Hydrol. Earth Syst. Sci. Discuss., 6, S285–S292, 2009
www.hydrol-earth-syst-sci-discuss.net/6/S285/2009/


Hydrology and Earth System Sciences Discussions



# *Interactive comment on* "Comment on "Biotic pump of atmospheric moisture as driver of the hydrological cycle on land" by A. M. Makarieva and V. G. Gorshkov, Hydrol. Earth Syst. Sci., 11, 1013–1033, 2007" *by* A. G. C. A. Meesters et al.

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Received and published: 12 March 2009

A key point of the DP consists in the question of how, why and whether the evaporative force can produce different (small and large) wind velocities rather than invariably leading to formation of hurricane wind speeds above any evaporating surface (see, e.g., lines 19-22 on p. 411 in the DP). As we have already mentioned in this discussion (see, e.g., S8, S51, S61-S62), the particular value of velocity produced by the evaporative force cannot be calculated without explicit consideration of turbulent friction forces that oppose the air motion. (The DP authors made no attempt to discuss this issue.)

A possible scheme of how this could be done for the case of stationary circulation was outlined on p. 1026 of Makarieva & Gorshkov (2007), where the power of the evaporative force was equated to the power of turbulent friction. Here we address this issue in greater detail.

A stationary constant value of velocity along some part of the streamline indicates absence of air acceleration, which corresponds to the equality between the pressure gradient force $f_A$ that accelerates air and the force of turbulent friction $f_T$ that opposes the air motion (forces are taken per unit area of the Earth's surface and have the dimension of $\mathrm{N\,m^{-2} = J\,m^{-3} = Pa}$):

$$f_A = \Delta p h_a / L = f_T. \tag{1}$$

Here $\Delta p$ is the horizontal pressure difference observed over distance $L$, $L$ is horizontal dimension of the considered circulation pattern, $\Delta p/L$ is the accelerating pressure gradient force per unit air volume, $h_a$ is characteristic height of the part of atmospheric column where the horizontal wind retains its direction. We will now explore the nature of the turbulent friction force $f_T$.

When considering motion of terrestrial animals and transport one takes into account two types of resistance forces. These are the surface friction force, which is independent of velocity and proportional to the weight of the body, and air resistance, which is proportional to the squared movement velocity $u$. Out of dimensional considerations these two friction terms per unit surface area can be written in the following form (Gorshkov, 1983):

$$f_T = \mu \rho g l + C_D \rho u^2. \tag{2}$$

Here $\rho$ is mass density of the body, $g = 9.8\ \mathrm{m\,s^{-2}}$ is acceleration of gravity, $l$ is the mean height of the body, $\mu \sim 1$ is the dimensionless coefficient of friction of rest, $C_D$ is the dimensionless drag coefficient reflecting the geometry of the moving body (Landau & Lifshitz, 1987, § 45). It is of the order of unity for spherical bodies; in the general case it is proportional to the ratio of the vertical cross-section area of the body to the area of its projection on the Earth's surface, i.e. to ratio $l/L$, where $l$ is height, $L$ is length of the body (Gorshkov, 1983). The second term in Eq. (2) grows up to the first one at velocities when Froude's number $Fr$ conforms to condition

$$Fr \equiv \frac{u^2}{gl} \geq \frac{1}{C_D}. \tag{3}$$

When velocity $u$ starts obeying to condition (3), total friction force $f_T$, Eq. (2), starts growing rapidly proportional to squared velocity. This condition sets the upper limit to transport velocity at which fuel is spent reasonably sparingly, as well as the upper limit to velocity of terrestrial locomotion in animals. E.g., animals give up terrestrial locomotion and switch to flight at $C_D^{-1} \geq 13$ (Gorshkov, 1983; 1984). We emphasize that the two terms in Eq. (2) are characterized by different physics and are independent of each other.

In meteorology turbulent friction is formally represented by the second term of Eq. (2) with drag coefficient $C_D$ being of the order of $10^{-3}$ (Garratt, 1977). Taking a characteristic horizontal pressure gradient on Earth to be in the order of $\Delta p/L \sim 1$ mbar $(100\,\text{km})^{-1} = 1\,\text{Pa}\,\text{km}^{-1}$, height $h_a \sim h_{\text{H}_2\text{O}} \approx 2$ km (e.g., Miller, 1964, Fig. 5), air density $\rho = 1.3\,\text{kg}\,\text{m}^{-3}$, global mean velocity $\overline{u} \sim 7\,\text{m}\,\text{s}^{-2}$ (Gustavson, 1979) and comparing $f_A$ and $f_T$ in Eq. (1) we have:

$$f_A \sim 2\,\text{N}\,\text{m}^{-2} \gg C_D \rho u^2 \sim 0.06\,\text{N}\,\text{m}^{-2}. \tag{4}$$

That is, the accelerating pressure gradient force exceeds the aerodynamic drag force by more than thirty times. This comparison shows that the account of aerodynamic turbulent friction $C_D \rho u^2$ is essentially insufficient for explaining the observed nearly constant horizontal velocities of the order of a few meters per second that can be maintained on the major part of the streamline over horizontal distances of the order several thousand kilometers. For example, air circulation over the Amazon river basin is characterized by wind velocities $u \sim 5\,\text{m}\,\text{s}^{-1}$ maintained practically year round over



HESSD

6, S285–S292, 2009

distances of $L \sim 2 \times 10^3$ km (Zhou & Lau, 1998). This implies that the air masses travel for several days inland at nearly constant velocity before they ascend and reverse the direction of their motion in the upper atmosphere. In the meantime, the force imbalance implied by Eq. (4) would correspond to an unrealistic acceleration $a \sim \Delta p/(L\rho) \sim 1.5 \times 10^{-4}$ m s$^{-2}$ $\approx 13$ m s$^{-1}$ day$^{-1}$ even if a conservative estimate of horizontal pressure gradient $\Delta p/L \sim 0.2$ Pa km$^{-1}$ (10 hPa over 5 thousand kilometers) is applied.

The problem can be solved by the proposition (Makarieva et al. 2009 ACPD 8: S8904, http://www.cosis.net/copernicus/EGU/acpd/8/S8904/acpd-8-S8904.pdf; Makarieva & Gorshkov 2009 HESSD 6: S59, http://www.cosis.net/copernicus/EGU/hessd/6/S59/hessd-6-S59.pdf) that there is another term in the cumulative friction force $f_T$ that is analogous to the first term in Eq. (2) and independent of horizontal velocity $u$. Total friction force can then be written as

$$f_T = f_{T0} + f_{Ta}, \quad f_{T0} = \rho g z_T, \quad f_{Ta} = C_D \rho u^2. \tag{5}$$

Here $f_{Ta}$ characterizes the aerodynamic turbulent friction in the lower part atmospheric column $zh_a$, while $f_{T0}$ represents surface turbulent friction and $z_T$ is the vertical scale of surface roughness. Let us discuss this term in greater detail.

Air pressure $p$ at the Earth's surface is approximately equal to the weight of atmospheric column. According to the equation of state it can be written as $p = \rho g h = 10^5$ J m$^{-3}$ (1 J m$^{-3}$ = 1 N m$^{-2}$), $h = RT/Mg = 8.4$ km is the atmospheric exponential scale height, $R = 8.3$ J mol$^{-1}$ K$^{-1}$ is the universal gas constant, $M = 29$ g mol$^{-1}$ is molar mass of air. Due to surface roughness, horizontal air movement leads to formation of turbulent eddies at the surface. The energy of these eddies is substracted from the kinetic energy of the mean horizontal air flow, resulting in its slow down. Interaction of turbulent eddies with each other and with the surface leads to their dissipation into yet smaller eddies and ultimately to dissipation to heat. Energy of the turbulent eddies is proportional to atmospheric pressure and to linear size $z_T$ of surface roughness. By



HESSD

6, S285–S292, 2009

Interactive Comment

analogy to the first term in Eq. (2), $f_{T0}$ can then be written as

$$f_{T0} = \rho g z_T = \mu \rho g h, \quad \mu \equiv z_T/h. \tag{6}$$

Since on global average $f_T = f_A \gg C_D \rho u^2$, Eq. (4), this means that $f_A \sim \rho g z_T$. Eq. (4) in this case gives $z_T \sim 0.2$ m and $\mu \sim 2 \times 10^{-5}$. In other words, compared to the case of the motion of solid bodies along a rough surface that is characterized by $\mu \sim 1$, Eq. (2), surface turbulent friction for air moving along a rough surface is vanishingly small (i.e., if the atmospheric column of the same weight were a solid body, its movement along the Earth's surface would be characterized by $\mu \sim 1$ instead of $\mu \sim 2 \times 10^{-5}$). However, the absolute value of surface turbulent friction $f_{T0}$ is large enough for it to be the major source of resistance in large-scale circulation patterns where horizontal wind velocities of the order of several meters per second are maintained along the major part of the horizontal streamline of the order of $10^3$ km:

$$f_A = f_T \approx f_{T0} \gg f_{Ta}. \tag{7}$$

Note that $z_T$ is not related to the vertical profile of wind velocity (logarithmic or any other) near the surface, but, instead, represents an independent linear scale corresponding to the actual linear size of surface inhomogeneities. Therefore, if one chooses an area with $z_T$ much smaller than the mean value for which Eq. (7) holds, then over such a surface term $f_{T0}$ may become negligible. Then the only term that can be empirically measured on such an area will be $f_{Ta}$ (see, e.g., Sheppard, 1947 on $C_D$ measurements on a smooth concrete field with $z_T \sim 10^{-3}$ m $\ll 0.2$ m).

Let us now explore the nature of the aerodynamic turbulent friction in the atmospheric column, $f_{Ta} = C_D \rho u^2$. It can be written in the standard form involving turbulent kinematic viscosity $\nu_T$ as $f_{Ta} = \nu_T du/dz$, where $z$ is height above the surface. Assuming that $\nu_T = h_{H_2O} w$ (Makarieva & Gorshkov, 2007), where $w$ is vertical wind velocity and $h_{H_2O} \approx 2$ km is the exponential scale height of the water vapor distribution, and taking into account that $du/dz \sim u/h_a \sim u/h_{H_2O}$, we have $f_{Ta} = \rho w u$. Integral continuity



HESSD

6, S285–S292, 2009

Interactive Comment

equation for circulation of horizontal length $L$ and height $h_a$ reads as $uh_a = wL$. From this equation and considering that, on the other hand, $f_{Ta} = C_D \rho u^2$, we derive an order of magnitude estimate for drag coefficient

$$C_D = ch_a/L, \quad c \sim 1. \tag{8}$$

This theoretical estimate agrees, in the order of magnitude, with the available observational evidence. At global mean velocity $\overline{u} = 7$ m s$^{-1}$ the observations give a mean $C_D \approx 1.2 \times 10^{-3}$ (Garratt, 1977). Taking mean cyclone radius $L \approx 600$ km (Simmonds, 2000), we obtain from Eq. (8) $c \approx 0.5$ and

$$C_D = h_a/2L. \tag{9}$$

Aerodynamic turbulent friction $f_{Ta} = C_D \rho u^2 = \rho uw/2$ can thus be interpreted as kinetic energy of turbulent eddies that are characterized by mean velocity $v_e = \sqrt{uw}$.

Equation (1) can now be re-written as

$$f_A = f_T = f_{T0} + f_{Ta}, \quad \Delta p h_a/L = \rho g z_T + (h_a/2L)\rho u^2. \tag{10}$$

From Eq. (10) one can see that $C_D$ (9) grows with decreasing linear size of the circulation and with increasing velocity $u$:

$$C_D = h_a/2L = \rho g z_T/(2\Delta p - \rho u^2). \tag{11}$$

If at $\Delta p \sim 10$ hPa $= 10^3$ J m$^{-3}$, $z_T = 0.2$ m and $u = 7$ m s$^{-1}$ it is equal to $C_D = 1.3 \times 10^{-3}$, at $u = 30$ m s$^{-1}$ it grows nearly twice to $C_D = 2.3 \times 10^{-3}$. This magnitude of change agrees very well with the available independent empirical estimates of $C_D$ (Garratt, 1977, Fig. 4). Note also that by measuring $\Delta p$, $u$ and $C_D$ it is in principle possible to determine $z_T$ from Eq. (11).



HESSD

6, S285–S292, 2009

Interactive Comment

Further on, Eq. (10) shows that velocity $u$ is also dependent on horizontal linear scale $L$ growing with decreasing $L$,

$$u = \sqrt{2(\Delta p/\rho - g z_T L/h_a)}. \tag{12}$$

At large $L$ in Eq. (12) the value of $u$ is determined as a small difference between two large terms, which makes it sensitive to the accuracy of estimates of all parameters entering the equation. It is, however, clear that $u$ (12) becomes fairly close to the theoretical maximum possible value of $u_{\text{max}} = \sqrt{2\Delta p/\rho}$ as soon as $L$ decreases by a few times compared to the large-scale circulation (where $f_T \sim f_{T0} \gg f_{Ta} = C_D \rho u^2$ and $g z_T L/h_a \sim 2\Delta p/\rho$), i.e. when $g z_T L/h_a$ becomes considerably less than $2\Delta p/\rho$. Thus, the proposed representation of friction, Eq. (5), explains why the highest wind velocities are normally observed within compact circulation events like hurricanes ($L \sim 10^2$ km) and tornadoes ($L \sim h_a$). Note that in hurricanes the air masses may undergo acceleration over the major part of the streamline; the inward acceleration zeroes fairly close to or immediately at the eyewall, where velocity $u$ approaches its maximum value and the condition $f_A \approx f_{Ta} \gg f_{T0}$ starts to hold.

We remind the reader that within the biotic pump theory at $L \gg h_a$ the horizontal pressure difference $\Delta p$ in the circulation pattern driven by the evaporative force is related to the vertical non-equilibrium pressure difference $\Delta p_v$ of water vapor as $\Delta p \approx \Delta p_v = f_E h_{\text{H}_2\text{O}} = p_v \times 0.82$, where $f_E$ is given by Eq. (16) of Makarieva & Gorshkov (2007), $p_v$ is partial pressure of water vapor at the surface.

**Acknowledgements.** This commentary is written by Anastassia Makarieva and Victor Gorshkov, but submitted to this discussion by V. Gorshkov, as the discussion platform does not allow for group comments.

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HESSD

6, S285–S292, 2009

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Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 6, 401, 2009.