**Hydrology and
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Tidal propagation in an oceanic island with sloping beaches

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Abstract

In this study, a new analytical solution for describing the tide-induced groundwater fluctuations in oceanic islands with finite length and different slopes of the beaches is developed. Unlike previous solutions, the present solution is not only applicable for a semi-infinite coastal aquifer, but also for an oceanic island with finite length and different sloping beaches. The solution can be used to investigate the effect of higher-order components and beach slopes on the water table fluctuations. The results demonstrate the effect of higher-order components increases with the shallow water parameter or amplitude parameter and the water table level increases as beach slopes decrease.

1 Introduction

Groundwater near the ocean usually fluctuates with the tides, which will significantly affect the costal processes such as saltwater intrusions, beach sediment transportations, chemical transformations, and biological activities. To understand and manage the behavior of coastal aquifers, it is required to accurately predict the dynamic groundwater hydraulics. Most studies for groundwater fluctuations in costal aquifers are based on Boussinesq equation with Dupuit assumption (Freeze and Cherry, 1979). Dagan (1967) first solved the non-linear governing equation and approximated the solution by an expansion based on a shallow flow approximation. His solution is applicable when the oscillation of the groundwater motion is small compared with the mean water level. Parlange et al. (1984) extended the work of Dagan (1967) and used a perturbation technique to derive an approximate solution of the non-linear equation. Their solution demonstrates that the nonlinear effects on tidal propagation are not negligible and the omission of the effects may lead to a significant error in predicting water table elevation. Previous studies for water table fluctuations in costal aquifers considered the case of vertical beach. For more realistic case of a sloping beach, Nielsen (1990) considered a movable shoreline boundary condition and derived an analytical solution

for groundwater fluctuations in costal aquifers with a sloping beach. Li et al. (2000) further overcame the inconsistency of the boundary condition in Nielsen's solution and utilized the same parameter in Nielsen (1990) to develop a solution for the problem using a concept of moving boundary. Since the slope of the beach was included in
5 the perturbation parameter in both models, their model may be applicable to certain range of the beach slope. Based on the reason mentioned above, Teo et al. (2003) used two perturbation parameters, shallow water parameter and amplitude parameter, to derive a higher-order solution for the tide-induced water table fluctuations in coastal aquifers to a sloping beach. They considered the costal aquifers with infinite extension
10 in horizontal direction; however, for oceanic islands, the horizontal domain is finite and the beach slopes may be different in both sides of the oceanic island.

    Considering the oceanic island in coastal area, Jiao et al. (2001) investigated impact of land reclamation on groundwater flow systems. They derived steady-state solutions to show how the groundwater level, groundwater divide and submarine groundwater
15 discharge would change with land reclamation near coastal aquifer. Hu et al. (2008) extended the work in Jiao et al. (2001) to develop the transient solution for groundwater flow induced by land reclamation in oceanic land. Rotzoll et al. (2008) presented an analytical solution derived from one-dimensional confined flow equation for hydraulic head distribution in a finite-length and asynchronous dual-tide aquifer. They analyzed
20 the tidal responses in the unconfined central Maui Aquifer and estimated the hydraulic parameters in the study area. Sun et al. (2008) considered an island aquifer system comprising a confined aquifer and an overlying semipermeable layer. They derived an analytical solution for groundwater head response in the island aquifer system subject to dual-tide and compared the solution with the existing analytical solutions.

25   Motivated by the literatures mentioned above, the objective of this study is to develop a model for describing the tide-induced groundwater fluctuations in unconfined aquifers, which, to the best of our knowledge, have never been presented for the case of oceanic islands with dual-tide and different slopes of the beaches.

2 Mathematical model and analytical solution

2.1 Mathematical statement

The configuration of tidal induced groundwater flow is shown in Fig. 1. Assuming the fluid is incompressible and inviscid, the potential head ϕ satisfies the Laplace's equation

$$\phi_{xx} + \phi_{zz} = 0, \quad 0 \le z \le h(x,t) \tag{1}$$

where $h(x,t)$ is the total tidal induced water table height. The tidal oscillations cause two moving boundary conditions at the coasts and can be expressed as

$$h(x_0(t),t) = D(1 + \alpha \cos \omega t) \quad \text{on} \quad x_0(t) = A \cot \beta_1 \cos \omega t \tag{2}$$

and

$$h(\lambda - x_\lambda(t),t) = D(1 + \alpha \cos \omega t) \quad \text{on} \quad x_\lambda(t) = A \cot \beta_2 \cos \omega t \tag{3}$$

for left-hand side (LHS) and right-hand side (RHS) of the oceanic island, respectively. The length of the oceanic island is denoted as λ in Fig. 1.

The $x_0(t)$ and $x_\lambda(t)$ are the horizontal extent of the tidal variation at the sloping beaches, the beach angles for LHS and RHS of the island are, respectively, denoted as β_1 and β_2. The dimensionless amplitude parameter $\alpha = A/D$ represents the ratio of the maximum tidal amplitude, A, to the average water table height, D, and ω is the tidal frequency. At the bottom of the island, the boundary condition is

$$\frac{\partial \phi}{\partial z} = 0, \quad z = 0 \tag{4}$$

The boundary at the water table can be represented as

$$\phi = h, \quad z = h \tag{5}$$

and the flow at the water table is modeled by the following equation (Batu, 1998)

$$n_e \frac{\partial \phi}{\partial t} = K \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial z} \right)^2 \right] - K \frac{\partial \phi}{\partial z}, \quad z = h \tag{6}$$

where n_e is the effective porosity and K is the saturated hydraulic conductivity. Both parameters are assumed constants.

2.2 Perturbation approach

The governing equation and boundary conditions are rewritten in dimensionless forms using the following non-dimensional variables

$$X = \frac{x}{L}, \quad Z = \frac{z}{D}, \quad H = \frac{h}{D}, \quad \Phi = \frac{\phi}{D}, \quad \varepsilon = \frac{D}{L}, \quad \lambda^* = \frac{\lambda}{L}, \quad T = \omega t \tag{7}$$

where $L = \sqrt{\frac{KD}{n_e \omega}}$ is a decay length scale of water table fluctuations and ε is defined as the shallow water parameter. The governing Eq. (1) becomes

$$\Phi_{ZZ} = -\varepsilon^2 \Phi_{XX} \tag{8}$$

and the boundary conditions (Eqs. 4–6) lead to

$$\Phi_Z = 0, \quad Z = 0 \tag{9}$$

$$\Phi = H, \quad Z = H \tag{10}$$

and

$$2\varepsilon^2 \Phi_T = \varepsilon^2 \Phi_X^2 + \Phi_Z^2 - \Phi_Z \tag{11}$$

where Φ_X, Φ_Z and Φ_T represent the first derivatives of Φ with respect to X, Z and T, respectively. In addition, Φ_{XX} and Φ_{ZZ} represent the second derivatives of Φ with respect to X and Z, respectively.

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The boundary conditions in Eqs. (2) and (3), respectively, becomes

$$H(X_0(T),T) = 1 + \alpha \cos T \quad \text{on} \quad X_0(T) = \alpha \varepsilon \cot \beta_1 \cos T \tag{12}$$

and

$$H(\lambda^* - X_\lambda(T),T) = 1 + \alpha \cos T \quad \text{on} \quad X_\lambda(T) = \alpha \varepsilon \cot \beta_2 \cos T \tag{13}$$

By introducing the new variables (Li et al., 2000; Teo et al., 2003)

$$X_1 = X - X_0(T) \quad \text{and} \quad T_1 = T \tag{14}$$

Then

$$\frac{\partial f}{\partial T} = \frac{\partial f}{\partial T_1} + \frac{\partial f}{\partial X_1}\frac{\partial X_1}{\partial T} = \frac{\partial f}{\partial T_1} + \alpha \varepsilon \cot \beta_1 \sin T_1 \frac{\partial f}{\partial X_1} \tag{15}$$

and Eqs. (12) and (13) can be, respectively, transformed to

$$H(X_L(T_1),T_1) = 1 + \alpha \cos T_1 \quad \text{on} \quad X_L(T_1) = 0 \tag{16}$$

and

$$H(X_R(T_1),T_1) = 1 + \alpha \cos T_1 \quad \text{on} \quad X_R(T_1) = \lambda^* - \alpha \varepsilon \cot \beta_2 \cos T_1 - \alpha \varepsilon \cot \beta_1 \cos T_1 \tag{17}$$

where X_L and X_R denote the moving boundary on the LHS and RHS of the island.

Assuming that the potential head Φ and water table level H can be expanded in powers of ε, respectively, as

$$\Phi = \sum_{n=0}^{\infty} \varepsilon^n \Phi_n \tag{18a}$$

and

$$H = \sum_{n=0}^{\infty} \varepsilon^n H_n \tag{18b}$$

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The detail of derivation for Eq. (8) with boundary conditions in Eqs. (9) and (10) is listed in Appendix A and the results up to second-order are

$$O(\varepsilon^0): \quad 2H_{0T_1} = (H_0 H_{0X_1})_{X_1} \tag{19a}$$

$$H_0(0,T_1) = H_0(X_R,T_1) = 1 + \alpha \cos T_1 \tag{19b}$$

$$O(\varepsilon^1): \quad 2(H_{1T_1} + \alpha \cot\beta_1 \sin T_1 H_{0X_1}) = (H_0 H_1)_{X_1 X_1} \tag{20a}$$

$$H_1(0,T_1) = H_1(X_R,T_1) = 0 \tag{20b}$$

$$O(\varepsilon^2): \quad 2(H_{2T_1} + \alpha \cot\beta_1 \sin T_1 H_{1X_1}) = \frac{1}{2}\left(H_1^2\right)_{X_1 X_1} + (H_0 H_2)_{X_1 X_1} + \frac{1}{3}\left(H_0^3 H_{0X_1 X_1}\right)_{X_1 X_1} \tag{21a}$$

$$H_2(0,T_1) = H_2(X_R,T_1) = 0 \tag{21b}$$

2.2.1 Zero-order ($O(\varepsilon^0)$) approximation

The perturbation expansion of H_0 in power of α can be expressed as

$$H_0 = 1 + \sum_{n=1}^{\infty} \alpha^n H_{0n} \tag{22b}$$

Equations (19a) and (19b) can be expanded in different order α as:

$$O(\varepsilon^0 \alpha^1): \quad 2H_{01T_1} = H_{01X_1 X_1} \tag{23a}$$

$$H_{01}(0,T_1) = H_{01}(X_R,T_1) = \cos T_1 \tag{23b}$$

$$O(\varepsilon^0 \alpha^2): \quad 2H_{02T_1} = H_{02X_1 X_1} + (H_{01}H_{01X_1})_{X_1} \tag{24a}$$

$$H_{02}(0,T_1) = H_{02}(X_R,T_1) = 0 \tag{24b}$$

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The derivation for the solutions of Eq. (23) is given in Appendix B and the result is

$$H_{01} = e^{-X_1}\cos(T_1 - X_1) + a_1\left[e^{X_1}\cos(T_1 + X_1) - e^{-X_1}\cos(T_1 - X_1)\right]$$
$$+ a_2\left[e^{X_1}\sin(T_1 + X_1) - e^{-X_1}\sin(T_1 - X_1)\right] \tag{25}$$

Similarly, the solution of Eq. (24) can be obtained as

$$H_{02} = -\frac{1}{4}\left(\delta_{11}e^{-2X_1} + \delta_{12}e^{2X_1} + 2\delta_{13}\cos 2X_1 + 2\delta_{14}\sin 2X_1\right)$$
$$-\frac{1}{2}e^{-2X_1}\left[\delta_{16}\cos 2(T_1 - X_1) + \delta_{15}\sin 2(T_1 - X_1)\right]$$
$$-\frac{1}{2}e^{2X_1}\left[\delta_{18}\cos 2(T_1 + X_1) + \delta_{17}\sin 2(T_1 + X_1)\right]$$
$$+ b_1 X_1 + b_2 + c_1 e^{\sqrt{2}X_1}\cos(2T_1 + \sqrt{2}X_1) + c_2 e^{\sqrt{2}X_1}\sin(2T_1 + \sqrt{2}X_1)$$
$$+ c_3 e^{-\sqrt{2}X_1}\cos(2T_1 - \sqrt{2}X_1) + c_4 e^{-\sqrt{2}X_1}\sin(2T_1 - \sqrt{2}X_1) \tag{26}$$

The definitions of symbols a_1, a_2, δ_{11}, δ_{12}, δ_{13}, δ_{14}, b_1, b_2, c_1, c_2, c_3 and c_4 in Eqs. (22) and (26) are given in Table 1. For the case that the costal aquifer has a half domain with $\lambda \to \infty$, a_1 and a_2 approach to zero. Equations (25) and (26) are identical to the solutions in Teo et al. (2003) when X_1 is sufficiently small for zero-order approximation in ε for order α^1 and α^2, respectively.

2.2.2 First-order ($O(\varepsilon^1)$) approximation

For $O(\varepsilon^1)$, H_1 can be expanded as

$$H_1 = \sum_{n=1}^{\infty} \alpha^n H_{1n} \tag{27}$$

The equation and boundary conditions can be arranged as

$$O(\varepsilon^1 \alpha^1): \quad 2H_{11T_1} = H_{11X_1X_1} \tag{28a}$$

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$$H_{11}(0,T_1) = H_{11}(X_R,T_1) = 0 \qquad (28b)$$

$$O(\varepsilon^1 \alpha^2): \quad 2H_{12T_1} + 2\cot\beta_1 \sin T_1 H_{01X_1} = H_{12X_1X_1} + (H_{01}H_{11})_{X_1X_1} \qquad (29a)$$

$$H_{12}(0,T_1) = H_{12}(X_R,T_1) = 0 \qquad (29b)$$

The solution for Eq. (28) is $H_{11}=0$. Substituting H_{01} in Eqs. (22) into (29), the solution of Eq. (29) is

$$
\begin{aligned}
H_{12} = \frac{1}{2}\cot\beta_1 &\left\{ \left[e^{-X_1}(\delta_{21}\cos X_1 - \delta_{22}\sin X_1) + e^{X_1}(\delta_{23}\cos X_1 + \delta_{24}\sin X_1) \right] + (d_1 X_1 + d_2) \right. \\
&+ \left[e^{-X_1}(\delta_{21}\cos(2T_1 - X_1) + \delta_{22}\sin(2T_1 - X_1)) + e^{X_1}(\delta_{23}\cos(2T_1 + X_1) + \delta_{24}\sin(2T_1 + X_1)) \right] \\
&\times 2\left[e^{\sqrt{2}X_1}(f_1\cos(2T_1 + \sqrt{2}X_1) + f_2\sin(2T_1 + \sqrt{2}X_1)) \right. \\
&\left. \left. + e^{-\sqrt{2}X_1}(f_3\cos(2T_1 - \sqrt{2}X_1) + f_4\sin(2T_1 - \sqrt{2}X_1)) \right] \right\}
\end{aligned}
\qquad (30)
$$

The definitions of symbols δ_{21}, δ_{22}, δ_{23}, δ_{24}, d_1, d_2, f_1, f_2, f_3 and f_4 in Eq. (30) are also listed in Table 1. Similarly, for the special case of semi-half costal aquifer, Eq. (30) is identical to the solutions in Teo et al. (2003) for first-order approximation in ε for order α^2.

2.2.3 Second-order ($O(\varepsilon^2)$) approximation

At $O(\varepsilon^2)$, H_2 is expanded to

$$H_2 = \sum_{n=1}^{\infty} \alpha^n H_{2n}$$

Consequently, the equation and boundary conditions in Eq. (21) is further adapted to

$$O(\varepsilon^2 \alpha^1): \quad 2H_{21T_1} = H_{21X_1X_1} + \frac{1}{3}H_{01X_1X_1X_1X_1} \qquad (31a)$$

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$$H_{21}(0,T_1) = H_{21}(X_R,T_1) = 0 \tag{31b}$$

$$O(\varepsilon^2\alpha^2): \quad 2H_{22T_1} + 2\cot\beta_1\sin T_1 H_{11X_1} = H_{22X_1X_1} + (H_{01}H_{21})_{X_1X_1} + \frac{1}{3}H_{02X_1X_1X_1X_1}$$
$$+ (H_{01}H_{01X_1X_1})_{X_1X_1} \tag{32a}$$

$$H_{22}(0,T_1) = H_{22}(X_R,T_1) = 0 \tag{32b}$$

The solutions of the boundary value problems shown above are

$$H_{21} = \frac{1}{2}\Big[g_1 e^{X_1}\cos(T_1+X_1) + g_2 e^{X_1}\sin(T_1+X_1)$$
$$+ g_3 e^{-X_1}\cos(T_1-X_1) + g_4 e^{-X_1}\sin(T_1-X_1)\Big]$$
$$+ \frac{1}{3}\Big\{ -X_1 e^{-X_1}\cos(T_1-X_1) - X_1 e^{-X_1}\sin(T_1-X_1)$$
$$+ (a_1-a_2)X_1\Big[e^{X_1}\cos(T_1+X_1) + e^{-X_1}\cos(T_1-X_1)\Big]$$
$$+ (a_1+a_2)X_1\Big[e^{X_1}\sin(T_1+X_1) + e^{-X_1}\sin(T_1+X_1)\Big]\Big\} \tag{33}$$

$$H_{22} = k_1 e^{\sqrt{2}X_1}\cos(2T_1+\sqrt{2}X_1) + k_2 e^{\sqrt{2}X_1}\sin(2T_1+\sqrt{2}X_1)$$
$$+ k_3 e^{-\sqrt{2}X_1}\cos(2T_1-\sqrt{2}X_1) + g_4 k^{-\sqrt{2}X_1}\sin(2T_1-\sqrt{2}X_1)$$
$$+ \varphi_1 X_1 e^{-2X_1}\cos(2T_1-2X_1) + \varphi_2 X_1 e^{-2X_1}\sin(2T_1-2X_1)$$
$$+ \varphi_3 X_1 e^{2X_1}\cos(2T_1+2X_1) + \varphi_4 X_1 e^{2X_1}\sin(2T_1+2X_1) \tag{34}$$

Table 1 also shows the definitions of symbols g_1, g_2, g_3, g_4, k_1, k_2, k_3, k_4, φ_1, φ_2, φ_3 and φ_4 in Eqs. (33) and (34). The present solutions second-order approximation in ε for order α^1 and α^2 can be reduced to the solutions in Teo et al. (2003) with the same order for a semi-half costal aquifer.

3 Results and discussion

Figure 2 shows the distribution of water table level (H) verse time ($T/2\pi$) for various order solutions at the horizontal distance $X=1$ and $\beta_1=\beta_2=45°$. This figure can be used to investigate the effects of the higher-order components on the water table fluctuations in an oceanic island with sloping beaches. In Fig. 2a, the solutions of water table level for $O(\alpha^2)$, $O(\varepsilon\alpha^2)$ and $O(\varepsilon^2\alpha^2)$ are close when $\alpha=0.2$ and $\varepsilon=0.3$. As demonstrated in Fig. 2b and c, the difference between zero-order and higher-order solutions increases with α or ε and significant differences are observed in Fig. 2d when $\alpha=0.4$ and $\varepsilon=0.5$.

Comparisons of higher-order solution in Teo et al. (2003) and this study are drawn in Fig. 3a and b, respectively, for $T=\pi/4$ and $\pi/2$ with various length of the oceanic island. These figures indicate that the present solution is getting close to Teo et al.'s solution as λ increases for different times. Therefore, the present solution is applicable for describing the water table level at short horizontal distance in a semi-infinite costal aquifer.

Since the beach slopes may have influence on the water table level, the distributions of water table level versus horizontal distance are illustrated in Fig. 4 for different time with various beach slopes. Graphically, the water table level increases as the beach slope decreases. In other words, the beach slope essentially affects the water table level in oceanic islands. The solution for a semi-half costal aquifer in Teo et al. (2003) is a special case of the present solution when the horizontal distance is not large.

4 Concluding remarks

Using the perturbation technique, an analytical solution is developed for describing the tide-induced groundwater fluctuations in oceanic islands with finite length and different slopes of the beaches. Two perturbation parameters, the shallow water parameter ε and the amplitude parameter α, were used in the present model to derive higher-order solution.

The difference between the zero-order and higher-order solution increases with ε or α and a significant difference is observed when both ε and α are large. It is found that the beach slopes significantly influence the tide-induced water table and the water table level increase as beach slopes decreases. The present solution is more general than that of Teo et al. (2003) and is capable of describing the case of a semi-infinite coastal aquifer when the horizontal distance is small.

Appendix A

Substituting Eq. (18a) and (18b) into the governing Eq. (8) and boundary conditions in Eqs. (9) and (10) leads to

$$\Phi_{0ZZ} + \varepsilon\Phi_{1ZZ} + \varepsilon^2\Phi_{2ZZ} + \ldots = -\varepsilon^2(\Phi_{0X_1X_1} + \varepsilon\Phi_{1X_1X_1} + \varepsilon^2\Phi_{2X_1X_1} + \ldots) \quad \text{(A1)}$$

$$\Phi_{0Z} + \varepsilon\Phi_{1Z} + \varepsilon^2\Phi_{2Z} + \ldots = 0 \quad \text{(A2)}$$

and

$$\Phi_0 + \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + \ldots = H_0 + \varepsilon H_1 + \varepsilon^2 H_2 + \ldots \quad \text{(A3)}$$

Equation (A1) can be rearranged in order of ε as:

$$O(\varepsilon^0): \quad \Phi_{0ZZ} = 0 \quad \text{(A4a)}$$

$$\Phi_{0Z} = 0 \quad \text{at} \quad Z = 0 \quad \text{(A4b)}$$

$$\Phi_0 = H_0 \quad \text{at} \quad Z = H \quad \text{(A4c)}$$

$$O(\varepsilon^1): \quad \Phi_{1ZZ} = 0 \quad \text{(A5a)}$$

$$\Phi_{1Z} = 0 \quad \text{at} \quad Z = 0 \quad \text{(A5b)}$$

$$\Phi_1 = H_1 \quad \text{at} \quad Z = H \quad \text{(A5c)}$$

$O(\varepsilon^2):\quad \Phi_{2ZZ} = -\Phi_{0X_1X_1}$ (A6a)

$\Phi_{2Z} = 0 \quad \text{at} \quad Z = 0$ (A6b)

$\Phi_2 = H_2 \quad \text{at} \quad Z = H$ (A6c)

Integrating Eq. (A4a) with respective to Z twice obtains

$\Phi_0 = C_0(X_1, T_1) + C_0^* Z$ (A7)

where C_0 and C_0^* are constants of integration. Based on Eqs. (A4b) and (A4c), one has

$\Phi_0 = H_0$ (A8)

Similarly, we can obtain

$\Phi_1 = H_1$ (A9)

From Eq. (A6a), one can get

$\Phi_2 = C_2(X_1, T_1) + C_2^* Z - \dfrac{Z^2}{2}\Phi_{0X_1X_1}$ (A10)

Substituting Eqs. (A6b) and (A6c) into Eq. (A10) leads to

$H_2 = C_2(X_1, T_1) - \dfrac{H^2}{2}\Phi_{0X_1X_1}$ (A11)

and therefore

$\Phi_2 = C_2(X_1, T_1) + \dfrac{H^2}{2}\Phi_{0X_1X_1} - \dfrac{Z^2}{2}\Phi_{0X_1X_1}$ (A12)

Based on Eq. (15), Φ_T in Eq. (11) can be expressed as

$\Phi_T = \Phi_{T_1} - \Phi_{X_1}\alpha\varepsilon\cot\beta_1\sin T_1$ (A13)

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Therefore, Eq. (11) can be expressed as

$$2(\Phi_{T_1} - \Phi_{X_1}\alpha\varepsilon\cot\beta_1\sin T_1) = \Phi_{X_1}^2 + \frac{1}{\varepsilon^2}\Phi_Z^2 - \frac{1}{\varepsilon^2}\Phi_Z \tag{A14}$$

Substituting Φ in Eq. (18a) and Eqs. (A8), (A9) and (A12) into Eq. (A14) result in the following equation

$$2\left[H_{0T_1} + \varepsilon(H_{1T_1} + \alpha\cot\beta_1\sin T_1 H_{0X_1}) + \varepsilon^2(H_{2T_1} + \alpha\cot\beta_1\sin T_1 H_{1X_1})\right]$$

$$= \left[H_{0X_1}^2 + 2\varepsilon H_{0X_1}H_{1X_1} + \varepsilon^2\left[H_{1X_1}^2 + 2H_{0X_1}(H_{2X_1} + H_0 H_{0X_1}H_{0X_1X_1})\right] + \dots\right]$$

$$+ \frac{1}{\varepsilon^2}\left(\varepsilon^4 H_{0X_1X_1}^2 H_0 + \dots\right)$$

$$+ \frac{1}{\varepsilon^2}\left[\varepsilon^2 H_0 H_{0X_1X_1} + \varepsilon^3(H_1 H_{0X_1X_1} + H_0 H_{1X_1X_1}) + \varepsilon^4(H_2 H_{0X_1X_1} + H_1 H_{1X_1X_1} + H_0 H_{2X_1X_1}\right.$$

$$\left. + H_0 H_0 H_{0X_1}H_{0X_1X_1X_1} + H_0 H_{0X_1}^2 H_{0X_1X_1} + \frac{1}{3}H_0^3 H_{0X_1X_1X_1X_1} + H_0^2 H_{0X_1X_1}^2 + \dots\right)\right] \tag{A15}$$

Then Eq. (A15) can be expressed in terms of different order of ε as shown in Eqs. (19) to (21).

Appendix B

The general solution of Eq. (23a) can be expressed as

$$H_{01} = \Lambda_1\exp((1+i)X_1)\exp(iT_1) + \Lambda_1\exp(-(1+i)X_1)\exp(iT_1) \tag{B1}$$

where Λ_1 and Λ_2 are complex numbers. Since the exponential function can be expressed in forms of triangular function, i.e., $\exp(iz) = \cos z + i\sin z$, Eq. (B1) can then be

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written as

$$H_{01} = a_1 \exp(X_1)\cos(T_1 + X_1) + a_2 \exp(X_1)\sin(T_1 + X_1) + a_3 \exp(-X_1)\cos(T_1 - X_1)$$
$$+ a_4 \exp(-X_1)\sin(T_1 - X_1) \tag{B2}$$

Substituting the boundary at $X_1 = 0$ in Eq. (23b) into Eq. (B2), we have

$$a_3 = 1 - a_1 \tag{B3}$$

and

$$a_4 = -a_2 \tag{B4}$$

Equation (B2) can be therefore expressed as

$$H_{01} = e^{-X_1}\cos(T_1 - X_1) + a_1 \left[e^{X_1}\cos(T_1 + X_1) - e^{-X_1}\cos(T_1 - X_1) \right]$$
$$+ a_2 \left[e^{X_1}\sin(T_1 + X_1) - e^{-X_1}\sin(T_1 - X_1) \right] \tag{B5}$$

Expanding the triangular functions in Eq. (B5) and sorting out in terms of sine and cosine functions, the relationships of the coefficients a_1 and a_2 can be obtained from the boundary condition of Eq. (23b) at $X_1 = X_R$ as

$$2a_1 \cos X_R \sinh X_R + 2a_2 \sin X_R \cosh X_R = 1 - \exp(-X_R)\cos(X_R) \tag{B6}$$

$$-2a_1 \sin X_R \cosh X_R + 2a_2 \cos X_R \sinh X_R = -\exp(-X_R)\sin(X_R) \tag{B7}$$

Furthermore, from Eqs. (B6) and (B7), the coefficients a_1 and a_2 can be solved as

$$a_1 = \frac{\sinh X_R \cos X_R + \exp(-X_R)(\cosh X_R \sin^2 X_R - \sinh X_R \cos^2 X_R)}{\cosh 2X_R - \cos 2X_R} \tag{B8}$$

and

$$a_2 = \frac{\cosh X_R \sin X_R - (\exp(-X_R)/2)(\cosh X_R + \sinh X_R)\sin 2X_R}{\cosh 2X_R - \cos 2X_R} \tag{B9}$$

Based on Eq. (B5), the solution of H_{01} can therefore be expressed in Eq. (25).

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Table 1. Definition of Symbols.

Symbol	Illustration
a_1	$\dfrac{\sinh X_R \cos X_R + e^{-X_R}(\cosh X_R \sin^2 X_R - \sinh X_R \cos^2 X_R)}{\cosh 2X_R - \cos 2X_R}$
a_2	$\dfrac{\cosh X_R \sin X_R - (e^{-X_R}/2)(\cosh X_R + \sinh X_R)\sin 2X_R}{\cosh 2X_R - \cos 2X_R}$
δ_{11}	$(1-a_1)^2 + a_2^2$
δ_{12}	$a_1^2 + a_2^2$
δ_{13}	$a_1 - a_1^2 - a_2^2$
δ_{14}	a_2
δ_{15}	$-2a_2(1-a_1)$
δ_{16}	$(1-a_1)^2 - a_2^2$
δ_{17}	$2a_1 a_2$
δ_{18}	$a_1^2 - a_2^2$
b_1	$\dfrac{1}{4X_R}\left(\delta_{11}e^{-2X_R} + \delta_{12}e^{2X_R} + 2\delta_{13}\cos 2X_R + 2\delta_{14}\sin 2X_R - 1\right)$
b_2	$\dfrac{1}{4}$
c_1	$\dfrac{\Delta_1}{2(\cosh 2\sqrt{2}X_R - \cos 2\sqrt{2}X_R)}$
c_2	$\dfrac{\Delta_2}{2(\cosh 2\sqrt{2}X_R - \cos 2\sqrt{2}X_R)}$
c_3	$\frac{1}{2}(\delta_{16}+\delta_{18}) - c_1$
c_4	$\frac{1}{2}(\delta_{15}+\delta_{17}) - c_2$

Table 1. Continued.

Symbol	Illustration
Δ_1	$\frac{1}{2}(\delta_{15}+\delta_{17})e^{-\sqrt{2}X_R}\sin 2\sqrt{2}X_R(\cosh\sqrt{2}X_R+\sinh\sqrt{2}X_R)$ $+(\delta_{16}+\delta_{18})e^{-\sqrt{2}X_R}(\cosh\sqrt{2}X_R\sin^2\sqrt{2}X_R-\sinh\sqrt{2}X_R\cos^2\sqrt{2}X_R)$ $+\sinh\sqrt{2}X_R\cos\sqrt{2}X_R\left[-(\delta_{15}e^{-2X_R}-\delta_{17}e^{2X_R})\sin 2X_R+(\delta_{16}e^{-2X_R}+\delta_{18}e^{2X_R})\cos 2X_R\right]$ $-\cosh\sqrt{2}X_R\sin\sqrt{2}X_R\left[(\delta_{15}e^{-2X_R}+\delta_{17}e^{2X_R})\cos 2X_R+(\delta_{16}e^{-2X_R}-\delta_{18}e^{2X_R})\sin 2X_R\right]$
Δ_2	$-\frac{1}{2}(\delta_{16}+\delta_{18})e^{-\sqrt{2}X_R}\sin 2\sqrt{2}X_R(\cosh\sqrt{2}X_R+\sinh\sqrt{2}X_R)$ $+(\delta_{15}+\delta_{17})e^{-\sqrt{2}X_R}(\cosh\sqrt{2}X_R\sin^2\sqrt{2}X_R-\sinh\sqrt{2}X_R\cos^2\sqrt{2}X_R)$ $+\sinh\sqrt{2}X_R\cos\sqrt{2}X_R\left[(\delta_{15}e^{-2X_R}+\delta_{17}e^{2X_R})\cos 2X_R+(\delta_{16}e^{-2X_R}-\delta_{18}e^{2X_R})\sin 2X_R\right]$ $+\cosh\sqrt{2}X_R\sin\sqrt{2}X_R\left[-(\delta_{15}e^{-2X_R}-\delta_{17}e^{2X_R})\sin 2X_R+(\delta_{16}e^{-2X_R}+\delta_{18}e^{2X_R})\cos 2X_R\right]$
δ_{21}	$-1+a_1+a_2$
δ_{22}	$1-a_1+a_2$
δ_{23}	a_1+a_2
δ_{24}	$-a_1+a_2$
d_1	$\frac{1}{X_R}\left[\delta_{21}\left(1-e^{-X_R}\cos X_R\right)+\delta_{22}e^{-X_R}\sin X_R+\delta_{23}\left(1-e^{X_R}\cos X_R\right)-\delta_{24}\,e^{X_R}\sin X_R\right]$
d_2	$-(\delta_{21}+\delta_{23})$
f_1	$\dfrac{\Delta_3}{2(\cosh 2\sqrt{2}X_R-\cos 2\sqrt{2}X_R)}$
f_2	$\dfrac{\Delta_4}{2(\cosh 2\sqrt{2}X_R-\cos 2\sqrt{2}X_R)}$
f_3	$-\frac{1}{2}(\delta_{21}+\delta_{23})-f_1$

Table 1. Continued.

Symbol	Illustration
f_4	$-\frac{1}{2}(\delta_{22}+\delta_{24})-f_2$
Δ_3	$-\frac{1}{2}(\delta_{22}+\delta_{24})e^{-\sqrt{2}X_R}\sin 2\sqrt{2}X_R(\cosh\sqrt{2}X_R+\sinh\sqrt{2}X_R)$ $-(\delta_{21}+\delta_{23})e^{-\sqrt{2}X_R}(\cosh\sqrt{2}X_R\sin^2\sqrt{2}X_R-\sinh\sqrt{2}X_R\cos^2\sqrt{2}X_R)$ $+\sinh\sqrt{2}X_R\cos\sqrt{2}X_R\left[\left(\delta_{22}e^{-2X_R}-\delta_{24}e^{2X_R}\right)\sin X_R-\left(\delta_{21}e^{-2X_R}+\delta_{23}e^{2X_R}\right)\cos X_R\right]$ $+\cosh\sqrt{2}X_R\sin\sqrt{2}X_R\left[\left(\delta_{22}e^{-2X_R}+\delta_{24}e^{2X_R}\right)\cos X_R+\left(\delta_{21}e^{-2X_R}-\delta_{23}e^{2X_R}\right)\sin X_R\right]$
Δ_4	$\frac{1}{2}(\delta_{21}+\delta_{23})e^{-\sqrt{2}X_R}\sin 2\sqrt{2}X_R(\cosh\sqrt{2}X_R+\sinh\sqrt{2}X_R)$ $-(\delta_{22}+\delta_{24})e^{-\sqrt{2}X_R}(\cosh\sqrt{2}X_R\sin^2\sqrt{2}X_R-\sinh\sqrt{2}X_R\cos^2\sqrt{2}X_R)$ $-\sinh\sqrt{2}X_R\cos\sqrt{2}X_R\left[\left(\delta_{22}e^{-2X_R}+\delta_{24}e^{2X_R}\right)\cos X_R+\left(\delta_{21}e^{-2X_R}-\delta_{23}e^{2X_R}\right)\sin X_R\right]$ $+\cosh\sqrt{2}X_R\sin\sqrt{2}X_R\left[\left(\delta_{22}e^{-2X_R}-\delta_{24}e^{2X_R}\right)\sin X_R-\left(\delta_{21}e^{-2X_R}+\delta_{23}e^{2X_R}\right)\cos X_R\right]$
g_1	$\dfrac{\Delta_5}{3(\cosh 2X_R-\cos 2X_R)}$
g_2	$\dfrac{\Delta_6}{3(\cosh 2X_R-\cos 2X_R)}$
g_3	$-g_1$
g_4	$-g_2$
Δ_5	$X_R[\cos 2X_R-\cosh 2X_R-(1-2a_1-2a_2)\sin 2X_R+(1-2a_1+2a_2)\sinh 2X_R]$
Δ_6	$X_R[\cos 2X_R-\cosh 2X_R+(1-2a_1+2a_2)\sin 2X_R+(1-2a_1-2a_2)\sinh 2X_R]$
k_1	$\dfrac{\Delta_7}{2(\cos(2\sqrt{2}X_R)-\cosh(2\sqrt{2}X_R))(\cosh\sqrt{2}X_R+\sinh\sqrt{2}X_R)^2}$
k_2	$\dfrac{\Delta_8}{2(\cos(2\sqrt{2}X_R)-\cosh(2\sqrt{2}X_R))(\cosh\sqrt{2}X_R+\sinh\sqrt{2}X_R)^2}$

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Table 1. Continued.

Symbol	Illustration
k_3	$-k_1$
k_4	$-k_2$
φ_1	$\frac{1}{6}\left(2-4a_1+2a_1^2+4a_2-4a_1a_2-2a_2^2\right)$
φ_2	φ_1
φ_3	$\frac{1}{6}\left(-2a_1^2+4a_1a_2+2a_2^2\right)$
φ_4	φ_3
Δ_7	$-(X_R(\cosh((-2+\sqrt{2})X_R)+\sinh((-2+\sqrt{2})X_R))(\varphi_1\cos((-2+\sqrt{2})X_R)$ $-\varphi_3\cos((-2+\sqrt{2})X_R)\cosh(2(2+\sqrt{2})X_R)+\varphi_2\sin((-2+\sqrt{2})X_R)$ $+\varphi_4\sin((-2+\sqrt{2})X_R)\cosh(2(2+\sqrt{2})X_R)+\cosh(2\sqrt{2}X_R)(-\varphi_1\cos((2+\sqrt{2})X_R)$ $+\varphi_2\sin((2+\sqrt{2})X_R))+\cosh(4X_R)(\varphi_3\cos((2+\sqrt{2})X_R)+\varphi_4\sin((2+\sqrt{2})X_R)$ $+\varphi_3\cos((2+\sqrt{2})X_R)\sinh(4X_R)+\varphi_4\sin((2+\sqrt{2})X_R)\sinh(4X_R)$ $-\varphi_1\cos((2+\sqrt{2})X_R)\sinh(2\sqrt{2}X_R)+\varphi_2\sin((2+\sqrt{2})X_R)\sinh(2\sqrt{2}X_R)$ $-\varphi_3\cos((-2+\sqrt{2})X_R)\sinh(2(2+\sqrt{2})X_R)+\varphi_4\sin((-2+\sqrt{2})X_R)\sinh(2(2+\sqrt{2})X_R)$
Δ_8	$-(X_R(\cosh((-2+\sqrt{2})X_R)+\sinh((-2+\sqrt{2})X_R))(-\varphi_2\cos((-2+\sqrt{2})X_R)$ $+\varphi_4\cos((-2+\sqrt{2})X_R)\cosh(2(2+\sqrt{2})X_R)+\varphi_1\sin((-2+\sqrt{2})X_R)$ $+\varphi_3\sin((-2+\sqrt{2})X_R)\cosh(2(2+\sqrt{2})X_R)+\cosh(2\sqrt{2}X_R)(\varphi_2\cos((2+\sqrt{2})X_R)$ $+\varphi_1\sin((2+\sqrt{2})X_R))+\cosh(4X_R)(-\varphi_4\cos((2+\sqrt{2})X_R)+\varphi_3\sin((2+\sqrt{2})X_R)$ $+\varphi_4\cos((2+\sqrt{2})X_R)\sinh(4X_R)+\varphi_3\sin((2+\sqrt{2})X_R)\sinh(4X_R)$ $+\varphi_2\cos((2+\sqrt{2})X_R)\sinh(2\sqrt{2}X_R)+\varphi_1\sin((2+\sqrt{2})X_R)\sinh(2\sqrt{2}X_R)$ $+\varphi_4\cos((-2+\sqrt{2})X_R)\sinh(2(2+\sqrt{2})X_R)+\varphi_3\sin((-2+\sqrt{2})X_R)\sinh(2(2+\sqrt{2})X_R)$
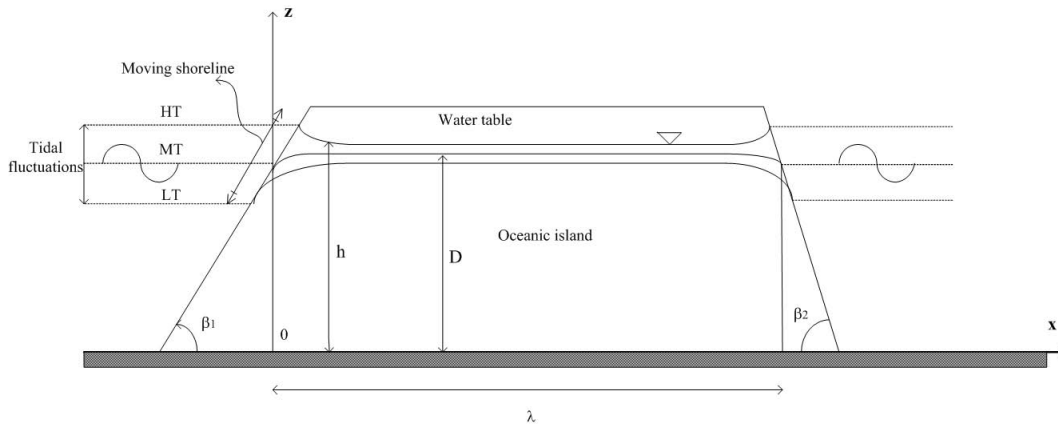
Fig. 1. The profile of tidal water table fluctuations in an oceanic island with sloping beaches.
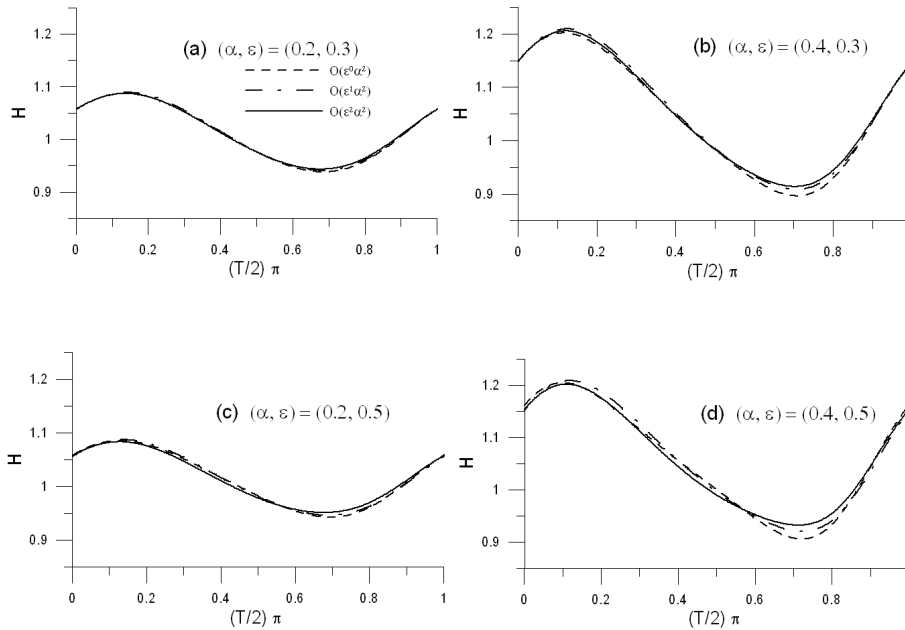
Fig. 2. Distribution of water table level (H) versus time ($T/2\pi$) in an oceanic island at $X=1$ with beach slopes $\beta_1=\beta_2=45°$.

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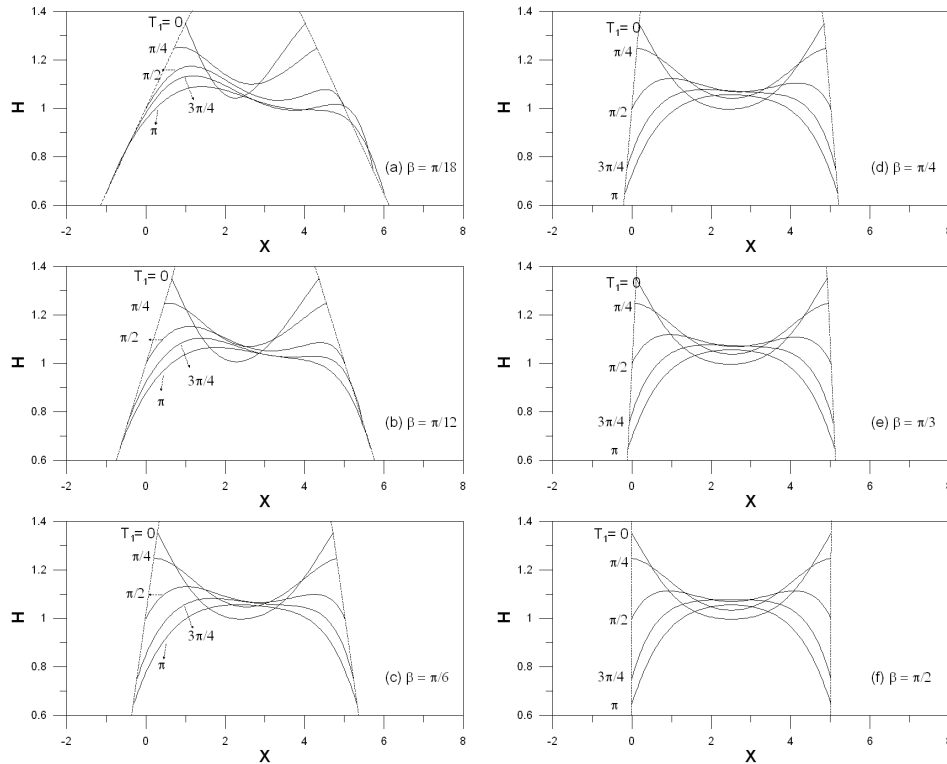| ◀◀	▶▶
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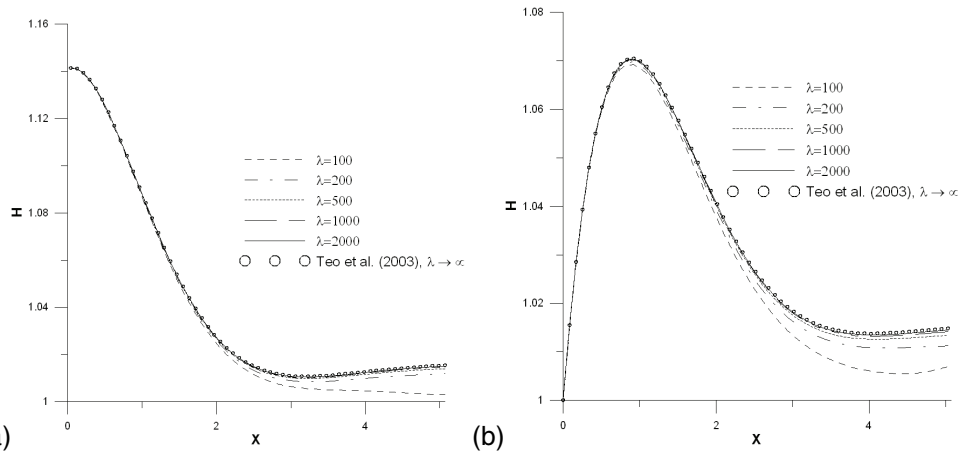
Fig. 3. Comparison of tide-induced water table level (H) in Teo et al. (2003) and the present solution for various length (λ) of the oceanic island ($\varepsilon=0.3$, $\alpha=0.2$, $\beta_1=\beta_2=45°$). **(a)** $T=\pi/4$; **(b)** $T=\pi/2$.

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Fig. 4. Distribution of water table level (H) versus horizontal distance (X) for various beach slopes ($\varepsilon=0.5$, $\alpha=0.35$).