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Interactive Comment

# *Interactive comment on* "Thermodynamics of the hydraulic head, pressure head, and gravitational head in subsurface hydrology, and principles for their spatial averaging" *by* G. H. de Rooij

**Anonymous Referee #4**

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The paper deals with the appropriate definition of averaged pressure for flow in porous media. The first part gives a revision of the thermodynamic derivation of fluid pressure and its treatment in averaging approaches on the pore scale. The second part reflects the definition of phase pressure on the macro-scale for flow in the unsaturated zone and suggests a volume averaging procedure for pressure and flow in macroscopically heterogeneous media.

The definition of phase pressure on large scales is an open point of discussion and to my understanding this open problem leads to misunderstandings when comparing large scale observations to model predictions. Therefore I think the paper addresses a very important issue. The first part of the paper, the review of thermodynamic definitions, gives a good overview to understand the problem. It is well written and very helpful that the ideas behind the theories of Gray and Miller are explained in such a clear way. However, I am not so convinced of the second part. I do not think that the definition of averaged pressure, flow and hydraulic conductivity in the paper solves inconsistencies of large scale pressure definitions. Also I do not see, how they differ from existing volume averaging theory. It did not become clear to me, what the statement of the second part is. Therefore I think that the paper should be revised before publication. If the second part would be rewritten in a way that problems with macroscopic pressure measurements are reviewed (as already done in parts for the retention curve), the paper would be a very good review paper.

Specific comments (only concerning the part from Section 2.2. on):

1) As written above, the statement of the upscaling part and the concept followed to define averaged pressures did not become clear to me. This is outlined in more detail below.

* It is written in the abstract that consistent upscaling equations are derived for various heads (lines 9-12 on page 1138). To my understanding, upscaled equations would be derived for a problem (a differential equation, including boundary conditions) and not for a variable. The macroscopic variables result from upscaling of a given problem. I think this is often mixed up in the paper.

* The volume averaged porosity and water content and the phase averaged potential is defined by eqns. (8) - (12). It is argued that the phase averaged potential is consistent to thermodynamically defined pressure, as it contains the total energy of the system. In Section 3 the point is made that the vertical coordinate has also to be volume averaged, as any additive term in the potential has to be averaged. Is this not always done this way in "classical" volume averaging theory, if the Darcy equation is averaged (as



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for example for the macroscopic two-phase problem in M. Quintard and S. Whitaker: Two-phase flow in heterogeneous porous media: The method of large-scale averaging, Transport in Porous Media 3, 357-413, 1988)? The approach in the volume averaging papers is always that the flow or transport equation is averaged, and not the variable. But the gravity term is averaged the same way as the pressure gradient term. A comparison of the definitions given here to the definitions made in volume averaging theory would be useful to clarify this question.

* It is outlined that it does not make sense to volume average the hydraulic conductivity tensor directly (page 1149, lines 1-3). However, I do not think that volume averaging of the hydraulic conductivity directly would be done in any upscaling approach. I think that all upscaling approaches would proceed from the Darcy equation, average the equation (stochastic average, volume average, or any average) and derive from that an effective conductivity (for example: P. Renard and G. de Marsily, Calculating equivalent permeability: A review, Advances in Water Resources 20, 253-278, 1997).

* It is argued that the volume averaged Darcy equation (or flux) does not yield an equation, which has the shape of a Darcy equation, as the potential gradient is coupled to the hydraulic conductivity via the integral. Again: is this not also found in all volume averaging approaches (Whitaker, The method of volume averaging, Kluwer, 1999 for the single phase flow problem and Quintard and Whitaker, 1988 for the two-phase problem)? It is also found in stochastic averaging, only that the integral over the area is there an integral over the ensemble. The problem of the coupling is the same, though. The next step would be to introduce a closure or approximations, which allow to decouple averaged head gradient from the rest and to define this way an effective conductivity. To obtain an effective conductivity that is independent on other variables or boundary conditions is only possible for simple conditions, for example, if boundaries are far away from the region of interest. For simple conditions it has, however, been demonstrated very often that the effective conductivity is approximately independent of the averaged pressure head (for example, P. King, The use of field theoretic methods



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for the study of flow in a heterogeneous porous medium, Physica A, 3935 - 3947, 1987).

* At the end of Section 2.2 the statement is made that the total energy concept does not hold for macroscopic pressure, as the phase averaged potential does not appear in the areal averaged Darcy equation (page 1150, lines 21-22). I cannot follow this argument. If for some reason one decides that the macroscopic variable of the problem has to be the phase averaged potential: why is it not possible to try to write the equation in a way (for example by multiplying and dividing by the water content) that the phase averaged pressure would be the macroscopic variable? It is very likely that the resulting problem would be a mess, and it will not look like a Darcy equation for sure, but I do not think that it is in principle impossible to derive an upscaled problem, which has the phase averaged pressure as macroscopic variable.

2) In Section 3 the phase average is applied to derive an averaged retention function. What is the explanation that this the most consistent or reasonable way to average the retention function? It would be the definition, which is consistent to the total energy concept. But would it not depend on the measurement device or the problem one is interested in, how the upscaled retention curve should be defined?

3) I do not understand eq. (14). If K_j,A is really the areally averaged hydraulic conductivity, the answer to whether an averaged gradient could be found that enforces eq. (14), I think, is no. The non-averaged Darcy equation cannot be brought into the shape of eq. (14) with any weighting function without dropping terms. Should not the question rather be if an equivalent hydraulic conductivity (K_eff, not K_A) can be found, so that under reasonable approximations, the averaged Darcy equation could have the shape of a Darcy equation?

4) Why is it is meaningless to volume average flux (page 2249, lines 4-5)? A volume average can be considered as a filter, and I do not find it inconsistent to filter the flux field. It might be that it is not a measurable quantity and to compare to measurements



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an average over the plane might be more appropriate. But in general I do not see what is wrong with volume averaging flux. In the paper a reference to the paper of Nordbotten is given, but an explanation would be helpful.

5) The sentence on page 1150, lines 18-21 is somewhat confusing. The coupling of the head and the conductivity for the averaged problem generates a nonlinearity, but the Richards equation is non-linear in its non-averaged form and not only due to coupling of head gradient and conductivity.

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