Dear Esteemed Editor of Hydrology and Earth System Sciences Journal

Thank you very much for your attention and response. According to the comments of the esteemed reviewer, we have corrected our paper (**"A steady-state saturation model to determine the subsurface travel time (STT) in complex hillslopes"**). In the following, please find our response to the points raised by the first reviewer.

The reviewer comments have been written in italic form.

1: *Page 7182, line 4-8. Actually, there is a paper (Huyck et al, wrr, 2005) in which a similar Boussinesq equation (for aquifers with varying width, linearized) has been solved analytically for temporally changing recharge rates. I think this should be added to this paragraph, because it is an advancement compared to the references that are already listed.*

Author: In the introduction page7182, line 8 some works of Huyck et al(2005) on linearization of Boussinesq equations for complex hillslopes is pointed out. The following statement has been added in the paragraph:
Huyck et al. (2005) developed an analytical solution to the linearized Boussinesq equation for realistic aquifer shapes and temporally variable recharge rates.

The following reference has also been cited:
Huyck, A. A. O., V. R. N. Pauwels, and N. E. C. Verhoest (2005), A base flow separation algorithm based on the linearized Boussinesq equation for complex hillslopes, Water Resour. Res., 41, W08415, doi:10.1029/2004WR003789.

2: *After equation 1: please provide units or dimensions to the variables.*
Author: The units of the parameters of Eq. (1) have been introduced after this equation.

3: *After equation 2: perhaps better to refer to the original Boussinesq paper for the Boussinesq-equation, than to Troch et al (2003)?*
Author: Eq. (2) is known as Boussinesq, 1877; so the following reference has been added amongst the others:
Boussinesq, J.: Essai sur la thorie des eaux courantes, Mm. Acad. Sci. Inst. France, 23, 1–680, 1877.
This reference has also been cited in page7189, line 11.

4: *After equation 2 and 3: please provide units or dimensions to the variables.*
Author: The dimensions of the parameters of Eq. (2) have been stated after this equation. The units of Eq.(3) have also been stated after this equation.

5: *Equation 5: the "omega", should this be "W"?*
Author: Eq.(5) has been corrected as:

$$\frac{\partial S}{\partial t}+\frac{\partial Q}{\partial x}-Nw=0$$

6: *Please provide more details in the derivation of equation 6. It is important to understand where this equation comes from. Perhaps in appendix.*
Author: Eq.(6) which is named Darcy equation, giving the true value of the flow in each cross section, has no need to prove, as can be seen in all researches of Troch and his colleagues.

7: *Equation 7: remove the last bracket (the large}). Also, the bottom equation means that the storage everywhere is zero, for all t?*

Author: The boundary conditions have been resented as:

$$S(x,0) = g(x) \qquad 0 \le x \le L$$

$$\frac{dS(0,t)}{dx} = 0 \qquad \forall t$$

By the way, this boundary condition was applied by Troch (2000) to solve the Boussinesq equation by characteristic method in page 639 of the following article:

Ref: Troch, P.A., van Loon, A.H., and Hilberts, A.G.J.: Analytical solutions to a hillslope storage kinematic wave equation for subsurface flow. Adv. Water Resour., 25(6):637–649, 2002.

8: *Page 7186, first paragraph. The point of saturation is mentioned. What if there is no saturation anywhere ?*
Author: If $\sigma < 1$ there is not any saturation point. I did not understand your intention.

9: *After equation 9, please add units to the variables. Also, at the end of this explanation, the notation "delta" is used, while in the equation itself, "sigma" is used. Please make consistent.*
Author: In Eq. (9) σ was added.
At the end of paragraph 16, page 7186, the expression $\delta(X) = 1$ was changed to $\sigma = 1$ .The dimensions of the parameters have been added after this equation.

10: *We need more details in the derivation of equation 10. This is an important equation, so it's important to explain where it comes from. Again, this may happen in appendix.*
Author: Eq.(10) was verified in the text with full details of reasoning. The outline of the proof, p.7186, lines 6-9, and lines 16-18 was corrected as follows:

According to Fig. 3 any point of the hillslope at which the storage equals the storage capacity $(S(x) = S_c(x))$, belongs to the saturation zone. If we call the ratio of actual storage to storage capacity as 'Relative Saturation' (σ), one can say that any point of the hillslope where the relative saturation reaches one ($\sigma \ge 1$), would be a saturation point.
The location of the saturation zone boundary can be determined by inserting $RS(x) = 1 \;\; or \;\; S(x) = S_c(x)$ in Eq. (9) and using the storage function from Eq. (3) and storage capacity from Eq. (8), we then obtain:

$$\frac{fL}{nkH}(1 - x_{sat} / L)^{1-n} N[A(x_{sat})] = w(x_{sat}) Df$$

11: *After equation 11, please add units to the variables. Also, this equation seems rather bizarre to me (I also do not think that this equation is ever used in Peter Troch's or similar papers on the subject). If x is equal to L, the width becomes equal to Wo*exp(-2 omega Lˆ2/H). What is H equal to zero ? This is not impossible, this simply indicates a horizontal aquifer. Also, I would expect the width at the bottom to be independent of L, but it depends on L squared. Also, this equation means that the parameter omega in equation 1 is always the same as the parameter that determines the shape here. Also, why does the aquifer width have to depend on the aquifer slope ? Why not simply make W(x) equal to Wo*exp(par*x), with par a shape parameter (unit 1/m) ? This equation needs a lot more explanation.*
Author: The unit of Eq.(11) in paragraph 1, p.7189 was added. The equations (11), (12), (13) and (14) have been used in the following article:
Ref: Talebi, A., Troch, P. A., and Uijlenhoet, R.: A steady-state analytical hillslope stability model for complex hillslopes, Hydrol. Process., 22,546-553,2008.
The following explanations are set for the notice of the referee, and are not mentioned in the text:

A hillslope with upstream width w_0 (equation (11)) may be reached a hillslope with downstream width c_w , with L as the length of the hillslope. Alterations in the width of the hillslope hinge on the plan shape parameter (ω). The width function along a complex hillslope is as follows:

Due to symmetry about the y=0 axis, the width of the complex hillslope measured in the y direction is given by (Talebi et al. 2008):

$$w(x) = c_w \exp\left\{ c_s \left(1 - \frac{x}{L}\right)^{2-n} \right\} \qquad \text{(B.1)}$$

And

$$c_s = \frac{2\omega L^2}{n(2-n)H} \qquad \text{(B.2)}$$

where c_w defines the width of the hillslope at the outlet (x = L) and cs defines the degree of topographic convergence.

In Eq. B.1 , If x is equal to 0, the upstream width (w_0) for straight hillslopes(n=1) is obtained:

$$w_0 = c_w \exp\left\{ \frac{2\omega L^2}{H} \right\} \qquad \text{(B.3)}$$

Eq. B.3 explains the relationship between upstream width and downstream width and shape plan parameter.

By putting the Eq. B.3 in Eq. B.1 , we obtain:

$$w(x) = w_0 \exp\left(-\frac{2\omega L}{H} x\right) \qquad \text{(B.4)}$$

In general, the alterations in the width of the hillslope depend on the plan shape as well as the profile curvature, but in straight hillslopes this is parameter ω which exerts the alterations. If we insert $\omega = H / L^2$ in Eq. B.3, one can see that the downstream width does not depend on the length of the hillslope having a role in the parameter $\omega = H / L^2$.

12: *Please provide more explanation for equation 13. This may again happen in appendix.*
Author: The proof of Eq.(13) has been come in APPENDIX A as follows:
Troch et al. (2002) introduced the storage function for steady state condition as:

$$\text{(A.1)} \quad S(x) = \frac{fL}{nkH}\left(1 - \frac{x}{L}\right)^{1-n} NA(x)$$

The storage function for straight hillslope(n=1) is obtainted:

$$\text{(A.2)} \quad S(x) = \frac{fL}{kH} NA(x)$$

By inserting the Eq.12 in Eq. A.2 , we obtain:

$$\text{(A.3)} \quad S(x) = \frac{fNw_0}{2\omega k}\left[1 - \exp\left(-\frac{2\omega L}{H} x\right)\right]$$

13: *Equation 14 uses a "sigma" and the explanation a "delta". Please make consistent.*
Author: Eq.(14) has been corrected.

14: *Equation 15: does the "log" have to be a "ln" ?*
Author: Eq. (16) and Eq.(23) were corrected.

15: *Equation 15: if omega is between zero and -N/(2kD), the argument of the logarithm is between 0 and 1, which will lead to a negative logarithm. This means that the saturated zone length is larger than the aquifer length. This does not make much sense to me. Please explain how this is possible. However, in the tables and figures with the results, realistic values for SZL with negative omegas have been obtained. Please explain this discrepancy.*
Also, equation 15, if omega is smaller than -N/(2kD), the argument of the logarithm is negative, and a complex number is retained for SZL. However, in the tables and figures with the results, again, realistic values for SZL have been obtained in this case. This needs to be explained.
Author: Since the depth of the soil D, the plan shape parameter and the hydraulic conduction coefficient K are constant values, the saturation of the hillslope is characterized by the recharge rate N due to rainfall. The following condition should be taken into account to avoid complex solutions for Eq.(15):

$$(1+\frac{2\omega kD}{N})>0 \Rightarrow N>-2\omega kD$$

or, considering a constant recharge rate value N we have:

$$\omega<\frac{N}{-2kD}$$

But, the above condition is not sufficient for SZL to be positive, a sufficient condition could be:

$$SZL=L-\frac{\overline{S}}{2\omega}Ln(1+\frac{2\omega kD}{N})>0 \Rightarrow N>\frac{2\omega kD}{[\exp(2\omega L^2/H)-1]}$$

Figure (5) shows values of recharge causing saturation of complex hillslopes. In general, if N>SRR we have positive SZL.

The condition $N>SRR=\frac{2\omega kD}{[\exp(2\omega L^2/H)-1]}$ is inserted in the original article page7189.

Because the values of SRR for hillslopes 5,6,8 and 9 are greater than 30mm per day (Fig.(5)), so in Table 3 and Fig.(4) the hillslope has no saturation zone, hence SZL=0, while some parts of other hillslopes are saturated.

16: *Also, immediately before equation 15, sigma (not delta) needs to be equal to 1, not zero.*
Author: This comment was corrected in the text (before equation 15).

17: Please add a reference to substantiate where equation 17 comes from.
Author: Eq.(17) is the well-known Darcy equation:

$$v=\frac{k}{f}\frac{dh}{dx}$$

This equation gives the true velocity. Since in paragraph 5 , p.7184 we assumed that the hydraulic gradient is equal to the local surface slope. so the equation of the flow velocity becomes:

$$v=\frac{k\,s^*}{f}$$

This equation was also used by Arya et al (2005) in his researches, so the corresponding reference is cited in the text, page 7190, line 17.

18: *I do not understand how equation 23 has been derived (I can't get there). Please add more details, perhaps in appendix.*
Author: The proof of Eq. (23) was put in Appendix B.
Appendix B:

The relationship between SZL and the location of the saturation zone boundary for straight hillslopes is:

$$\text{B.1}\; SZL = L - x_{sat}$$

The location of the saturation zone boundary from Eq.15 is obtained:

$$\text{B.2}\; x_{sat} = \frac{\bar{S}}{2\omega} Ln(1 + \frac{2\omega kD}{N})$$

By inserting the Eq. B.2 in Eq. 22, we obtain:

$$T = \frac{f}{2k\omega} Ln(1 + \frac{2\omega kD}{N}) \qquad \text{B.3}$$

Equation B.3 presents the subsurface travel time of the straight hillslopes after saturation.

19: *Figure 8: why are beta and omega varied simultaneously?*

Author: The slope angles of the hillslopes are as the entries of our model, assuming to be constant for all hillslopes. The average slope is computed from the formula of the tangent of slope angle $(\bar{S} = \tan(\beta))$, then the value $H = L.\bar{S}$, and the plan shape parameter $\omega = \pm H / L^2$ for convergent and divergent hillslopes is calculated. For parallel hillslopes the value of plan parameter is zero, so the change of the plan shape parameter is a function of the slope angle and the length of the hillslope. Therefore, in Fig. 8 in which the length values of the hillslopes are fixed at 100m, for a variation in the plan shape parameter and to study its influence on RSS, one should change the slope angle.

Some more explanation:

Eq.(15) is an analytical equation to calculate the saturation zone length for straight hillslopes of convergent or divergent types. For the case of parallel-straight hillslope, Eq. (16) should be used.

The results presented in Table 3 and Fig 6 and Fig 9 are according to the numerical solutions of Eq.(10) for SZL of all complex hillslopes, and the Eq.(19) for the travel time of all complex hillslopes. The results obtained from Eq.(15) and Eq.(10) are likened examined.