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A stochastic approach for the description of the water balance dynamics in a river basin

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Abstract

The present paper introduces an analytical approach for the description of the soil water balance dynamics over a schematic river basin. The model is based on a stochastic differential equation where the rainfall forcing is interpreted as an additive noise in the soil water balance. This equation can be solved assuming known the spatial distribution of the soil moisture over the basin transforming the two dimensional problem in a one dimensional one. This assumption is particularly true in the case of humid and semihumid environments, where spatial redistribution of soil moisture becomes dominant producing a well defined pattern. The model allowed to derive the probability density function of the saturated portion of a basin and of its relative saturation. This theory is based on the assumption that the water storage capacity varies across the basin following a parabolic distribution and the basin has homogeneous soil texture and vegetation cover. The methodology outlined the role played by the basin shape in the soil water balance dynamics.

1 Introduction

Dynamics of soil moisture in time and space is governed by complex and dynamical interactions between climate, soil and vegetation. Its spatial distribution over a river basin provides a crucial link between hydrological and ecological processes through its controlling influence on runoff generation, groundwater recharge, transpiration, carbon assimilation, etc. The interrelationship between ecological and geophysical determinants of surface water balance is at the forefront of a number of outstanding issues in ecohydrological science (e.g., Rodríguez-Iturbe and Porporato, 2004; Montaldo et al., 2005).

Recent research has received significant input for the description of this variable through the experimental campaigns carried out by using extended measurements through portable time domain reflectometry TDR (e.g., Tarrawarra and Mahurangi ex-

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periments) and active or passive microwave sensors (e.g., Monsoon, 1990; Washita 1992, 1994 and SGP 1997, 1999). These experiments have increased our understanding of the temporal variability and of the spatial structure of the soil moisture fields and of the importance of physical characteristics such as soil texture, vegetation and topographic patterns for soil moisture variability (Western et al., 2002; Kim and Barros, 2002; Wilson et al., 2004; Jawson and Niemann, 2007).

The dynamics of soil moisture at a point has been extensively investigated by numerous authors using stochastic differential equations to derive its steady-state probability density function (e.g., Rodríguez-Iturbe et al., 1999; Laio et al., 2001; Porporato et al., 2004; Rigby and Porporato, 2006). This theory has been useful to investigate on the interactive manner by which resource availability are manifested within various ecological systems observed in nature (e.g., Scanlon et al., 2005; Caylor et al., 2005) and to describe the vegetation water stress in a probabilistic framework (Porporato et al., 2001).

Recent studies have extended the theoretical description of the soil moisture to the spatial scale introducing a space-time soil moisture model driven by stochastic space-time rainfall forcing described by a sequence of circular cell of Poisson rate (Isham et al., 2005; Rodríguez-Iturbe et al., 2006; Manfreda et al., 2006). The methodology explicitly accounts for soil characteristics, vegetation patterns, and rainfall dynamics neglecting topographical effects and the upper bound due to saturation. This soil moisture model can be considered representative of a relatively flat landscape under semiarid climatic conditions.

The description of the soil moisture evolution over a river basin is at the moment a challenging topic that may represent a bridge in the ecohydrological and hydrological research. Some examples in this direction are given in the paper by Botter et al. (2007a) where the probability density functions of the slow components of the runoff are derived using a river basin schematization with uniform macroscopic parameters governing the soil water balance neglecting the spatial heterogeneity of soil properties. The same authors extended the previous work introducing the spatial heterogeneity of the basin summing the runoff contributions provided by different subbasins with spatially averaged soil properties (Botter et al., 2007b).

The present work represents an attempt to fill such a gap through the definition of the probability distribution of the relative saturation of a basin characterized by variable water storage capacity. The proposed scheme includes a number of approximations, but it leads to an interesting framework for the derivation of the main statistics of basin scale variables. Among others, our interest focused on the behavior of saturated areas over a basin that may be responsible of the dynamics of riparian vegetation as well as runoff generation.

The theory is based on the conceptual model Xinanjiang that describes watershed heterogeneity using a parabolic curve for the distribution of the water storage capacity (Zhao et al., 1980). The Xinanjiang model was first developed in 1973 and published in English in 1980. It is a well-known lumped watershed model and has been widely used in China. Furthermore, the adopted relationship between the extent of saturated areas and the volume stored in the catchment has driven the evolution of a number of more recent models such as the Probability Distributed Model (Moore and Clarke, 1981; Moore, 1985, 1999) and the ARNO model (Todini, 1996).

The present paper provides a description of the model characteristics and the mathematical framework to derive the probability density functions of soil water content at basin scale in Sects. 2 and 3. Results of the theory are discussed in Sect. 4 that precedes the conclusions.

2 Model description

2.1 Rainfall model

The spatial heterogeneity of rainfall is neglected assuming uniform distribution of rainfall occurring at random in time over the entire basin. Such an assumption may be more or less reliable depending on climatic characteristics of rainfall forcing and basin

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size. In general, one should expect that it becomes more realistic for river basins of small sizes.

Rainfall occurrences are modelled by a sequence of instantaneous pulses that occur in a Poisson process of rate λ in time. Each pulse is characterized by a random total depth Y exponentially distributed with mean $m_Y = \alpha$ that may be considered as the mean daily rainfall since the model is interpreted at the daily time scale (see Rodríguez-Iturbe et al., 1999).

In the following, we will refer to a normalized version of the density function of rainfall depths described as

$$f_H(h) = \gamma e^{-\gamma h} \tag{1}$$

where $\gamma = w_{\max}/\alpha$ and $w_{\max}$ is the maximum value of the water storage capacity in the basin.

2.2 The variability of the soil water storage capacity over the basin

The water storage capacity of the soil is certainly one of the most significant parameter for a correct description of soil moisture dynamics. In fact, it is the main factor controlling the temporal dynamics of the process as shown by Manfreda and Rodríguez-Iturbe (2006). For this reason, in the present work, the soil thickness is assumed to vary over the basin according to a given distribution. For sake of simplicity, the remaining sources of heterogeneity like pattern of vegetation and soil texture variability will be neglected in the present work assuming that the soil texture as well as the vegetation are uniform over the watershed.

The watershed heterogeneity is described using a parabolic curve for the water storage capacity of the soil (Zhao et al.,1980)

$$\frac{f}{F} = 1 - \left(1 - \frac{W}{w_{\max}}\right)^b \tag{2}$$

where f/F represents the fraction of the basin with water storage capacity $\leq W$, w_{max} represents the maximum value of the water storage capacity in the basin and b is a shape parameter that according to Zhao (1992) assumes values between 0.1–0.4 increasing with the characteristic dimension of the basin. An example of the Eq. (2) is given in Fig. 1 where the same function is plotted for different values of b. Parameter b affects the spatial heterogeneity of W that increases with larger values of b, while the basin assumes uniform distribution of soils when b=0.

Furthermore, the recent work of Chen et al. (2005) outlined that the above expression can be estimated directly from elevation data. In particular, they use the spatial distribution of the wetness index (W_I) (Kirkby, 1975), as suggested by Gou et al. (2000), to estimate the so called index of runoff difficulty through a normalized function of the wetness index, $(\max[W_I]-W_I)/(\max[W_I]-\min[W_I])$.

The total water storage capacity of the basin is obtained integrating $(1-f/F)$ between $W=0$ and w_{max}, obtaining

$$WM = \frac{w_{max}}{1+b}. \tag{3}$$

In order to obtain a water balance equation with only one state variable, it is necessary to make the hypothesis that the soil water distribution is known over the basin. In particular, it is possible to assume that the soil water content is redistributed within the basin cumulating in the areas with lower soil depth. Under these hypotheses, the conceptual schematization of the basin is sketched in Fig. 2, where both the soil water content distribution and the soil water capacity are described. From this graph, it is also clear that the relative saturated areas, a, are described by the same relationship given in Eq. (2) where a correspond to the ratio f/F.

The relative saturation of the basin is a significant variable to interpret the basin dynamics and it will be considered, from now on, the state variable of the system along with the saturated portion of the basin, a. The relative saturation of the basin may be defined as the ratio between the total water content of the basin divided by the total storage capacity of the basin,

$$s = \frac{\int_{\text{basin}} \theta Z_r}{\int_{\text{basin}} n Z_r}, \tag{4}$$

where θ is the soil water content, n is the porosity, and Z_r is the root depth. From now on the product nZ_r will be called water storage capacity, W.

The watershed-average soil moisture storage at time t, is the integral of 1-f/F between zero and the actual value of the water level in the basin scheme, wm_t,

$$W_t = \int_0^{wm_t} \left(1 - \frac{f}{F}\right) dW = WM \left(1 - \left(1 - \frac{wm_t}{w_{\max}}\right)^{1+b}\right). \tag{5}$$

Under the described schematization, the relative saturation of the basin, s, expressed as the ratio between watershed-average soil moisture storage and the total available volume can be defined as

$$s = \frac{W_t}{WM} = \left(1 - \left(1 - \frac{wm_t}{w_{\max}}\right)^{1+b}\right). \tag{6}$$

2.3 The soil water losses

The function describing the soil water losses represents the deterministic part of the stochastic equation describing the soil water balance. It depends on the local value of the soil water content and the maximum rate of soil water losses. The main contributions to soil losses are given by: the actual evapotranspiration and the soil leakage. A possible approximation for the sum of this two terms is given by a linear function where the soil losses are assumed to be proportional to the soil water content in a point

$$L(s,x) = V\zeta(t,x), \tag{7}$$

where $L(s,x)$ is the soil water loss relative to the soil saturation $\zeta(t,x)$ at time t in the point x in space, and V is the water loss coefficient.

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It is worth nothing to remark that the same loss function has been used by numerous authors (e.g., Entekhabi and Rodríguez-Iturbe, 1994; Pan et al., 2003; Porporato et al., 2004; Rodríguez-Iturbe et al., 2006) essentially for two reasons: first of all, it represents a reasonable approximation for the sum of the actual evapotranspiration and the leakage and second it represents a useful simplification in a mathematical framework.

Since the adopted soil loss function is a linear one, it may be generalized at the basin scale using the product between the relative basin saturation, s, and the water loss coefficient. It follows

$$L_b(wm_t) = Vs = V\left(1-\left(1-\frac{wm_t}{w_{\max}}\right)^{1+b}\right). \tag{8}$$

For analytical purposes, the soil water losses can be expressed as a function of the ratio $R=\frac{wm_t}{w_{\max}}$ using an approximated expression in exponential form. In this case, the soil water losses are expressed as

$$L_b\left(R=\frac{wm_t}{w_{\max}}\right) \cong V\left(\frac{e^{-kR}-1}{e^{-k}-1}\right) \tag{9}$$

where k is a coefficient that has been used to fit the above with Eq. (8). The two functions were fitted imposing the condition that they subtend the same area between R=0 and 1. Using this assumption, one may obtain the following expression

$$b = \frac{2-2e^k+k+e^k k}{e^k-k-1}, \tag{10}$$

that may be solved numerically in k providing an estimate of k as a function of the parameter b. This yields $k \cong b/\left(\frac{b}{7}-\frac{1}{3}\right)$.

The Eq. (9) is represented in Fig. 3 for different values of the parameter b that varies from 0.1 up to an hypothetical value of 1.5. This graph shows how the relationship between the soil losses at the basin scale become more non-linear with the increase of the values of b.

3 The water balance equation

The water balance equation needs to be written at the basin scale in order to derive the relative dynamics of soil water. This problem can tackled working with the mass conservation equation of total water, W_t, or with the water level in the parabolic reservoir. In the first case, the rainfall forcing represents a multiplicative noise, while in the second case it is an additive noise. This last approach is consequently preferable for analytical purposes.

Using the above approximations for the rainfall forcing and for the spatial distribution of the soil water storage capacity, the soil water balance over the basin can be described through the following stochastic differential equation in wm_t

$$\frac{dwm_t}{dt} = I - Vs = I - Vs, \tag{11}$$

where I represents an additive term of infiltration and water losses are assumed to be proportional to the relative saturation of the basin s. The advantage to solve the water balance equation in wm_t is that the infiltration rate can be summed as an additive term of the stochastic differential equation. The water level wm_t in the basin schematization increases as long as the infiltration does not exceed the maximum water storage capacity of the basin $w_{\max}$. The schematization, in fact, accounts for the upper bound imposed by the soil saturation.

In the present scheme, the runoff generation occur for saturation excess and also the soil moisture is redistributed within the basin obtaining a behavior comparable with a Dunne mechanism where the subsurface flow and direct precipitation on saturated areas (saturated overland flow) are dominant runoff generation mechanisms, at least in humid areas (e.g., Hibbert, 1967; Dunne and Black, 1970).

According to Eq. (2), the water level in the parabolic reservoir proposed to describe the soil water storage capacity can be related to the fraction of saturated areas, a, as

$$wm_t = (1 - (1 - a)^{\frac{1}{b}})w_{\max}, \tag{12}$$

or to the relative saturation of the basin using Eq. (6).

The water balance equation can be solved using the standardized variable

$$R = \frac{wm_t}{w_{\max}} \tag{13}$$

where $R \in [0, 1]$.

It is convenient to standardize the soil water loss rate

$$\rho(R) = \beta \left(\frac{e^{-kR} - 1}{e^{-k} - 1} \right), \tag{14}$$

where $\beta = V/(w_{\max})$ is the normalized soil water loss coefficient.

The water balance equation becomes

$$\frac{dR}{dt} = \frac{Y}{w_{\max}} - \rho(R). \tag{15}$$

Following Rodríguez-Iturbe et al. (1999), the probability density function (PDF) of R can be obtained and solved analytically for steady-state conditions. The PDF of R, obtained using the simplified loss function $\rho(R)$ in the water balance equation above, becomes

$$p(R) = \frac{C}{\rho(R)} e^{-\gamma R + \lambda \int \frac{1}{\rho(R)} du} = \frac{C e^{k(R-1)-R\gamma} \left(e^{k}-1\right) \left(e^{kR}-1\right)^{\frac{\lambda(1-e^{-k})}{k\beta}-1}}{\beta}, \tag{16}$$

where C is a constant of integration that may be computed simply imposing the normalizing condition, $\int_0^1 p(R) dR = 1$. Thus, C assumes the following value

$$C = 1 \Big/ \int_0^1 \frac{e^{k(-1+R)-R\gamma} \left(e^{k}-1\right) \left(e^{kR}-1\right)^{-1+\frac{\lambda - e^{-k}\lambda}{k\beta}}}{\beta} dR \tag{17}$$

$$C = \frac{\beta}{\frac{\Gamma\left[1-\frac{\gamma}{k}\right]\Gamma\left[\frac{\lambda-e^{-k}\lambda}{k\beta}\right]}{ke^{k}\Gamma\left[\frac{k\beta-\beta\gamma+\lambda-e^{-k}\lambda}{k\beta}\right]} + \frac{F_1\left[1-\frac{\gamma}{k},1+\frac{\left(e^{-k}-1\right)\lambda}{k\beta},2-\frac{\gamma}{k},e^{k}\right]}{e^{\gamma}(\gamma-k)} \frac{1}{(-1)^{\frac{\lambda-e^{-k}\lambda}{k\beta}}\left(e^{k}-1\right)}}. \tag{18}$$

where $\Gamma[.]$ is the complete Gamma Function and $F_1[.,.,.,.]$ is the Hypergeometric Function (Abramowitz and Stegun, 1964).

3.1 Probability density function of saturated areas of the basin

Under these hypotheses, it is possible to define the probability distribution of saturated areas given the climatic forcing and the geomorphologic characteristics of the basin. In particular, the probability density function of a can be obtained from the probability density function of R as

$$p_A(a) = p_R(f^{-1}(a))\frac{df^{-1}(a)}{da}. \tag{19}$$

To this end, it is necessary to clarify the relationship between R and a that may be obtained from Eq. (2) where the ratio f/F may be also interpreted as the saturated portion of the basin. It follows

$$R = f^{-1}(a) = 1 - (1-a)^{\frac{1}{b}}. \tag{20}$$

The derivative of $f^{-1}(a)$ is

$$\frac{df^{-1}(a)}{da} = \frac{1}{b}(1-a)^{\frac{1}{b}-1}. \tag{21}$$

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Consequently, using the Eq. (19) one obtains the following expression for the probability density function of the relative saturated areas of a basin

$$p_A(a) = \frac{(1-a)^{\frac{1}{b}-1}}{b\beta} C e^{\gamma\left((1-a)^{\frac{1}{b}}-1\right)-(1-a)^{\frac{1}{b}}k} \cdot \left(e^k - 1\right)\left(e^{k-(1-a)^{\frac{1}{b}}k} - 1\right)^{\frac{\lambda - e^{-k}\lambda}{k\beta}-1}. \tag{22}$$

3.2 Probability density function of the relative saturation of the basin

The relative saturation of the basin can be easily characterized at this point using the probability distribution of Eq. (16). Then, one can use the relationship between R and s,

$$R = g^{-1}(s) = (1 - (1 - s)^{\frac{1}{1+b}}), \tag{23}$$

to obtain the derived probability distribution of s. To this end, the same approach used in the previous paragraph should be used where the we also need the derivative of the function $g^{-1}(s)$

$$\frac{dg^{-1}(s)}{ds} = \frac{(1-s)^{\frac{1}{1+b}-1}}{1+b}. \tag{24}$$

The probability density function for the relative saturation of the basin at the steady state can be described by the following expression

$$p(s) = \frac{(1-s)^{\frac{-b}{1+b}} C}{(1+b)\beta}\left(e^k - 1\right)\left(e^{k-k(1-s)^{\frac{1}{1+b}}} - 1\right)^{\frac{\lambda - e^{-k}\lambda}{k\beta}-1} \cdot e^{-k(1-s)^{\frac{1}{1+b}}+\gamma\left((1-s)^{\frac{1}{1+b}}-1\right)}. \tag{25}$$

4 Results and discussion

The model proposed here is a minimalist representation of soil moisture dynamics at basin scale. In the following applications we show that it provides a realistic description of the basin water balance under a wide range of conditions.

In order to show the dynamics of the relative saturated portion of the basin, a numerical simulation of the described model was performed over a temporal window of 100 years using different values of b and w_{max}. A realization of the process is given in Fig. 4 considering a limited temporal window of 900 days. Different parameters of the soil water storage capacity distribution may change dramatically the dynamics of the system and this is even more clear in the PDFs described in the following. Furthermore, the simulation has also been used for comparison with the theoretical distributions obtaining a very good agrement as one may observe in Fig. 5, where two probability density function of the relative saturation of the basin and the relative portion of the saturated areas are compared with the PDFs obtained via numerical simulation.

Figure 6 describes a sequence of probability distributions of the relative saturation of the basin and of the saturated areas assuming different set of parameters for the basin characteristics and with fixed climatic conditions. It may be immediately appreciated how the relative structure of the basin plays a fundamental role in the dynamics of s and a. It is interesting to note that on one hand the reduction of the maximum water storage capacity, w_{max}, increases the variability of both relative basin saturation and saturated areas, on the other hand the distribution of the w dictated by the parameter b does not provide the same effect on s and a. In fact, the increase of the exponent b does not apparently affect the variance of s, but at the same time increases the variability of the saturated areas.

The mean and the standard deviation of the saturated areas, a, are described in Fig. 7 as a function of the water loss coefficient V. Generally, the saturated areas of the basin decreases in mean with the increase of the water loss coefficient. A different behavior is observed for the standard deviation that reach the maximum value when the soil water losses coefficient is equal to the mean daily rainfall. One may also observe that the presence of a more heterogeneous soil water storage capacity (b=0.4) induces a higher mean but also a higher variability.

The parameters $w_{\max}$ and b may also affect the partition between runoff and soil water losses that is described in Fig. 8 as a function of the Poisson rate of rainfall λ. The general signal is an increase of the soil losses with the increase of the incoming rainfall. Of course some differences may be observed for the different basin configuration considered herein. It may be noticed that the increase of the parameter b, representing the spatial heterogeneity of the water storage capacity, tends to reduce the expected value of the soil water losses, but certainly in this case the controlling parameter is the maximum value of the water storage capacity $w_{\max}$.

5 Conclusions

In the present paper, a new approach is introduced to describe analytically the relative soil saturation and the dynamics of saturated areas within a river basin. The method provides a simplified description of river basin characteristics, but includes the effect of spatial variability of water storage capacity adopting the same schematization used by Zhao et al. (1980) for the Xinanjiang model.

In summary, this approach allowed to:

- Derive the probability density function of the saturated portion of a basin also called runoff source areas that represent a significant variable in the dynamics of a river basin (e.g., Fiorentino et al., 2006). Furthermore, the model introduced may be easily adopted to derive the probability density function of runoff production.

- Derive the probability density function of the relative saturation of a river basin characterized by a given climatic forcing and structure of the soil water storage capacity.

- Identify the role of climatic and physical features of the dynamics of the river basin in a humid environment, because all the adopted parameters are physically meaningful.
- Define a theoretical framework useful also for developer and numerous users of the Xinanjiang model and similar models.

The model has not been applied to a real case yet, but a specific experiment has been designed for this theory in order to derive the statistics of the averaged soil moisture over a basin hillslope in order to compare the derived PDFs with a real case. Also, the proposed scheme can be used to derive the probability density function and the cumulative probability of the runoff production at basin scale taking into to account to relevant phenomena like the non-linearity in the rainfall-runoff generation mechanisms and the saturation effect of the basin.

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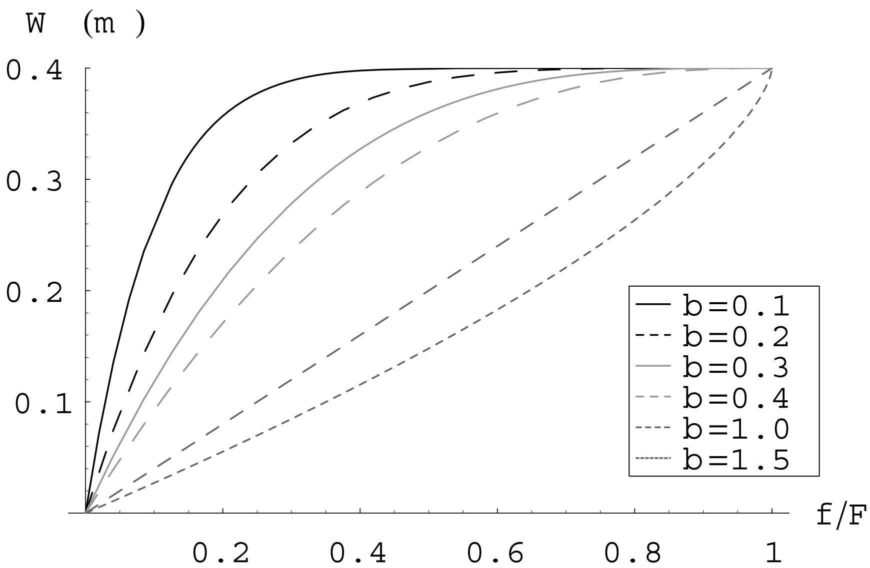

Fig. 1. Distribution of the water storage capacity of a river basin assuming w_{max}=40 cm in both cases, while the parameter *b* changes from 0.1 to 1.5.

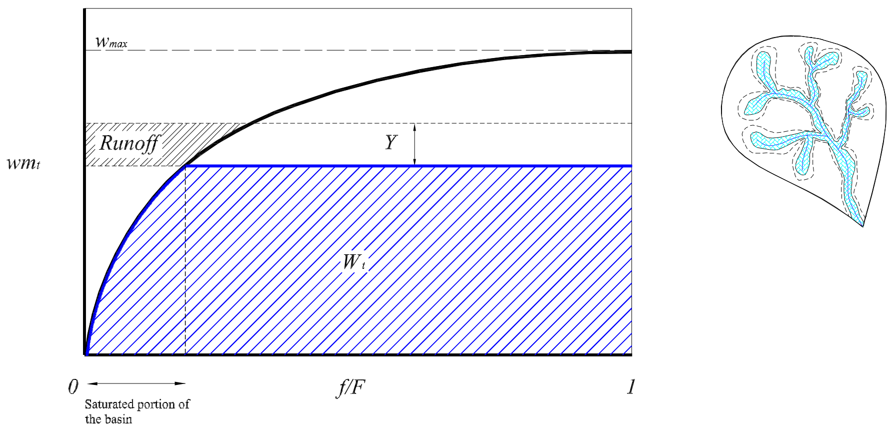

Fig. 2. Schematization of the basin structure and soil water content distribution.

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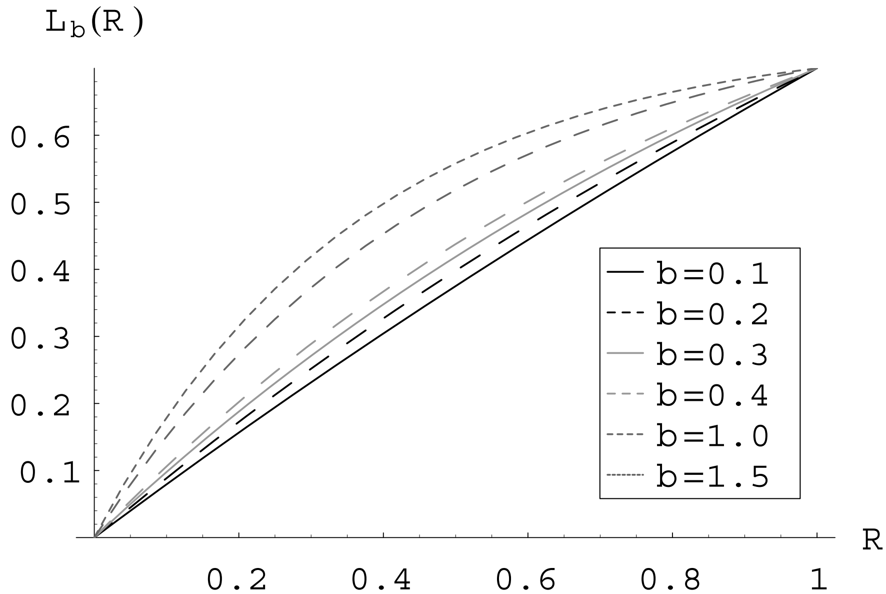

Fig. 3. Soil water losses computed at the basin scale as a function of the ratio $R = \frac{wm_t}{w_{\max}}$.

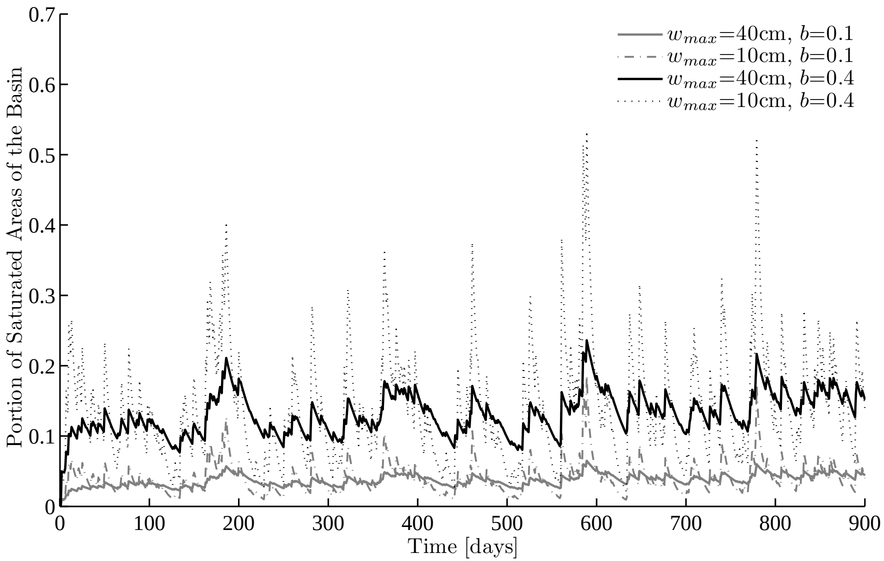

Fig. 4. Temporal dynamics of saturated areas for different values of w_{max} and b reproduced by a numerical simulation performed at the daily time-scale. The remaining parameters are λ=0.3, α=1.0 cm and V=0.7 cm/d.

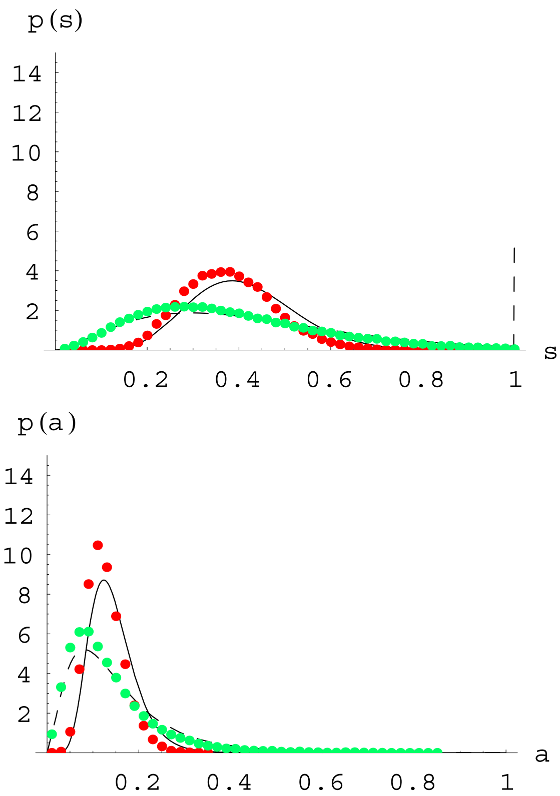

Fig. 5. Comparison between the probability density function of the relative saturation of the basin obtained (upper graph) and relative saturated areas (bottom) with the theoretical distribution given in Eqs. (22) and (25) and numerical simulation (full circles). The parameters adopted are w_{max}=40 cm (continuous line) and 10 cm (dashed line), while the remaining are b=0.4, λ=0.3, α=1.0 cm and V=0.7 cm/d.

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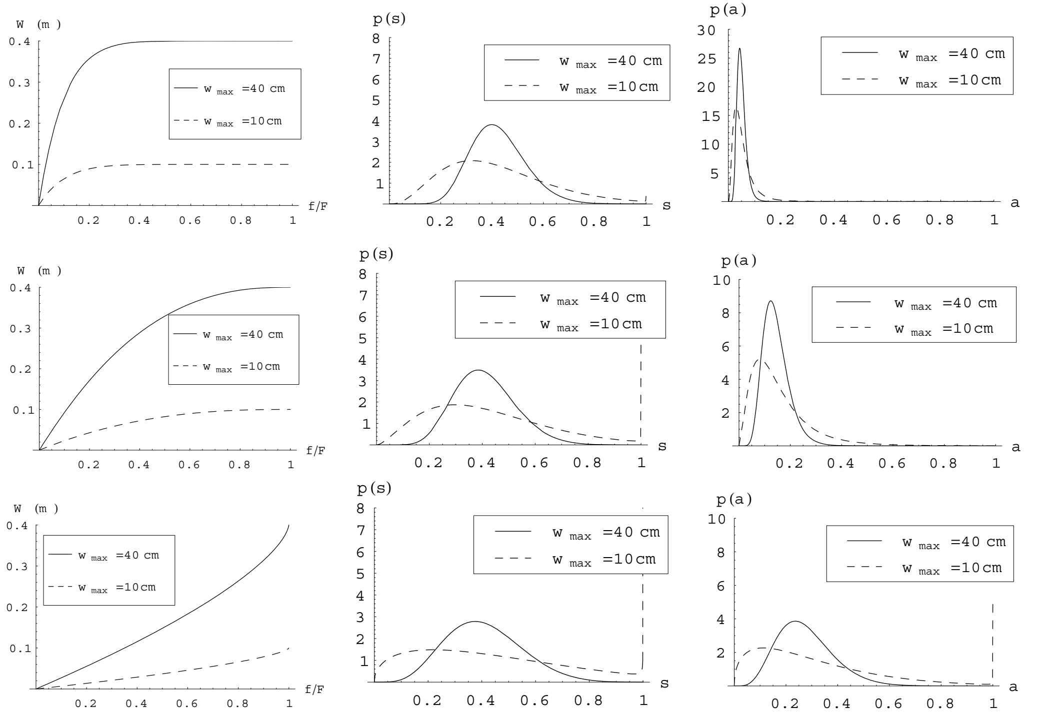

Fig. 6. Probability density functions of the relative saturation (second column) and of the saturated areas (third column) of a river basin assuming w_{max} equal to 40 cm and 10 cm, while the parameter b varies between 0.1, 0.4 and 1.5 in the top down order. The remaining parameters are λ=0.3, α=1.0 cm and V=0.7 cm/d. In the first column, the soil water storage capacity distribution is represented for the corresponding set of parameters w_{max} and b on each row.

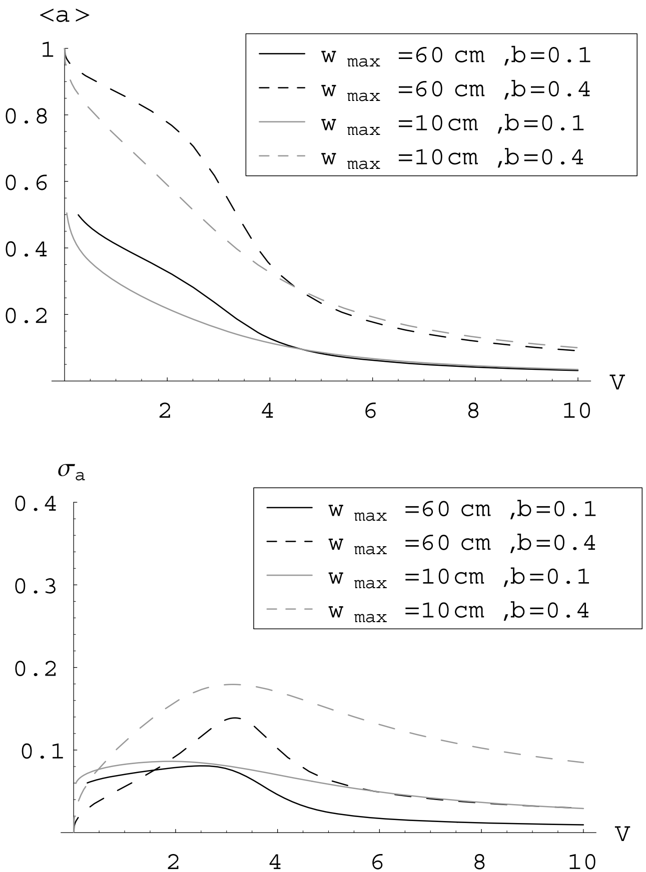

Fig. 7. Expected value and standard deviation of the saturated areas of the basin as a function of the soil water loss coefficient V for different values of w_{max} and b. Remaining parameters are the same of Fig. 5.

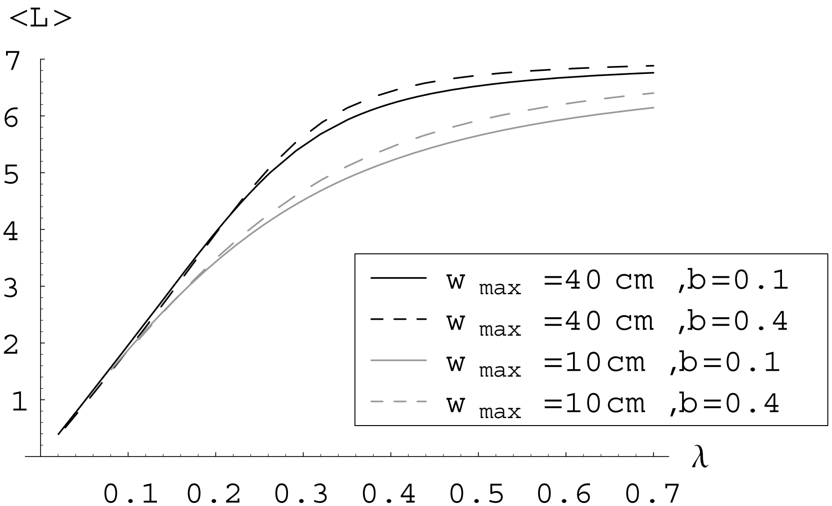

Fig. 8. Expected value of the soil water losses $<L>$ as a function of the parameter λ assuming different values for w_{max} and b.The remaining parameters are λ=0.3, α=2.0 cm and V=0.7 cm/d.