

Dear Esteemed Editor of Hydrology and Earth System Sciences Journal

Thank you very much for your attention and response. According to the comments of the esteemed reviewer, we have corrected our paper (“**A steady-state saturation model to determine the subsurface travel time (STT) in complex hillslopes**”). In the following, please find our response to the points raised by the second reviewer.

The reviewer comments have been written in italic form.

Question 1. *Their English is very poor. Grammatical errors, spelling, technical words.*

Author: The English was edited by native English.

Question 2. *You did not mention, but $S(x) = 0$ at $x = 0$ is the boundary condition?*

Author: You are right. This is the boundary condition. This means that in $x=0$ (upstream), $S(x) = 0$.

Question 3. *Their math and physics are very poor. There are many errors in the computations.*

Author: The math and physics of equations have been presented under supervision of two professors and two assistant prof of Hydrology and all processes and formulations have been checked again and have been corrected and improved.

Question 4: *The unit of N in Eq. (5) is length³/time. But the unit of N in Eq. (25) is length/time.very strange?*

Author: Eq. 5 has been corrected as follows:

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} - N(t)w(x) = 0 \quad (5)$$

In the text also the $N\omega$ was changed to $N(t)w(x)$, where $w(x)$ is hillslope width.

Moreover, the dimensions of Eq. 5 are as follows:

N : [length/time], S : [length²], Q : [length³/time], $w(x)$: [length]

This means that both hands of Eq. 5 have the same dimension (length²/time). In Eq. 25, the dimension of N is also [length/time].

Question 5: *You used the steady solution of Eq. (5). This is not related with Eq. (2). Why do you call hsb equation? Eq. (2) requires the upstream and downstream boundary conditions. But Eq. (5) requires the only upstream boundary condition. The seepage flow is generally influenced by the downstream boundary condition.*

Author: you are right. Eq. 2 was substituted by hillslope-storage kinematic wave equation in section 2.2. In fact, we have not used hsb equation in our calculation (as you had mentioned). This means that the saturation zone length of the complex hillslopes was calculated numerically by using the hillslope-storage kinematic wave equation for subsurface flow, so an analytical equation was presented for calculating the saturation zone length of the straight hillslopes and all plan shapes geometries.

Question 6: *I cannot understand Eq. (7). At $t = 0$ $S(x; 0) = g(x)$ and 0 ? This indicates $g(x) = 0$?*

Question 7: *But Eq. (7) does not contain $g(x)$. Where is $g(x)$? If Eq:(7)is the analytical solution of Eq:(5); the solution should contain t :*

Author (6&7): As we have mentioned in the text, $g(x)$ represents the initial soil moisture storage along the hillslope. You can follow details based on the following equations:

Ref: Troch, P.A., van Loon, A.H., and Hilberts, A.G.J.: *Analytical solutions to a hillslope storage kinematic wave equation for subsurface flow. Adv. Water Resour., 25(6):637–649, 2002*

Along the hillslope the continuity equation reads:

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} - N(t)w(x) = 0 \quad (1a)$$

where $N(t)$ is the recharge to the saturated layer. Clearly we have complete saturation whenever $S(x,t) \geq S_c(x)$. Let us further assume that the flow rate Q is related to the storage $S(x,t)$ through a kinematic form of Darcy's equation:

$$Q = -k \frac{S}{f} \frac{\partial z}{\partial x} \quad (2a)$$

where z is the elevation of the bedrock above a given datum. Combining (2a) with the continuity equation for given recharge N and assuming no spatial variability in k and f , one obtains a quasi-linear wave equation in terms of soil moisture storage

$$a(x) \frac{\partial S}{\partial x} + \frac{\partial S}{\partial t} = c(x, S),$$

where

$$a(x) = -\frac{kz'(x)}{f},$$

$$c(x, S) = Nw(x) + \frac{kz''(x)}{f} S \quad (3a)$$

and $z'(x)$ and $z''(x)$ are first and second derivatives of the bedrock profile curvature function $z(x)$ with respect to x . Eq. (3a) is a quasi-linear wave equation that can be solved analytically with the method of characteristics. Eq. (3a) can then be written as a set of ordinary differential equations :

$$\frac{dx}{dt} = a(x) = -\frac{kz'}{f}, \quad (4a)$$

$$\frac{dS}{dx} = \frac{c(x, S)}{a(x)} = -\frac{f}{kz'} N(t)w(x) - \frac{z''}{z'} S \quad (5a)$$

(4a) describes a family of characteristic curves in the (x,t) plane, and (5a) describes how the storage propagates along each curve. Fig. 1 illustrates the method of characteristics in the (x,t) plane. In the context of subsurface flow, it is reasonable to assume the following initial and boundary conditions:

$$S(x, 0) = g(x), \quad 0 \leq x \leq L,$$

$$\frac{dS(0, t)}{dx} = 0 \quad \forall t,$$

where $g(x)$ represents the initial soil moisture storage along the hillslope. For subsurface flow and rainfall events with constant intensity and duration T , it is reasonable to assume that the subsurface flow does not reach equilibrium at $t=T$; therefore, four domains can be distinguished (partial equilibrium, see also Fig.1; the characteristic curve passing through the origin does not reach $x=L$ before $t=T$). Domain 1 (D1) is defined by the characteristic curve $t=t(x,0)$ and the lines $t=0$ (initial condition), $t=T$ (end of rainfall event), and $x=L$ (outlet of hillslope). To obtain $s(x,t)$ in D1 we solve (4a) and (5a) subject to the following initial and boundary conditions:

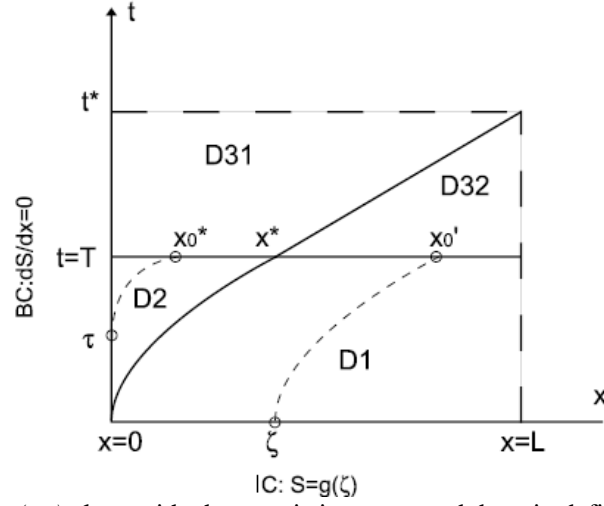


Fig.1: (x,t) plane with characteristic curves and domain definition.

$$t(\zeta) = 0 \quad 0 \leq \zeta \leq L,$$

$$S(\zeta) = g(\zeta) \quad 0 \leq \zeta \leq L,$$

where ζ is a parameter representing the intersection of a characteristic curve with the x -axis. The solution in domain D1 is then given by:

$$t = \frac{fL^2}{(2-n)nkH} \left[(1 - \zeta/L)^{2-n} - (1 - x/L)^{2-n} \right], \quad (6a)$$

$$S(x, t) = g(\zeta) \left[\frac{(1 - \zeta/L)}{(1 - x/L)} \right]^{n-1} + \frac{fL}{nkH} (1 - x/L)^{1-n} N[A(x) - A(\zeta)], \quad (7a)$$

By substituting $\zeta = 0$ and $S = g(\zeta) = 0$ in equations 6a and 7a, we have:

$$T = \frac{fL^2}{(2-n)nkH} \left[1 - (1 - x^*/L)^{2-n} \right], \quad (8a)$$

$$(9a) \quad S(x) = \frac{fL}{nkH} \left(1 - \frac{x}{L}\right)^{1-n} NA(x)$$

Now, eq. 8a shows the equilibrium time of the complex hillslope that is the same with the eq. 19 in our paper (last version). This means that the boundary condition is $S(x=0)=0$. In fact, based on Eq. 9a: $x = 0 \rightarrow A(x) = 0 \rightarrow S(x) = 0$.

Question10: The unit of the right side of Eq. (8) is time/length. The unit of left side of Eq. (8) is length². very strange!

Author: Dimension of parameters in eq. 8 (last version) is as follows:

$$S(x) = \frac{fL}{nkH} \left(1 - \frac{x}{L}\right)^{1-n} NA(x)$$

[length/time] N
 [length²]S
 [length²]A
 [length/time] k
 [length] H
 [length]x,L
 - :f,n

length². This means that the dimension of both sites of Eq. 5 is

$$\left[\frac{\text{length}}{\text{time}} \right] * \text{length} * \left[\frac{\text{length}}{\text{time}} \right] * \text{length}^2 = \text{length}^2$$

Question 11: hsB is the hill-storage Boussinesq equation or Boussinesq hydrological equation?

Author: hsb is the hillslope-storage Boussinesq model that has been presented by Troch et al., 2003

REF: Troch, P.A., van Loon, A.H., and Paniconi, C.: Hillslope-storage Boussinesq model for subsurface flow and variable source areas along complex hillslopes: 1. Formulation. Water Resour. Res., 39(11):1316, doi: 10.1029/2002WR001728, 2003.

Question 12: I cannot follow their manuscript. Please do over from the beginning very carefully.

Author: Now, by presenting many explanations, the manuscript and especially formulation have become clear.

Question 13: Where is your original points in your manuscript?

Author: As we have mentioned in the text (page 7182), original point in this paper is investigation of subsurface travel time of complex hillslopes (hillslopes with different plan shape and profile curvature). We have presented our objectives as follows:

- (i) introduce an equation for subsurface travel time of all complex hillslopes with regard to parameters such as the saturation zone length, total length, soil porosity, profile curvature, soil hydraulic conductivity, and average bedrock slope,
- (ii) calculate the saturation zone length of nine basic hillslopes in steady-state conditions,
- (iii) explore the effects of different factors such as the soil depth, the recharge rate, bedrock slope angle on travel time and saturation zone length,

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