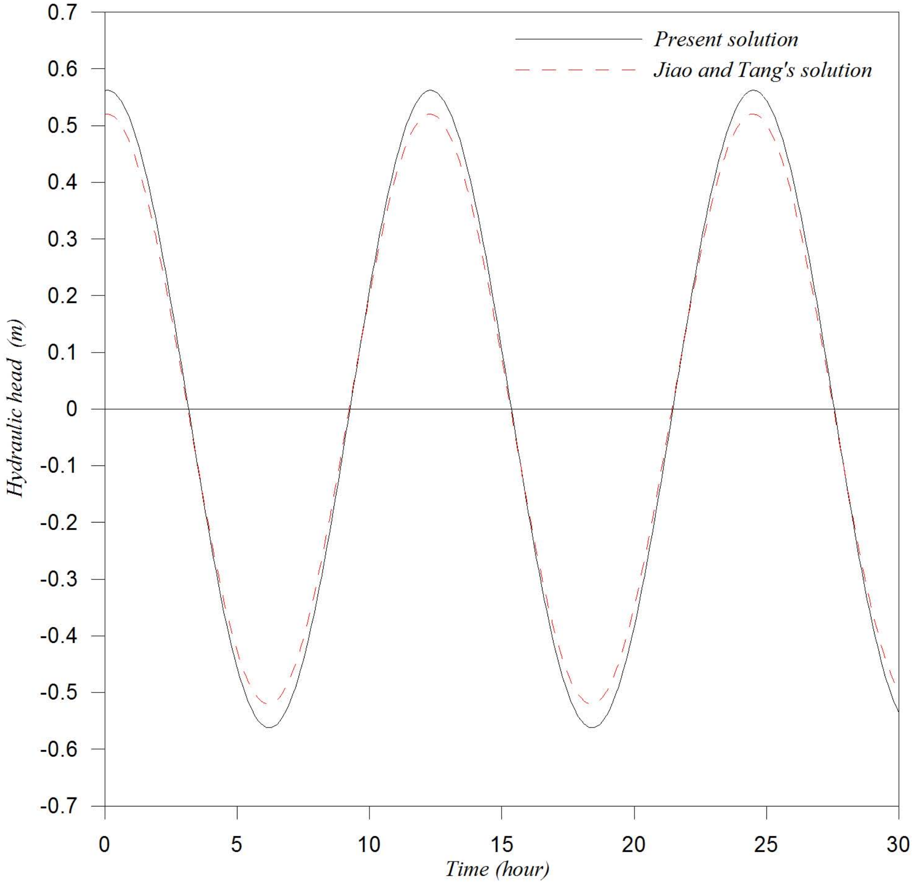

Fig. 10. Predicted hydraulic heads by the present solution and Jiao and Tang's solution (1999).

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