Hydrol. Earth Syst. Sci. Discuss., 6, C2303–C2306, 2009
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Hydrology and Earth System Sciences Discussions

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6, C2303–C2306, 2009

Interactive Comment

Interactive comment on "Technical Note: Linking soil – and stream-water chemistry based on a riparian flow-concentration integration model" *by* J. Seibert et al.

J. Seibert et al.

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Received and published: 8 October 2009

We want here to directly respond to the error found in the equations by reviewer #3. First of all we thank reviewer #3 for making us aware of this obvious error in our equation. While this is an embarrassing error, luckily the error is rather a typographical mistake than a substantial error. The apparent error in equation 5 arises when replacing the profile depth z by the argument stated in equation 3. The actual error is not in equation 5 but in equation 3 where we transposed a and b.

Please see next pages for a detailed response including the equations.

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 6, 5603, 2009.

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Equation 3 was erroneously written as

$$z = b^{-1} \ln\left({aQ}/{b} \right) \qquad \text{(3.erronous)}$$

The correct version of equation 3 is

$$z = b^{-1} \ln\left({bQ}/{a} \right) \qquad \text{(3.\underline{correct})}$$

Inserting the correct expression for *z* from equation 3 into equation 2 results in the sequence of steps outlined in equation 5. Equation 5 and all following equations are thus correct and it is equation 3 (and the authors) that are to blame.

Equation 3 implies a lower integration limit of minus infinity ($Z_0 \to -\infty$). This did not cause any error in the end because after deriving Eq. 5 the lower integration bound was set to $Z_0 \to -\infty$ (and thus Q_0=0) anyway. However, to make the derivation presented in the manuscript consistent, this lower integration bound should be defined earlier (i.e. prior to the development of equations 3, 4 and 5).

We will of course correct Equation 3 and make the above change in the revised manuscript.

Below is a more detailed derivation that we undertook to recheck the equations. Please see the article for explanations of the variables and for the assumptions behind the individual steps in the derivation below.

The water flux at a certain depth z is

$$q = a \cdot e^{bz}$$

Q is linked to q and z (with the lower integration limit being minus infinity) by

$$Q = \int_{-\infty}^{z} q\,du = \int_{-\infty}^{z} a \cdot e^{bu}\,du = {a}/{b} \cdot e^{bz}$$

Expressing z as a function of Q results in

Fig. 1.

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$$z = b^{-1} \ln\left(b/a \cdot Q \right) \qquad \text{(3.correct)}$$

(This is how equation 3 should have looked in the first place)

Equation 4 is found by forming the derivative of z by Q of equation 3:

$$\frac{dz}{dQ} = \frac{d\left[b^{-1} \ln\left(b/a \cdot Q\right)\right]}{dQ} = b^{-1} \cdot b/a \cdot \frac{1}{b/a \cdot Q} = \frac{1}{bQ} = (bQ)^{-1} \Rightarrow dz = (bQ)^{-1} dQ \qquad (4)$$

Inserting (3) and (4) into (2) one can find (doing some additional rearrangement steps here) the third step as it is in the article:

$$L = ac_0 \int_{Z_0}^{Z_1} e^{(b+f)z} dz = ac_0 \int_{Q_0}^{Q_1} e^{(b+f)b^{-1} \ln(b/a \cdot Q)} \cdot (bQ)^{-1} dQ = ac_0 \int_{Q_0}^{Q_1} \left(e^{\ln(b/a \cdot Q)}\right)^{\frac{b+f}{b}} \cdot (bQ)^{-1} dQ$$

$$= ac_0 \int_{Q_0}^{Q_1} \left(\frac{bQ}{a}\right)^{\frac{b+f}{b}} \cdot (bQ)^{-1} dQ$$

Following the development above leads to (with a lot more intermediate steps):

$$L = ac_0 \int_{Q_0}^{Q_1} \left(\frac{bQ}{a}\right)^{\frac{b+f}{b}} \cdot (bQ)^{-1} dQ = ac_0 \int_{Q_0}^{Q_1} \left(\frac{b}{a}\right)^{\frac{b+f}{b}} \cdot Q^{\frac{b+f}{b}} \cdot \frac{1}{b} \cdot \frac{1}{Q} dQ$$

$$= \frac{a}{b} c_0 \cdot \left(\frac{b}{a}\right)^{\frac{b+f}{b}} \cdot \int_{Q_0}^{Q_1} Q^{\frac{b+f}{b}-1} dQ = \frac{a}{b} \cdot \left(\frac{a}{b}\right)^{-\frac{b+f}{b}} \cdot c_0 \cdot \int_{Q_0}^{Q_1} Q^{\frac{b+f}{b}-1} dQ$$

$$= \left(\frac{a}{b}\right)^{1-\frac{b+f}{b}} \cdot c_0 \cdot \frac{b}{b+f} \cdot \left[Q^{\frac{b}{b+f}} \right]_{Q_0}^{Q_1}$$

This is the same as the last step of equation 5.

Fig. 2.

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