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Hydrology and Earth System Sciences Discussions

Interactive comment on "Numerical analysis of Richards' problem for water penetration in unsaturated soils" *by* A. Barari et al.

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Reply to the reviewer 1:

1- There are many references in the paper relating to the analysis of Richards' problem. Since, it is apparent, most of them are in Hydrology and porous media journals. This paper is in the field of Mathematical Hydrology which is fitted completely with the scope of a hydrology journal. Please see one of the last articles in numerical analysis of Richard s' problem:

Nasseri, M., Shaghaghian, M.R., Daneshbod, Y. and Seyyedian, H. (2008). An analytical Solution of Water Transport in Unsaturated Porous Media. Journal of Porous

Media, Vol. 11, Issue 6, pp. 591-601. In the aforementioned article, the authors have investigated the Richard s' problem by using Adomian Decomposition Method (ADM) but the strength of current research (HPM and VIM) is using the easier procedures and less computations than ADM. Please see just some references of the article to show the fitness with hydrology scope:

Ju, S. H. and Kung, K.J.S. (1997). Mass types, element orders and solution schemes for Richards' equation, In: Computers and Geosciences, Vol. 23, No. 2, pp. 175-187. Arampatzis, G., Tzimopoulos, C., Sakellariou-Makrantonaki, M., and Yannopoulos, S. (2001). Estimation of unsaturated flow in layered soils with the finite control volume method, Irrigation and drainage, Vol.50, pp. 349-358. Kavetski, D., Binning, P. and Sloan, S.W. (2002). Non-iterative time stepping schemes with adaptive truncation error control for the solution of Richards' equation. Water Resources Research, Vol. 38, No. 10, pp. 1211-1220. Baca, R.G., Chung, J.N. and Mulla, D.J. (1997). Mixed transform finite element method for solving the nonlinear equation for flow in variably saturated porous media. International Journal for Numerical Methods in Fluids, Vol. 24, pp. 441-455. Bergamaschi, L. and Putti, M. (1999). Mixed finite element and Newton-type linearizations for the solution of Richards' equation. International Journal for Numerical Methods in Engineering, Vol. 45, pp. 1025-1046. Milly, P.C.D. (1985). A mass conservation procedure for time-stepping in models of unsaturated flow. Advances in Water Resources, Vol. 8, pp. 32-36. Miller, C.T., Abhishek, C. and Farthing, M.W.(2005). A spatially and temporally adaptive solution of Richards' equation. Advances in Water Resources, vol. 29, Issue 4, pp. 525-545. Varado, N., Braud, I., Ross, P.J. and Haverkamp, R. (2006). Assessment of an efficient numerical solution of the 1D Richards equation on bare soil. Journal of Hydrology, Vol. 323, Issues 1-4, pp. 244-257. Farthing, M.W., Kees, C.E., Coffey, T.S., Kelley, C.T., Miller, C.T. (2003). Efficient steady-state solution techniques for variably saturated groundwater flow, Advances in Water Resources, Vol. 26, Issue 8, PP. 833-849. Chounet LM, Hilhorst D, Jouron C, Kelanemer Y, Nicolas P. (1999). Simulation of water flow and heat transfer in soils by means of a mixed finite element method. Adv Water Resour, 22(5):445–60. Forsyth

PA, Wu Y-S, Pruess K. (1995). Robust numerical methods for saturated–unsaturated flow with dry initial conditions in heterogeneous media. Adv Water Resour, 18:25–38. Jones JE, Woodward CS. (2001). Newton–Krylov-multigrid solvers for large-scale, highly heterogeneous, variably saturated flow problems. Adv Water Resour, 24(7):763–74. Kavetski D, Binning P, Sloan SW. (2001). Adaptive time stepping and error control in a mass conservative numerical solution of the mixed form of Richards_ equation. Adv Water Resour, 24:595–605. Kees CE, Miller CT. (2002). Higher order time integration methods for two-phase flow. Adv Water Resour, 25(2):159–77. Huang K, Mohanty BP, van Genuchten M.Th.(1996). A new convergence criterion for the modified Picard iteration method to solve the variably saturated flow equation, Journal of Hydrology, 178: 69-91

2- The articles relating to the analytical investigation of RE problem as a strongly nonlinear problem, are so limited. Actually, the authors made a detailed literature review in this field and presented the available numerical and analytical solutions for Richard s' problem. About the analytical methods, Please see the following references:

Witelski, T.P. (1997). Perturbation analysis for wetting fronts in Richards' equation. Transport in Porous Media, Vol. 27, No. 2, pp. 121-134. Wazwaz, A.M. (2005). Traveling wave solutions for generalized forms of Burgers, Burgers-KDV and Burgers-Huxley equations. Applied Mathematics and Computation, Vol. 169, pp. 639-656. Nasseri, M., Shaghaghian, M.R., Daneshbod, Y. and Seyyedian, H. (2008). An analytical Solution of Water Transport in Unsaturated Porous Media. Journal of Porous Media, Vol. 11, Issue 6, pp. 591-601.

In the mentioned articles, the authors investigated the problem using ADM and perturbation methods. There is detailed description in the article about the merits of HPM and VIM to perturbation methods. Moreover, they require less computation in comparison to ADM. Since, recently, many complex and strongly nonlinear problems in the field of Porous Media and Fluid Mechanic are investigating using theses methods. For example: H. Bararnia, E. Ghasemi, S. Soleimani, A.Barari, D.D.Ganji, HPM- Pade'

Method on Natural Convection of Darcian Fluid about a Vertical Full Cone Embedded in Porous Media, 2009, Journal of Porous Media, in press

H. Bararnia, Mo. Miansari, A. Barari, G. Domairry , Full heat transfer and flow analysis for the Falkner–Skan wedge flow , International Journal of Modern Physics B, 2009, in press.

Jafar Biazar, Zainab Ayati, Hamideh Ebrahimi, 2009, Homotopy Perturbation Method for General Form of Porous Medium Equation, 12(11), Journal of Porous Media, 1121-1127.

A. Mehmood, A. Ali, An Application of He's Homotopy Perturbation Method in Fluid Mechanics, International Journal of Nonlinear Sciences and Numerical Simulation, 10(2), 239-246 Mo.Miansari, Me.Miansari, A.Barari, G.Domairry, Analysis of Blasius Equation for Flat-Plate Flow with Infinite Boundary Value, International Journal for Computational Methods in Engineering Science and Mechanics, 2009, in press. F. Fouladi, E. Hosseinzadeh, A.Barari, G. Domairry, Highly Nonlinear Temperature Dependent Fin Analysis by Variational Iteration Method, Heat Transfer Research, 2009, in press. E. Hosseinzadeh, A. Barari, F. Fouladi , G. Domairry, Numerical Analysis of Forth-Order Boundary Value Problems in Fluid Mechanics and Mathematics, Thermal Science Journal, 2009, in press. M. Esmaeilpour, D.D. Ganji, Application of He's homotopy perturbation method to boundary layer flow and convection heat transfer over a flat plate, Physics Letters A, Volume 372, Issue 1, 10 December 2007, Pages 33-38. D.D. Ganji, A. Sadighi, Application of homotopy-perturbation and variational iteration methods to nonlinear heat transfer and porous media equations, Journal of Computational and Applied Mathematics, Volume 207, Issue 1, 1 October 2007, Pages 24-34.

3- The authors are thankful for the kind issues of reviewer. I recommend the kind reviewer to consider some of the aforementioned articles in the field of Hydrology and Fluid Mechanics. Some of the relevant articles would be added to the final version of article. 4- There is not limitation for higher n values and surely, it is possible to achieve more accurate nonlinear model and results. But, as it is clear, by considering the higher values of n, the computations takes so long and the equations length would be so long to mention. But, as it is proved, the methods used are so qualified for nonlinear problem because of high accuracy and less computation than the other methods. Just by a short search in the Scopus, kind reviewer can find that, the most high cited articles in the Engineering field belong to papers of Prof. J.H.He and Prof.Ganji on the application of HPM and VIM methods in complex and nonlinear problems in engineering sciences. 5- The equations solved one form of RE, only similar to the general form of Burgers' equation. Moreover, the equation is obtained through transform, and the original form of the equation is in fact RE. The equation considered in the paper pointed out by the respected reviewer is a KdV- Burger equation which is different from the one considered in the present paper. Experience with solving the general form of Burgers' equation will help in solving similar forms of RE, which has been pointed out in the paper.

6- Solving a nonlinear problem by means of HPM begins with a trial-function of the same order of the original problem with some unknown parameters, followed by constructing a linear differential equation whose solution is the chosen trial-function. The next step is to construct such a homotopy that when the homotopy parameter p=0, it becomes the above constructed linear equation; and when p=1, it turns out to be the original nonlinear equation. The changing process of p from zero to unity is just that of the trial-function (initial solution) to the exact solution. To approximately solve the problem, the solution is expanded into a series of p, just like that of the classical perturbation method. Generally, one iteration is enough, but the solution is always obtained with three steps at most. The respected reviewer can refer to the following two papers to see that the equations considered are highly nonlinear ones:

- Witelski, T.P. (1997). Perturbation analysis for wetting fronts in Richards' equation. Transport in Porous Media, Vol. 27, No. 2, pp. 121-134. - Wazwaz, A.M. (2005). Traveling wave solutions for generalized forms of Burgers, Burgers-KDV and Burgers-Huxley equations. Applied Mathematics and Computation, Vol. 169, pp. 639-656.

7- Actually, using the high order RE can achieve the better results but it would be so difficult to analysis the highly nonlinear problems using analytical and numerical methods. In the current research the aim of authors is to present approximate solutions by two new and powerful but simple analytical methods with clear merits to the available methods. The high accuracy is apparent from the figures for the highly nonlinear ones. The successful application of the novel methods considered in solving one dimensional case will pave the way for future research towards exploring more complex situations. This is the subject of ongoing research by the authors. 8- Please see : Nasseri, M., Shaghaghian, M.R., Daneshbod, Y. and Seyyedian, H. (2008). An analytical Solution of Water Transport in Unsaturated Porous Media. Journal of Porous Media, Vol. 11, Issue 6, pp. 591-601.

Please also note the Supplement to this comment.

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 6, 6359, 2009.