Hydrol. Earth Syst. Sci. Discuss., 5, 1967–2003, 2008
www.hydrol-earth-syst-sci-discuss.net/5/1967/2008/

Hydrology and Earth System Sciences Discussions

Sensitivity analysis of Takagi-Sugeno-Kang rainfall-runoff fuzzy models

A. P. Jacquin[1] and A. Y. Shamseldin[2]

[1]Departamento de Obras Civiles, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile
[2]Department of Civil and Environmental Engineering, The University of Auckland, Private Bag 92019, Auckland, New Zealand

Received: 3 June 2008 – Accepted: 16 June 2008 – Published: 18 July 2008

Correspondence to: A. P. Jacquin (alejacquin@yahoo.com)

Published by Copernicus Publications on behalf of the European Geosciences Union.

Abstract

This paper is concerned with the sensitivity analysis of the model parameters of the Takagi-Sugeno-Kang fuzzy rainfall-runoff models previously developed by the authors. These models are classified in two types of fuzzy models, where the first type is intended to account for the effect of changes in catchment wetness and the second type incorporates seasonality as a source of non-linearity. The sensitivity analysis is performed using two global sensitivity analysis methods, namely Regional Sensitivity Analysis and Sobol's variance decomposition. The data of six catchments from different geographical locations and sizes are used in the sensitivity analysis. The sensitivity of the model parameters is analysed in terms of several measures of goodness of fit, assessing the model performance from different points of view. These measures include the Nash-Sutcliffe criteria, volumetric errors and peak errors. The results show that the sensitivity of the model parameters depends on both the catchment type and the measure used to assess the model performance.

1 Introduction

The problems encountered during the calibration of rainfall-runoff models can be broadly classified as those associated with parameter insensitivity and those arising from parameter interactions. The sensitivity analysis of a rainfall-runoff model permits to detect these parameter insensitivities and interactions, determining the relative importance of the different model parameters in the performance of the model. If the result of the sensitivity analysis indicates that some model parameters are unimportant in determining the model performance, then it is possible to fix them to some chosen appropriate values, thus reducing the dimensionality of the search space for subsequent model calibration (Saltelli et al., 2004). Most typically, sensitivity analysis is performed by studying the characteristics of the model response surface, which is basically the multidimensional surface defined by the model parameters and the ob-

HESSD

5, 1967–2003, 2008

jective function values (e.g. Sorooshian and Gupta, 1995; Xiong and O'Connor, 2000). Nevertheless, the sensitivity of the model predictions to other input factors, such as land use (Nandakumar and Mein, 1997; Hundecha and Bárdossy, 2004) or initial soil moisture conditions (e.g. Zehe and Blöschl, 2004; Zehe et al., 2005), is also possible.

Parameter insensitivity refers to the case where the objective function values are not largely affected by variations in the values of one or more parameters. However, this does not mean that the time series of discharge estimations does not vary with changes in these parameters (Wagener et al., 2002) or that the parameter is redundant in the model structure (O'Connor, 2005). Firstly, it is possible that even though the model output is affected by the values taken by some parameters, the chosen objective function gives little emphasis to the response modes associated with them. In this case, the sensitivity of the model to these apparently insensitive parameters can be observed by analysing the variations in measures of model performance other than the chosen objective function (Wagener et al., 2002). Secondly, it is possible that the model output seems to be itself insensitive to the value of one or more of the model parameters, because the model components related to them are not activated by the calibration input data (Sorooshian and Gupta, 1995; Beven, 2001). In order to prevent this situation, it is necessary to ensure that the data chosen for model calibration is informative/representative, in the sense that it encompasses a wide range of conditions in which the model is expected to operate.

In addition to this, the model structure itself may be such that the response surface suffers from parameter interactions at a local and/or a global scale. Parameter interactions at a local scale occur when simultaneous changes in two or more parameters seem to compensate with respect to the value of the objective function, creating elongated valleys along which the parameter vector may move without evident variations in the height of the model response surface. Another problem that often affects the model response surface of rainfall-runoff models is that of multiple local optima, which can be seen as a kind of parameter interaction at a global scale. From the point of view of the identification of insensitive model parameters, the importance of parameter interactions is that a parameter which does not individually affect the model performance can still have strong influence through interactions with others (Saltelli et al., 2004).

The purpose of this paper is to study the sensitivity of the parameters of the Takagi-Sugeno-Kang (Takagi and Sugeno, 1985; Sugeno and Kang, 1988) rainfall-runoff fuzzy models previously developed by Jacquin and Shamseldin (2006). Takagi-Sugeno-Kang fuzzy models involve complex non-linear relationships between the model output and the model parameters; thus, it is expected that the model response surface is affected both by interactions at a local scale and multiple optima. In this case, the application of traditional local sensitivity analysis methods (i.e. the examination of changes in the model output due to changes in the parameters values in the vicinity of the some nominal/optimal parameter set) is not a suitable alternative for determining whether or not a particular model parameter is important. Accordingly, in this study the sensitivity of the model parameters is analysed using global sensitivity analysis methods, namely Regional Sentitivity Analysis (Spear and Hornberger, 1980; Hornberger and Spear, 1981) and Sobol's variance decomposition (Sobol, 1993). In the authors' present knowledge, there are currently no studies dealing with the sensitivity analysis of fuzzy-based rainfall-runoff models using such methods.

2 Sensitivity analysis methods

2.1 Local versus global methods

Local sensitivity analysis (LSA) methods measure the sensitivity of a quantity Y under examination to small variations in the model parameters, with respect to some chosen nominal values (Beven, 2001). Classical LSA methods are based on the calculation of the derivatives

$$S_p = \frac{\partial Y}{\partial \theta_p}, \tag{1}$$

where Y represents the output quantity under examination (e.g. some measure of model performance) and θ_p represents a model parameter. These derivatives are usually approximated by finite differences, i.e. by evaluation of the change ΔY that results from a small change $\Delta\theta_p$ in the parameter θ_p, while the remaining components of the parameter vector remain constant at their nominal values. Applications of LSA methods in hydrological modelling include the work of Mein and Brown (1978), Gupta and Sorooshian (1985) and Castaings et al. (2005), among others.

There are two main drawbacks of LSA methods that make them inappropriate for the case of model structures affected by parameter interactions, as frequently noted in the literature (Saltelli et al., 2004; Fieberg and Jenkings, 2005; Pappenberger et al., 2008). In the first place, the local estimates of parameter sensitivity obtained with these methods do not provide any information about the effect of variations of the models parameters across their feasible ranges. In addition to this, LSA methods are unable to detect the effect of parameter interactions, because only one parameter is varied at a time.

Global sensitivity analysis (GSA) methods attempt to answer the question of whether or not a particular parameter θ_p is, overall, an important factor in determining the value of the quantity Y. For this purpose, GSA methods estimate to what extent the value of Y is affected by variations in the value of each parameter θ_p across its feasible range. Furthermore, some of these methods also analyse the effect of simultaneous changes in the values of the remaining parameters, thus accounting for parameter interactions in the model structure. Several GSA methods are described in the literature, including regression analysis (e.g. Saltelli et al., 2004), the method of Morris (Morris, 1991), Regional Sentitivity Analysis (Spear and Hornberger, 1980; Hornberger and Spear, 1981) and Sobol's variance decomposition (Sobol, 1993).

2.2 GSA methods applied in this study

2.2.1 Regional Sensitivity Analysis

Regional Sensitivity Analysis (RSA) is a GSA method widely used in hydrological modelling (e.g. McIntyre, N.R. et al., 2003; Mertens et al., 2005; Pappenberger et al., 2008; Tang et al., 2007). Monte Carlo sampling is used for obtaining a large sample of parameter sets and the corresponding values of the output quantity Y. The parameter sets in the sample are then classified as either behavioural or non-behavioural, according to some a priori fixed criterion concerning the value of Y. Thus, the sample of parameter sets is split into a behavioral (S) and a non-behavioural sub-sample (S^*). For each model parameter θ_p, the empirical cumulative probability distribution from each sub-sample is calculated. In the case of a sensitive parameter, the probability distribution from the behavioural sub-sample greatly differs from that of the non-behavioural sub-sample. In contrast, these probability distributions are essentially the same if the performance of the model is relatively insensitive to variations of the parameter θ_p alone. The cumulative probability distribution from the behavioural set ($F_S(\theta_p)$) and the non-behavioural set ($F_{S^*}(\theta_p)$) are compared using a Kolmogorov-Smirnov test.

As pointed out by Saltelli et al. (2004), RSA has the drawback of being unable to deal with parameter interactions, because comparing $F_S(\theta_p)$ and $F_{S^*}(\theta_p)$ does not account for the effect of simultaneous variations of the remaining model parameters. In fact, Spear and Hornberger (1980) clarify that the equality of the distributions $F_S(\theta_p)$ and $F_{S^*}(\theta_p)$ is a necessary but not a sufficient condition for the insensitivity of the parameter θ_p. That is, great differences between $F_S(\theta_p)$ and $F_{S^*}(\theta_p)$ always prove the sensitivity of the parameter θ_p. However, the similarity of $F_S(\theta_p)$ and $F_{S^*}(\theta_p)$ does not necessarily imply that the parameter θ_p is unimportant, because it could still have relevance through interactions with other parameters.

In this study, the RSA method is applied in the manner proposed by Wagener et al. (2001a). A Monte Carlo sample of parameter sets is produced and sorted according to the value of the output quantity Y under analysis. The sorted sample is subsequently split into 10 sub-samples of equal size and, for every model parameter, the cumulative probability distribution within each sub-sample is ploted. Visual comparison of the cumulative probability distributions associated with the different sub-samples allows the detection of sensitive parameters, which are necessarily associated with visible discrepancies between these probability distributions. If these discrepancies are not observed, it is both possible that the parameter is overall insensitive or that it only affects the model performance through interactions.

2.2.2 Sobol's variance decomposition

Sobol's variance decomposition (SVD) is a GSA method that is receiving increasing attention from hydrologists (e.g. Francos et al., 2003; Kanso et al., 2005; Wang et al., 2006; Ratto et al., 2007; Tang et al., 2007). SVD has the advantage over RSA of being able to deal with parameter interactions in the model structure. Even though a more detailed description of SVD can be found in the dedicated literature (e.g. Chan et al., 2000; Saltelli et al., 2000), its basic features are given in what follows.

The SVD method uses the model output variance $V[Y]$ as a measure of the variability of the quantity Y, which may depend, in principle, on the values assigned to all the individual model parameters θ_p. The output variance $V[Y]$ is calculated by exploration of the whole feasible space of the parameter set. If the model parameters are not correlated, the output variance $V[Y]$ can be decomposed in the following sum (Sobol, 1993)

$$V[Y] = \sum_p V_p + \sum_p \sum_{q>p} V_{pq} + \ldots + V_{12,\ldots,P}, \tag{2}$$

where term V_p represents the portion of the variance of Y that is due to changes in the parameter θ_p alone. Higher order terms indicate the portion of the total variance exclusively due to interactions between two or more parameters; for example, the term V_{pq} quantifies the joint contribution of θ_p and θ_q to the variance of Y, minus V_p and V_q.

There are two sensitivity indices provided by the SVD method that will be used in this study, namely the first-order effects and the total effects. The first-order effect of θ_p in Y, defined as (Sobol, 1993)

$$S_p = \frac{V_p}{V[Y]}, \tag{3}$$

measures to what extent the parameter θ_p individually affects the output quantity Y, independently of other model parameters. As mentioned by Saltelli et al. (2004), this means that evaluating first-order effects provides similar information to that of the RSA method. In reference to Eq. (2), the first-order effect S_p represents the fraction of the output variance $V[Y]$ that would be removed if the value of the parameter θ_p could be fixed (e.g. Saltelli et al., 2004). The total effect S_{Tp} of the parameter θ_p is given by (Homma and Saltelli, 1996)

$$S_{Tp} = \frac{V_p}{V[Y]} + \frac{\sum\limits_{q \neq p} V_{pq}}{V[Y]} + \frac{\sum\limits_{q \neq p} \sum\limits_{r > q} V_{pqr}}{V[Y]} + \ldots, \tag{4}$$

which is esentially the sum of the first-order effect and all the higher order terms in Eq. (2) that involve θ_p. Therefore, the total effect S_{Tp} indicates the overall importance of the parameter θ_p in the variability of Y, including both its first-order effect and interactions with other parameters. Mathematically, the total effect S_{Tp} measures the fraction of the output variance $V[Y]$ that would remain if the value of θ_p was unknown, but the true values of the remaining parameters could be fixed (e.g. Saltelli et al., 2004). In the case of non-correlated parameters, the total effect S_{Tp} is greater than or equal to the first-order effect S_p.

The analysis of first-order effects and total effects allows a straightforward diagnose of parameter sensitivities (Saltelli et al., 2004). If the total effect of a parameter is small, it can be concluded that the parameter is not important in determining the value of Y; by contrast, large total effects are necesarily associated with influential parameters. In addition to this, the difference between the total effect and the first-order effect of a parameter indicates to what extent the parameter is involved in interactions with others parameters. Finally, a large first-order effect proves that a parameter is influential on its own, independently of interactions with other parameters, while a small first-order effect found together with a large total effect shows that the parameter affects the output Y mainly through interactions with other parameters.

Variance decompositions similar to that of Eq. (2) can be written by grouping the parameters into subsets (see e.g. Saltelli et al., 2004). In that case, the first-order effect of a group of parameters indicates to what extent the parameters in the group affect the output quantity Y, excluding the effect of interactions with parameters in other groups. The total effect of a group of parameters includes both the first-order effect of the group and the interactions with parameters outside the group. Thus, the total effect of a group of parameters is a measure of the overall importance of the group of parameters in the variability of Y.

Although Monte Carlo methods can be used for the purpose of exploring the feasible space of the parameter set when calculating the variance terms in Eq. (2), these may be very computationally demanding (Saltelli et al., 2004; Tang et al., 2007). In the case of non-correlated parameters, the FAST method (Cukier et al., 1973, 1978) is a sampling strategy for the calculation of first-order effects at a lower computational cost. Saltelli et al. (1999) further developed this latter method into the Extended FAST method, which allows the simultaneous calculation of first-order and total effects.

3 Models description

3.1 Takagi-Sugeno-Kang fuzzy models

The fundamental elements of fuzzy sets theory were first proposed more than four decades ago (Zadeh, 1965), but applications of related modelling tools in hydrology are relatively recent (see e.g. Demicco and Klir, 2004). Applications of fuzzy methods in the hydrological context include modelling groundwater flow phenomena (Bárdossy and Disse, 1993; Bárdossy et al., 1995; Dou et al., 1999), and the interdependence between global circulation and precipitation (Özelkan et al. 1998; Pongracz et al. 2001; Zehe et al., 2006), for example. In the narrower context of river flow forecasting, fuzzy methods have been used for parameter estimation (Seibert, 1999; Yu and Yang, 2000), uncertainty analysis (Özelkan and Duckstein, 2001; Bárdossy et al., 2006; Jacquin and Shamseldin, 2007) and the development of rainfall-runoff models (Hundecha and Bárdossy, 2001; Vernieuwe et al., 2005), among other applications.

Fuzzy inference systems, or fuzzy models, are non-linear models that intend to describe the input-output relationship of a real system using a set of fuzzy IF-THEN rules and the inference mechanisms of fuzzy logic. In the case of Takagi-Sugeno-Kang (TSK) fuzzy inference systems, each fuzzy rule represents a local model of the real system under consideration (Takagi and Sugeno, 1985). The rules of a TSK system with input vector $X = (X_1, X_2, \ldots, X_K)$ and output Y have the general form

$$\text{IF } (X_1 \text{ is } A_{1,m}) \text{ AND } (X_2 \text{ is } A_{2,m}) \text{ AND} \ldots \text{AND } (X_K \text{ is } A_{K,m}) \text{ THEN } Y = f_m(X) \tag{5}$$

where the linguistic terms $A_{k,m}$ in the rule antecedents (i.e. the IF parts of the rules) represent fuzzy sets (Zadeh, 1965) with membership functions, $\mu_{k,m}(x_k)$, which are used to partition the domains of the input variables into overlapping regions. The functions f_m in the rule consequents (i.e. the THEN parts of the rules) are usually first-order polynomials having the form

$$f_m(X_1, X_2, \ldots, X_K) = b_{0,m} + b_{1,m}X_1 + b_{2,m}X_2, \ldots, b_{K,m}X_K. \tag{6}$$

For a given input $X = x = (x_1, x_2, \ldots, x_K)$, the degree of fulfilment (DOF) of each rule evaluates the compatibility of the input $X = x = (x_1, x_2, \ldots, x_K)$ with the rule antecedent and ultimately determines the contribution of the rule's response $y = f_m(x_1, x_2, \ldots, x_K)$ to the overall model's output. In the case of Gaussian type membership functions, whose analytical expression is given by

$$\mu_{k,m}(x_k) = \exp\left[-\frac{(x_k - c_{k,m})^2}{2\sigma_{k,m}{}^2}\right], \tag{7}$$

each membership function has two parameters, namely the centre $c_{k,m}$ and the spread $\sigma_{k,m}$. The degree of firing is frequently evaluated using the product operator, in which case it can be expressed as

$$\mathrm{DOF}_m(x)=\mu_{A_{1,m}}(x_1)\cdot\mu_{A_{2,m}}(x_2)\cdot\ldots\cdot\mu_{A_{K,m}}(x_K). \tag{8}$$

Finally, the overall output of a normalised first-order TSK fuzzy model with M rules is calculated according to

$$y=\frac{\sum_{m=1}^{M}\mathrm{DOF}_m(x_1,x_2,\ldots,x_K)\cdot\left[b_{0,m}+b_{1,m}x_1+b_{2,m}x_2+\ldots+b_{K,m}x_K\right]}{\sum_{m=1}^{M}\mathrm{DOF}_m(x_1,x_2,\ldots,x_K)}. \tag{9}$$

3.2 Rainfall-runoff fuzzy models under investigation

The rainfall-runoff models under investigation, previously proposed by Jacquin and Shamseldin (2006), correspond to TSK type fuzzy inference systems having the discharge in the catchment outlet as output variable. A brief description of these models is given in what follows, but further details on their interpretation and similarities with existing rainfall-runoff models can be found in the work by Jacquin and Shamseldin (2006).

The models can be classified in two types, each intended to account for different kinds of dominant non-linear effects in the rainfall-runoff relationship. Fuzzy models type 1 are intended to incorporate the effect of changes in the prevailing soil moisture content, while fuzzy models type 2 address the phenomenon of seasonality. Each fuzzy model type consists of five model structures of increasing complexity, where the most complex fuzzy models $\mathrm{TSK}_{1.5}$ and $\mathrm{TSK}_{2.5}$ include all the model components found in the remaining fuzzy models of the respective type. The rules of the fuzzy models are given by

$$\mathrm{TSK}_{\mathrm{mtype}.1}:\ \mathrm{IF}\ (V_{\mathrm{mtype}}\ \mathrm{is}\ A_m)\ \mathrm{THEN}\ Q^n=b_{0,m}, \tag{10}$$

$$\mathrm{TSK}_{\mathrm{mtype}.2}: \text{ IF } (V_{\mathrm{mtype}} \text{ is } A_m) \text{ THEN } Q^n = b_{1,m} RI^n, \tag{11}$$

$$\mathrm{TSK}_{\mathrm{mtype}.3}: \text{ IF } (V_{\mathrm{mtype}} \text{ is } A_m) \text{ THEN } Q^n = b_{0,m} + b_{1,m} RI^n, \tag{12}$$

$$\mathrm{TSK}_{\mathrm{mtype}.4}: \text{ IF } (V_{\mathrm{mtype}} \text{ is } A_m) \text{ THEN } Q^n = b_{2,m} \cdot \sum_{j=1}^{L} h_{j,m} P^n_{i-j+1}, \tag{13}$$

$$\mathrm{TSK}_{\mathrm{mtype}.5}: \text{ IF } (V_{\mathrm{mtype}} \text{ is } A_m) \text{ THEN } Q^n = b_{0,m} + b_{2,m} \cdot \sum_{j=1}^{L} h_{j,m} P^n_{i-j+1}, \tag{14}$$

where mtype represents the model type, i.e. type 1 or type 2. In all cases, the output variable in the rule consequents is given by the normalised discharge Q^n, calculated as the quotient between the discharge at the catchment outlet Q and the maximum discharge Q_{max} observed during the calibration period.

The choice of antecedent input variable V_{mtype} depends on the fuzzy model type under consideration. In the case of fuzzy models type 1, this corresponds to a normalised rainfall index RI^n, intended to give an indication of the prevailing soil moisture conditions in the catchment. Accordingly, the rule consequents of fuzzy models type 1 can be seen as local models of the rainfall-runoff relationship, valid for some fuzzily defined range of soil moisture content. At each time step i, the output of an auxiliary Simple Linear Model (SLM) of Nash and Foley (1982) is used to calculate the current rainfall index value RI_i from to the convolution summation

$$RI_i = G^a \cdot \sum_{j=1}^{L} P_{i-j+1} \cdot h^a_j, \tag{15}$$

where P_j is the rainfall measurement at time step j, L is the memory length of the catchment, G^a is the gain factor of the auxiliary SLM and h^a_j is the jth ordinate of the discrete pulse response function of the auxiliary SLM. The rainfall index RI_i is subsequently divided by its maximum value RI_{max} found during the calibration period, in order to obtain the normalised rainfall index RI_i^n (i.e. $RI^n = RI/RI_{max}$). With the aim of keeping the number of parameters to a minimum, the discrete pulse response ordinates h_j^a of the auxiliary SLM are obtained in parametric form using the gamma distribution model of Nash (1957). Fuzzy models type 2 use the time of the year t (in days) as input information to the rule antecedents. This is accomplished by calculating a normalised time of the year t^n, given by

$$t^n = t/365, \tag{16}$$

which is ultimately used as antecedent input variable. Each rule consequent of a type 2 fuzzy model can be seen as a model of the rainfall-runoff relationship that is associated with a particular season (fuzzily defined period) of the year.

Gaussian type membership functions, defined in Eq. (7), are chosen for modelling the antecedent fuzzy sets. However, in the case of fuzzy models type 1, the analytical expression of the leftmost (rule 1) and rightmost (rule M) membership function are modified in the following manner

$$\mathrm{DOF}_1(RI^n) = \mu_1(RI^n) = \begin{cases} 1, RI^n < c_1 \\ \exp\left[-\frac{(RI^n - c_1)^2}{2\sigma_1^2}\right], RI^n \geq c_1 \end{cases}, \tag{17}$$

$$\mathrm{DOF}_M(RI^n) = \mu_M(RI^n) = \begin{cases} \exp\left[-\frac{(RI^n - c_M)^2}{2\sigma_M^2}\right], RI^n \leq c_M \\ 1, RI^n > c_M \end{cases}, \tag{18}$$

while the membership function of the antecedent fuzzy sets of fuzzy models type 2 are given by

$$\mathrm{DOF}_m(t^n) = \mu_m(t^n) = \exp\left[-\frac{\left(\min\left\{\left|t^n - c_m\right|, 1 - \left|t^n - c_m\right|\right\}\right)^2}{2\sigma_m^2}\right] \tag{19}$$

in all cases. Details on the justification for Eqs. (17) to (19) can be found in the study by Jacquin and Shamseldin (2006). In both type 1 and type 2 fuzzy models, the description of each rule antecedent requires two parameters, namely the centres c_m and the spreads σ_m of the membership function, as shown in Table 1.

As seen in Eq. (10) to Eq. (14), the type of function f_m found in the rule consequents depends on the model structure being considered. Accordingly, the parameters found in the rule consequents vary among the different model structures, as shown in Table 1. As discussed in the work by Jacquin and Shamseldin (2006), the fuzzy models $TSK_{1.5}$ and $TSK_{2.5}$ are the most complex among fuzzy models type 1 and 2, respectively, because they include all the model components found in the remaining fuzzy models of the respective type. In particular, the rule consequents of the fuzzy models $TSK_{1.5}$ and $TSK_{2.5}$, as seen in Eq. (14), are first-order polynomials on the most recent normalized rainfall values P_j^n, which include a free term $b_{0,m}$ in addition to first-order terms. These fuzzy models allow a different pulse response function for each rule, characterized by gamma distribution parameters n_m and $(nK)_m$.

The sensitivity of the parameters of fuzzy models $TSK_{1.5}$ and $TSK_{2.5}$ is studied, in order to establish whether the parameters associated with a particular model component (e.g. the free terms in the rule consequents) are not important in determining the model performance. As mentioned earlier in the introduction, this situation would indicate that these parameters can be excluded from the search of the behavioural regions of the parameter space in a model calibration problem, by assigning them convenient values within their feasible ranges (Saltelli et al., 2004). In the case of the fuzzy models described here, this could be equivalent to considering a simpler model structure (e.g. fuzzy model $TSK_{1.3}$ instead of $TSK_{1.5}$), by removing the unimportant model component. The analysis is performed on fuzzy models with three rules, i.e. the same number of rules used in the study by Jacquin and Shamseldin (2006).

4 Methodology

4.1 Catchments and data

Table 2 shows the location of the catchments and the length of the data sets. The rainfall-runoff relationship of three of the test catchments, namely Sunkosi-1, Yanbian and Brosna, is affected by significant seasonal effects; in the case of the remaining catchments, intrinsic non-linearity due to changes in soil moisture contents has greater importance. The data used in this study consists of daily averaged values of precipitation and daily average discharge at the catchment's outlet. As shown in Table 2, the available data are divided into a calibration and a verification period for split-record simulations.

4.2 Measures of model performance

The sensitivity of the parameters of the fuzzy models $TSK_{1.5}$ and $TSK_{2.5}$ with respect to the model performance is analysed in terms of several measures of goodness of fit, assessing different aspects of the agreement between the observed and the simulated hydrograph. Each performance measure represents an output quantity Y, whose variability (with respect to the parameters of the model) is to be examined.

The first performance measure under examination is the R^2 efficiency criterion of Nash and Sutcliffe (1970), given by the following expression

$$R^2 = \frac{\mathrm{MSE}_0 - \mathrm{MSE}}{\mathrm{MSE}_0}, \tag{20}$$

where the initial mean squared error MSE_0 corresponds to the mean of the squares of the differences between the observed discharges and the long term mean during the calibration period. The mean squared error MSE is calculated as the mean of the squares of the differences between the model estimates and the observed discharges. The model efficiency R^2 is a decreasing function of the MSE, achieving a maximum value of unity if the model discharge estimates perfectly fit the observed discharges.

Another measure of goodness of fit used in this study is the deviation of runoff volumes, or relative error of the volumetric fit (REVF), given by

$$\mathrm{REVF}=1-\frac{\sum Q_{i}*}{\sum Q_{i}}, \quad (21)$$

where Q_i* and Q_i represent the model estimated and the observed discharge, respectively, at time step i. Positive REVF values indicate underestimation of discharge volumes, while negative REVF values are obtained when volumes are being overestimated.

The last measure of model performance considered in this study is the average relative error to the peak (REP), given by

$$\mathrm{REP}=\sum_{i=1}^{N_p}\frac{|Qp_i-Qp_i*|}{N_p Qp_i} \quad (22)$$

where N_p is the number of selected flow peaks, Qp_i represents a peak in the observed hydrograph, and Qp_i* is the model estimated discharge for the same time step as Qp_i. The REP would be equal to zero in the ideal case of a perfect estimation of all the selected flow peaks; increasing REP values indicate deterioration in the ability of the model to estimate the peak discharges. In this study, the discharge peaks retained for the calculation of the REP values are those exceeding the 90% of the calibration discharge data.

4.3 Computational experiments

4.3.1 Application of the RSA method

A random sample of 40 000 parameter sets is generated, both for the fuzzy model $\mathrm{TSK}_{1.5}$ and for $\mathrm{TSK}_{2.5}$. The feasible space for sample generation is defined by the parameter bounds established in Table 3. Except in the case of the free terms $b_{0,m}$, these bounds are the same as those imposed by Jacquin and Shamseldin (2006) for the calibration of the fuzzy models. In the case of the free terms $b_{0,m}$, whose values were not bounded during the calibration of the fuzzy models, the bounds shown in Table 3 are defined in such a manner that they are 50% wider than the range of $b_{0,m}$ values estimated by calibration of $TSK_{1.5}$ and $TSK_{2.5}$ for the test catchments. The performance of the fuzzy models is evaluated using the measures of goodness of fit indicated in Sect. 4.2. Probability distribution plots, produced using the software MCAT (Wagener et al., 2001b), are used for visually detecting sensitive parameters in the manner explained in Sect. 2.2.1.

4.3.2 Application of the SVD method

The SVD method is applied by splitting the model parameters in groups, with the purpose of clearly highlighting the importance of each model component in the performance of the fuzzy models. The groups considered are:

1. antecedent centres c_m,
2. antecedent spreads σ_m,
3. polynomial free terms $b_{0,m}$,
4. polynomial coefficients $b_{2,m}$,
5. gamma distribution parameters $n_{m,}$and
6. gamma distribution parameters $(nK)_m$.

In this case, the first-order effect of a group estimates to what extent the parameters in the group affect the model performance, excluding the effect of interactions with parameters outside the group. Similarly, the total effect sensitivity index of a group of parameters provides an estimation of the overall importance of the group in the performance of the fuzzy models, including interactions with parameters outside the group.

The samples of model parameters and the sensitivity indices of the groups of parameters defined above are obtained with the sensitivity analysis software SIMLAB 2.2 (European Union Joint Research Centre, 2004), using the Extended FAST sampling method (Saltelli et al., 1999). The bounds used for producing the samples of parameter sets are the same as those specified above for the case of the RSA method. The number of parameter sets in the sample is 9750.

5 Results

5.1 RSA results

As an example of the application of the method, Figure 1 shows the results of RSA when applied to the fuzzy model $TSK_{1.5}$ in the Sunkosi-1 catchment, using the efficiency R^2 as a measure model performance. Only the plots corresponding one fuzzy rule are shown, but similar ones are obtained for the remaining rules. From the analysis of this figure, it can be concluded that the model efficiency R^2 is sensitive to univariate changes in the parameters c_m, σ_m, and, particularly, $b_{0,m}$. The remaining parameters of $TSK_{1.5}$ are either non-important for determining the efficiency R^2, or their influence arises from interactions with other parameters.

Table 4 presents a qualitative classification of the parameters of the fuzzy models, similar to that presented by Tang et al. (2007), based on visual analysis of plots such as those in Fig. 1. It can be observed that parameter sensitivities of fuzzy models $TSK_{1.5}$ and $TSK_{2.5}$ differ, and that the sensitivity of the parameters of these fuzzy models depends on the type of catchment (seasonal or non-seasonal) where the models are applied. For example, the performance of $TSK_{1.5}$ does not seem to be greatly affected by univariate changes in the parameters $b_{2,m}$. In the case of the fuzzy model $TSK_{2.5}$, however, the values of all three measures of model performance are sensitive to the parameters $b_{2,m}$, although only when $TSK_{2.5}$ is applied in the seasonal catchments. Table 4 also shows that the sensitivity of a parameter depends on the measure of model performance being considered. Comparison of the columns corresponding to R^2 and REP reveals that these measures of model performance show sensitivity to the same type of parameters. This result is not surprising, because these measures of model performance emphasize the model errors in the high flow zone. In contrast, parameter sensitivities of the performance measure REVF are different from the other two.

In any case, a feature that is common to all models, catchments and measures of model performance is that the RSA method shows the polynomial free terms $b_{0,m}$ as the most sensitive parameters. In addition to this, the gamma distribution parameters n_m and $(nK)_m$ are not revealed as sensitive in any fuzzy model or catchment.

5.2 SVD results

Tables 5 and 6 show the first-order effects for the fuzzy models $TSK_{1.5}$ and $TSK_{2.5}$, respectively. Similarly, Tables 7 and 8 show the total effects corresponding to $TSK_{1.5}$ and $TSK_{2.5}$, respectively. It seems important to point out that the sensitivity indices obtained for the calibration and the verification period are consistently close, which suggests that the results obtained in this analysis are independent on the period of data used for evaluation. The following discussion gives the conclusions obtained from the analysis of Tables 5 to 8, according to the criteria outlined in Sect. 2.2.2.

5.2.1 First-order effects

As explained in Sect. 2.2.2, the first-order effect of a group of parameters represents the sensitivity of the quantity Y under examination to changes in the parameters of the group, without considering the effect of interactions with parameters outside the group. Conceptually, the results of the RSA method provide similar information concerning the sensitivity of parameters. Therefore, it is not surprising that the analysis of the first-order effects in Tables 5 and 6 confirms the main results of the RSA method, presented in the previous section. In the first place, the highest first-order effects in Tables 5 and 6

HESSD
5, 1967–2003, 2008

A. P. Jacquin and A. Y. Shamseldin

Title Page
Abstract | Introduction
Conclusions | References
Tables | Figures
◀◀ | ▶▶
◀ | ▶
Back | Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

correspond to the group of parameters $b_{0,m}$, which are the only parameters classified as very sensitive (VS) by the RSA method. In addition to this, the first-order effects of the groups of parameters classified as sensitive (S) by the RSA method are lower than in the previous case, although still non-negligible. Finally, the first-order effects of those groups of parameters for which the RSA method did not reveal any sensitivity are negligible in all cases.

5.2.2 Total effects

Recalling the discussion in Sect. 2.2.2, the total effect of a group of parameters measures the sensitivity of the model response to the parameters in the group, including possible interactions with parameters in other groups. Accordingly, the total effects shown in Tables 7 and 8 are necessarily higher than the first-order effects in Tables 5 and 6, which exclude the effect of interactions outside the group. These interactions with other groups are revealed by great differences between the group's total effects and first-order effects. The following discussion analyses the sensitivity of each group of parameters according to their total effects and the existence of interactions between groups.

To start with, the total effects shown in Table 7 indicate that the free terms $b_{0,m}$ are the most influential for determining the performance of the fuzzy model $TSK_{1.5}$, with respect to all three measures of model performance. It can also be seen that the sensitivity of the antecedent centres c_m is also very high. In all catchments and measures of model performance, the differences between the total effects and the first- order effects of both groups are large, which is not observed in the remaining groups. This situation indicates that, among all the groups of parameters, only the groups c_m and $b_{0,m}$ are involved in strong interactions, and that this interactions occur mainly between them. Another feature shown in Table 7 is that the performance of the fuzzy model $TSK_{1.5}$ can be moderately affected by the ancedent spreads σ_m. In particular, this group obtains total effects ranging between 0.16 and 0.28 for the measures of model performance R^2 and REP, with the highest total effects being observed in the seasonal catchments. However, this sensitivity is not observed in the case of the performance measure REVF, for which the total effects of this group are below 0.13 in all cases. Finally, analysis of Table 7 reveals that the remaining groups of parameters, namely the coefficients $b_{2,m}$ and the gamma distribution parameters n_m and $(nK)_m$, are not influential for determining the performance of the fuzzy model $TSK_{1.5}$, because the total effects of these groups are consistently small. More concretely, for all catchments and measures of model performance, the total effects of these groups do not exceed 0.14.

The total effects shown in Table 8 indicate that the free terms $b_{0,m}$ are the parameters with the highest influence in the performance of $TSK_{2.5}$, as indicated by all three measures of model performance and in all the catchments. Unlike the case of $TSK_{1.5}$, the total effects of the antecedent centres c_m are generally low. However, a moderate effect is seen when the model $TSK_{2.5}$ is applied in the seasonal catchments and the measure of model performance REP is used, in which case the total effects of the centres c_m range between 0.31 and 0.48. Similarly, the coefficients $b_{2,m}$ do not greatly affect the performance of $TSK_{2.5}$, except for the moderate effects seen in the seasonal catchments for the case of the measure of model performance REP. The last group having some importance in determining the performance of $TSK_{2.5}$ is that of the antecedent spreads σ_m, for which the total effects range between 0.23 and 0.47. Finally, Table 8 also reveals that the gamma distribution parameters n_m and $(nK)_m$ are not influential for to the performance of the fuzzy model $TSK_{2.5}$. In generall, the total effects of these groups remain below 0.18. The only exception is seen in the Brosna catchment, where the total effect of the parameters $(nK)_m$ in the measure of model performance REP reaches 0.29.

6 Conclusions

This study has analysed the sensitivity of the parameters of the Takagi-Sugeno-Kang rainfall-runoff fuzzy models proposed by Jacquin and Shamseldin (2006). These models can be classified in two model types, each consisting of five model structures of increasing complexity. The fuzzy models $TSK_{1.5}$ and $TSK_{2.5}$ are the most complex within type 1 and type 2, respectively, and they include all the model components found in the simpler fuzzy models of the corresponding group. Two global sensitivity analysis methods were applied, namely the RSA and SVD methods. The RSA method has the disadvantage of not being able to detect sensitivities arising from parameter interactions. By contrast, the SVD method is suitable for analysing models where the model response surface is expected to be affected by interactions at a local scale and/or local optima, such as the case of the rainfall-runoff fuzzy models of Jacquin and Shamseldin (2006).

In the case of the fuzzy model $TSK_{1.5}$, it was found that the performance of the model is quite sensitive to the antecedent centres c_m , although most of this influence is due to interactions with the parameters $b_{0,m}$. It was also observed that the antecedent spreads σ_m have a modest importance in determining the model performance of $TSK_{1.5}$. Similarly, it was observed that the antecedent parameters c_m and σ_m have a generally low influence in the performance of $TSK_{2.5}$. These situations imply that the actual definition of the antecedent fuzzy sets is not, on its own, a determinant factor for the goodness of fit of the fuzzy models $TSK_{1.5}$ and $TSK_{2.5}$. It would be possible to fix the antecedent parameters prior to the calibration of the fuzzy models without an important deterioration of model performance, provided that the values of the remaining parameters are conveniently adjusted.

It was also observed that, in general, the coefficients $b_{2,m}$ do not greatly affect the performance of $TSK_{1.5}$ and $TSK_{2.5}$. By contrast, the free terms $b_{0,m}$ were identified as the parameters with the highest influence in the performance of $TSK_{1.5}$ and $TSK_{2.5}$. Moreover, these parameters exhibit quite high (and also the highest) first-order effects in all cases, modifying the model performance independently of interactions with parameters in other groups. As pointed out by Saltelli et al. (2004), the identification of appropriate values for parameters with a high first-order effect should be a priority during the process of model calibration. Nevertheless, Jacquin and Shamseldin (2006) showed that removing the parameters $b_{0,m}$, by moving from the fuzzy models $TSK_{x.3}$

and $TSK_{x.5}$ to the simpler $TSK_{x.2}$ and $TSK_{x.4}$, respectively, does not have a significant impact in the performance of the optimised rainfall-runoff fuzzy model. This situation indicates that finding the "true" values of the free terms $b_{0,m}$ does not necessarily improve the goodness of fit, as long as these parameters are all assigned zero values.

This apparent contradiction between the findings of Jacquin and Shamseldin (2006) and the results of the sensitivity analysis presented in this study can be explained after a more careful consideration of the facts. First, the large total and first-order effects of the parameters $b_{0,m}$ indicate that allowing a free variation of these parameters across their feasible range does produce important changes in the goodness of fit of the fuzzy models $TSK_{1.5}$ and $TSK_{2.5}$. In fact, assigning very inappropriate values to these parameters may result in an important deterioration of model performance; for example, assigning highly negative values to the parameters $b_{0,m}$ in all of the rules would result in highly negative discharge estimates in cases where the most recent rainfall segment is null. However, it is still possible that a relatively good (and nearly optimal) model response can be obtained by fixing the values of the parameters $b_{0,m}$ as zero and calibrating the remaing model parameters accordingly.

Finally, the results of this study indicate that the gamma distribution parameters n_m and $(nK)_m$ are definitely unimportant in determining the goodness of fit of the fuzzy models $TSK_{1.5}$ and $TSK_{2.5}$. These results are in agreement with the findings of Jacquin and Shamseldin (2006), in the sense that allowing a different pulse response for each rule consequent (i.e. moving from the fuzzy models $TSK_{x.2}$ and $TSK_{x.3}$ to $TSK_{x.4}$ and $TSK_{x.5}$, respectively) does not necessarily improve the performance of the optimised fuzzy model. In order to reduce the dimensionality of the optimisation problem associated with the calibration of the model, these parameters could be excluded from the search of the behavioural regions of the parameter space, by assigning to them fixed values within their feasible ranges. For example, the values of the parameters n_m and $(nK)_m$ of all the fuzzy rules could be given the same values as those of the auxiliary SLM. As seen in Sect. 3.2, this would be equivalent to abandoning the models $TSK_{1.5}$ and $TSK_{2.5}$ in favour of the more parsimonious $TSK_{1.3}$ and $TSK_{2.3}$, respectively.

Further work could explore the application of the GSA methods applied in this paper to other hydrological models based on soft computing methods. For example, it would be convenient to perform sensitivity analysis of other fuzzy model structures. Similarly, it would also be interesting to use this methods method for investigating the relative importance of the parameters of typical neural network based rainfall-runoff models. The application of the method would allow identifying those parameters whose values can be fixed without important deterioration in model performance and those parameters whose appropriate calibration is most important.

References

Bárdossy, A., Bronstert, A., and Merz, B.: 1-, 2- and 3-dimensional modelling of water movement in the unsaturated soil matrix using a fuzzy approach, Adv. Water Resour., 18(4), 237–251, 1995.

Bárdossy, A. and Disse, M.: Fuzzy rule-based models for infiltration, Water Resour. Res., 29(2), 373–382, 1993.

Bárdossy, A., Mascellani, G., and Franchini, M.: Fuzzy unit hydrograph, Water Resour. Res., 42(2), W02401, doi:10.1029/2004WR003751, 2006.

Beven, K. J.: Rainfall-runoff modelling: the primer, John Wiley and Sons, Chichester, UK, 360 pp., 2001.

Castaings, W., Dartus, D., Honnorat, M., Le Dimet, F. X., Loukili, Y., and Monnier, J.: Automatic differentiation: a tool for variational data assimilation and adjoint sensitivity analysis for flood modelling, in: Automatic differentiation: applications, theory, and tools, edited by: Bücker, H. M., Corliss, G., Hovland, P., Naumann, U., and Norris, B., Springer, Chicago, USA, 249–262, 2005.

Chan, K., Tarantola, S., Saltelli, A., and Sobol, I. M.: Variance-based methods, in: Sensitivity Analysis, edited by: Saltelli, A., Chan, K., and Scott, E. M., John Wiley and Sons, Chichester, UK, 167–197, 2000.

Cukier, R. I., Fortuin, C. M., Shuler, K. E., Petschek, A. G., and Schaibly, J. H.: A Study of the sensitivity of coupled reaction systems to uncertainties in rate coefficients, I: Theory, J. Chem. Phys., 59(8), 3873–3878, 1973.

Cukier, R. I., Levine, H. B., and Shuler, K. E.: Nonlinear sensitivity analysis of multiparameter model systems, J. Comput. Phys., 26, 1–42, 1978.
Demicco, R. and Klir, G.: Fuzzy Logic in Geology, Academic Press, Amsterdam, The Netherlands, 347 pp., 2004.
Dou, X., Wouldt, W., and Bogardi, I.: Fuzzy rule-based approach to describe solute transport in the unsaturated zone, J. Hydrol., 220, 74–85, 1999.
European Union Joint Research Centre: SIMLAB: simulation environment for uncertainty and sensitivity analysis (version 2.2), Applied Statistics Sector, Institute for Systems, Informatics and Safety, European Union Joint Research Centre, Ispra, Italy, 2004.
Fieberg, J. and Jenkings, K. J.: Assessing uncertainty in ecological systems using global sensitivity analyses: a case example of simulated wolf reintroduction effects on elk, Ecol. Model., 187, 259–280, 2005.
Francos, A., Elorzab, F. J., Bouraouia, F., Bidoglioa, G., and Galbiatia, L.: Sensitivity analysis of distributed environmental simulation models: understanding the model behaviour in hydrological studies at the catchment scale, Reliab. Eng. Syst. Safe., 205–218, 2003.
Gupta, V. K. and Sorooshian, S.: The automatic calibration of conceptual catchment models using derivative-based optimization algorithms, Water Resour. Res., 21(4), 473–485, 1985.
Homma, T. and Saltelli, A.: Importance measures in global sensitivity analysis of model output, Reliab. Eng. Syst. Safe, 52, 1–17, 1996.
Hornberger, G. M. and Spear, R. C.: An approach to the preliminary analysis of environmental systems, J. Environ. Manage., 12, 7–18, 1981.
Hundecha, Y., Bárdossy, A., and Theisen, H.-W.: Development of a fuzzy logic-based rainfall-runoff model, Hydrolog. Sci. J., 46(3), 363–376, 2001.
Hundecha, Y. and Bárdossy, A.: Modeling of the effect of land use changes on the runoff generation of a river basin through parameter regionalization of a watershed model, J. Hydrol., 292, 281–295, 2004.
Jacquin, A. P. and Shamseldin, A. Y.: Development of rainfall-runoff models using Takagi-Sugeno-Kang fuzzy inference systems, J. Hydrol., 329, 154–173, 2006.
Jacquin, A. P. and Shamseldin, A. Y.: Development of a possibilistic method for the evaluation of predictive uncertainty in rainfall-runoff modelling, Water Resour. Res., 43(4), W04425, doi:10.1029/2006WR005072, 2007.
Kanso, A., Chebboa, G., and Tassina, B.: Application of MCMC-Global sensitivity analysis method for model calibration to urban runoff quality model, in: Sensitivity Analysis of Model

Output, edited by: Hanson, K. M. and Hemez, F. M., Los Alamos National Laboratory, 17–26, available at: http://library.lanl.gov, 2005.
McIntyre, N. R., Wagener, T., Wheater, H., and Chapra, S. C.: Risk-based modelling of surface water quality: a case study of the Charles River, Massachussetts, J. Hydrol., 274, 225–247, 2003.
Mein, R. G. and Brown, B. M.: Sensitivity of optimized parameters in watershed models, Water Resour. Res, 14(2), 299–303, 1978.
Mertens, J., Madsen, H., Kristensen, M., Jacques, D., and Feyen, J.: Sensitivity of soil parameters in saturated zone modelling and the relation between effective, laboratory and in situ estimates, Hydrol. Process., 19(8), 1611–1633, doi:10.1002/hyp.5591, 2005.
Morris, M. D.: Factorial sampling plans for preliminary computational experiments, Technometrics, 33(2), 161–074, 1991.
Nandakumar, N. and Mein, R. G.: Uncertainty in rainfall-runoff model simulations and the implications for predicting the hydrologic effects of land-use change, J. Hydrol., 192, 211–232, 1997.
Nash, J. E.: The form of the instantaneous unit hydrograph, IASH Publication, 45(3), 114–118, 1957.
Nash, J. E. and Foley, J. J.: Linear models of rainfall-runoff systems, in: Proceedings of the International Symposium on Rainfall-Runoff Modeling, Mississippi State University, USA, 18–21 May 1981, 51–66, 1982.
Nash, J. E. and Sutcliffe, J. V.: River flow forecasting through conceptual models, Part 1: A discussion of principles, J. Hydrol., 10, 282–290, 1970.
O'Connor, K. M.: River flow forecasting, in: River Basin Modelling for Flood Risk Mitigation, edited by: Knight, D. W. and Shamseldin, A. Y., Taylor and Francis, London, UK, 197–213, 2005.
Özelkan, E. C. and Duckstein, L.: Fuzzy conceptual rainfall-runoff models, J. Hydrol., 253, 41–68, 2001.
Özelkan, E. C., Galambosi, A., Duckstein, L., and Bárdossy, A.: A multi-objective fuzzy classification of large scale atmospheric circulation patterns for precipitation modelling, Appl. Math. Comput., 91, 127–142, 1998.
Pappenberger, F., Beven, K. J., Ratto, M., and Matgen, P.: Multi-method global sensitivity analysis of flood inundation models, Adv. Water Resour., 31(1), 1–14, 2008.
Pongracz, R., Bartholy, J., and Bogardi, I.: Fuzzy rule-based prediction of monthly precipitation,

Phys. Chem. Earth Pt. B, 26(9), 663–667, 2001.
Ratto, M., Young, P. C., Romanowicz, R., Pappenberger, F., Saltelli, A., and Pagano, A.: Uncertainty, sensitivity analysis and the role of data based mechanistic modelling in hydrology, Hydrol. Earth Syst. Sci., 11, 1249–1266, 2007, http://www.hydrol-earth-syst-sci.net/11/1249/2007/.
Saltelli, A., Tarantola, S., and Campolongo, F.: Sensitivity analysis as an ingredient of modelling, Stat. Sci., 15(4), 377–395, 2000.
Saltelli, A., Tarantola, S., Campolongo, F., and Ratto, M.: Sensitivity analysis in practice: a guide to assessing scientific models, John Wiley and Sons, Chichester, UK, 219 pp., 2004.
Saltelli, A., Tarantola, S., and Chan, K.: A quantitative, model independent method for global sensitivity analysis of model output, Technometrics, 41(1), 39–56, 1999.
Seibert, J.: Regionalization of parameter of a conceptual rainfall-runoff model, Agr. Forest Meteorol., 98-99, 279–293, 1999.
Sobol, I. M.: Sensitivity analysis for non-linear mathematical models, Math. Mod. Comput. Exp., 1(4), 407–414, 1993.
Sorooshian, S. and Gupta, V. K.: Model calibration, in: Conceptual Models of Watershed Hydrology, edited by: Singh, V. P., Water Resources Publications, Colorado, USA, 23–67, 1995.
Spear, R. C. and Hornberger, G. M.: Eutrophication in Peel Inlet, II: Identification of critical uncertainties via generalised sensitivity analysis, Water Resour. Res., 14, 43–49, 1980.
Sugeno, M. and Kang, G. T.: Structure identification of fuzzy model, Fuzzy Set. Syst., 28, 15–33, 1988.
Takagi, T. and Sugeno, M.: Fuzzy identification of systems and its application to modeling and control, IEEE T. Syst. Man Cyb., SMC-15(1), 116–132, 1985.
Tang, Y., Reed, P., Wagener, T., and van Werkhoven, K.: Comparing sensitivity analysis methods to advance lumped watershed model identification and evaluation, Hydrol. Earth Syst. Sci., 11, 793–817, 2007, http://www.hydrol-earth-syst-sci.net/11/793/2007/.
Vernieuwe, H., Georgieva, O., De Baets, B., Pauwels, V. R. N., Verhoest, N. E. C., and De Troch, F. P.: Comparison of data-driven Takagi-Sugeno models of rainfall-discharge dynamics, J. Hydrol., 302, 173–186, 2005.
Wagener, T., Boyle, D., Lees, M. J., Wheater, H. S., Gupta, H. V., and Sorooshian, S.: A framework for development and application of hydrological models, Hydrol. Earth Syst. Sci., 5, 13–26, 2001a,

http://www.hydrol-earth-syst-sci.net/5/13/2001/.

Wagener, T., Lees, M. J., and Wheater, H. S.: MCAT: Monte-Carlo Analysis Toolbox (version 3), Civil and Environmental Engineering Department, Imperial College of Science Technology and Medicine, London, UK, 2001b.

Wagener, T., Wheater, H. S., and Gupta, H. V.: Identification and evaluation of watershed models, in: Calibration of Watershed Models, edited by: Duan, Q., Gupta, H. V., Sorooshian, S., Rousseau, A. N., and Turcotte, R., American Geophysical Union, Washington, USA, 29–47, 2002.

Wang, X., Frankenberger, J. R., and Kladivko, E. J.: Uncertainties in DRAINMOD predictions of subsurface drain flow fr and Indiana silt loam using the GLUE methodology, Hydrol. Process., 20(14), 3069–3084, 2006.

Xiong, L. and O'Connor, K. M.: Analysis of the response surface of the objective function by the optimum parameter curve: how good can the optimum parameter values be?, J. Hydrol., 234, 187–207, 2000.

Yu, P.-S. and Yang, T. C.: Fuzzy multi-objective function for rainfall-runoff model calibration, J. Hydrol., 238, 1–14, 2000.

Zadeh, L. A.: Fuzzy Sets, Inform. Control, 8(3), 338-353, 1965.

Zehe, E. and Blöschl, G.: Predictability of hydrologic response at the plot and catchment scales: the role of initial conditions, Water Resour. Res., 40(10), W10202, doi:10.1029/2003WR002869, 2004.

Zehe, E., Becker, R., Bárdossy, A., and Plate, E.: Uncertainty of simulated catchment runoff response in the presence of threshold processes: role of initial soil moisture and precipitation, J. Hydrol., 315, 183–202, 2005.

Zehe, E., Singh, A. K., and Bárdossy, A.: Modelling of monsoon rainfall for a mesoscale catchment in North-West India, I: assessment of objective circulation patterns, Hydrol. Earth Syst. Sci., 10, 797–806, 2006, http://www.hydrol-earth-syst-sci.net/10/797/2006/.

HESSD

5, 1967–2003, 2008

Table 1. Parameters involved in the TSK rainfall-runoff fuzzy models proposed by Jacquin and Shamseldin (2006).

Model	c_m	σ_m	$b_{0,m}$	$b_{1,m}$	$b_{2,m}$	n_m	$(nK)_m$
$TSK_{X.1}$	✓	✓	✓				
$TSK_{X.2}$	✓	✓		✓			
$TSK_{X.3}$	✓	✓	✓	✓			
$TSK_{X.4}$	✓	✓			✓	✓	✓
$TSK_{X.5}$	✓	✓	✓		✓	✓	✓

Table 2. Location of the catchments and length of the data sets used in the experiments, including the definition of calibration and verification periods for split sampling tests.

Catchment	Country	No. of years in data set	Calibration period		Verification period	
			starting date	No. of years	starting date	No. of years
Sunkosi-1	Nepal	8	01/01/1975	6	01/01/1981	2
Yanbian	Central China	8	01/01/1978	6	01/01/1984	2
Shiquan-3	Central China	8	01/01/1973	6	01/01/1979	2
Brosna	Central Ireland	10	01/01/1969	8	01/01/1977	2
Bird Creek	Oklahoma, USA	8	01/10/1955	6	01/10/1961	2
Wollombi Brook	New Sout Wales, Australia	5	01/01/1963	4	01/01/1967	1

HESSD

5, 1967–2003, 2008

Sensitivity analysis of rainfall-runoff fuzzy models

A. P. Jacquin and A. Y. Shamseldin

Table 3. Feasible ranges of the parameters of the fuzzy models $TSK_{1.5}$ and $TSK_{2.5}$.

Parameter	Lower bound	Upper bound
c_m	0	1
σ_m	0.02	0.25
$b_{0,m}$	−1.315	1.456*
	−0.162	0.641**
$b_{2,m}$	0	P_{max}/Q_{max}
n_m	0.5	10
$(nK)_m$	0.5	30

* Bounds applicable to $TSK_{1.5}$
** Bounds applicable to $TSK_{2.5}$

HESSD
5, 1967–2003, 2008

A. P. Jacquin and A. Y. Shamseldin

Table 4. Parameters of the fuzzy models deemed sensitive or very sensitive by the RSA method according to the measures of model performance R^2, REVF and REP.

Catchment type	Parameter group	Fuzzy model $TSK_{1.5}$			Fuzzy model $TSK_{2.5}$		
		R^2	REVF	REP	R^2	REVF	REP
Seasonal	c_m	S	S	S	–	–	–
	σ_m	S	–	S	S	–	S
	$b_{0,m}$	VS	VS	VS	VS	VS	VS
	$b_{2,m}$	–	–	–	S	S	S
	n_m	–	–	–	–	–	–
	$(nK)_m$	–	–	–	–	–	–
Non-seasonal	c_m	S	–	S	–	–	–
	σ_m	–	–	–	–	–	–
	$b_{0,m}$	VS	VS	VS	VS	VS	VS
	$b_{2,m}$	–	–	–	–	–	–
	n_m	–	–	–	–	–	–
	$(nK)_m$	–	–	–	–	–	–

HESSD

5, 1967–2003, 2008

Table 5. First-order effect sensitivity indices of the parameters of the fuzzy model $TSK_{1.5}$ in the measures of model performance R^2, REVF and REP.

Catchment	Parameter group	Calibration			Verification		
		R^2	REVF	REP	R^2	REVF	REP
Sunkosi-1	c_m	0.04	0.02	0.09	0.05	0.02	0.09
	σ_m	0.04	0.00	0.04	0.04	0.00	0.04
	$b_{0,m}$	0.58	0.59	0.40	0.56	0.56	0.38
	$b_{2,m}$	0.01	0.01	0.03	0.01	0.01	0.02
	n_m	0.01	0.00	0.01	0.01	0.01	0.01
	$(nK)_m$	0.01	0.00	0.01	0.01	0.01	0.01
Yanbian	c_m	0.06	0.02	0.08	0.06	0.02	0.08
	σ_m	0.03	0.01	0.06	0.03	0.01	0.06
	$b_{0,m}$	0.51	0.47	0.42	0.52	0.48	0.46
	$b_{2,m}$	0.01	0.00	0.01	0.01	0.00	0.01
	n_m	0.01	0.01	0.02	0.01	0.01	0.02
	$(nK)_m$	0.01	0.01	0.01	0.01	0.01	0.01
Shiquan-3	c_m	0.07	0.02	0.09	0.07	0.02	0.10
	σ_m	0.02	0.01	0.02	0.02	0.01	0.03
	$b_{0,m}$	0.45	0.38	0.45	0.45	0.38	0.45
	$b_{2,m}$	0.01	0.01	0.01	0.01	0.01	0.01
	n_m	0.01	0.01	0.01	0.01	0.01	0.02
	$(nK)_m$	0.01	0.01	0.01	0.01	0.01	0.01
Brosna	c_m	0.08	0.02	0.08	0.08	0.02	0.08
	σ_m	0.04	0.01	0.06	0.05	0.01	0.06
	$b_{0,m}$	0.48	0.53	0.44	0.48	0.55	0.44
	$b_{2,m}$	0.01	0.01	0.02	0.01	0.01	0.02
	n_m	0.01	0.00	0.02	0.01	0.00	0.02
	$(nK)_m$	0.01	0.00	0.01	0.01	0.00	0.01
Bird Creek	c_m	0.06	0.02	0.09	0.06	0.02	0.08
	σ_m	0.02	0.01	0.03	0.02	0.01	0.02
	$b_{0,m}$	0.45	0.38	0.45	0.45	0.37	0.45
	$b_{2,m}$	0.01	0.01	0.01	0.01	0.01	0.01
	n_m	0.01	0.01	0.01	0.01	0.01	0.01
	$(nK)_m$	0.01	0.01	0.01	0.01	0.01	0.01
Wollombi Brook	c_m	0.06	0.02	0.08	0.06	0.02	0.07
	σ_m	0.02	0.01	0.03	0.02	0.01	0.03
	$b_{0,m}$	0.45	0.37	0.47	0.45	0.38	0.46
	$b_{2,m}$	0.01	0.01	0.01	0.01	0.01	0.01
	n_m	0.01	0.01	0.01	0.01	0.01	0.01
	$(nK)_m$	0.01	0.01	0.01	0.01	0.01	0.01

Table 6. First-order effect sensitivity indices of the parameters of the fuzzy model $TSK_{2.5}$ in the measures of model performance R^2, REVF and REP.

Catchment	Parameter group	Calibration			Verification		
		R^2	REVF	REP	R^2	REVF	REP
Sunkosi-1	c_m	0.01	0.00	0.10	0.01	0.00	0.07
	σ_m	0.03	0.00	0.09	0.04	0.00	0.06
	$b_{0,m}$	0.59	0.62	0.22	0.57	0.63	0.27
	$b_{2,m}$	0.03	0.04	0.08	0.01	0.03	0.08
	n_m	0.00	0.00	0.01	0.00	0.00	0.01
	$(nK)_m$	0.00	0.00	0.02	0.00	0.00	0.01
Yanbian	c_m	0.01	0.00	0.09	0.01	0.00	0.09
	σ_m	0.02	0.00	0.10	0.02	0.00	0.10
	$b_{0,m}$	0.62	0.64	0.32	0.62	0.64	0.30
	$b_{2,m}$	0.01	0.02	0.07	0.01	0.02	0.06
	n_m	0.00	0.00	0.01	0.00	0.00	0.01
	$(nK)_m$	0.00	0.00	0.02	0.00	0.00	0.03
Shiquan-3	c_m	0.00	0.00	0.01	0.00	0.00	0.01
	σ_m	0.02	0.00	0.01	0.02	0.00	0.02
	$b_{0,m}$	0.64	0.66	0.58	0.64	0.66	0.53
	$b_{2,m}$	0.00	0.00	0.01	0.00	0.00	0.01
	n_m	0.00	0.00	0.00	0.00	0.00	0.00
	$(nK)_m$	0.00	0.00	0.01	0.00	0.00	0.01
Brosna	c_m	0.00	0.00	0.06	0.01	0.00	0.08
	σ_m	0.03	0.00	0.13	0.03	0.00	0.14
	$b_{0,m}$	0.53	0.58	0.25	0.50	0.57	0.23
	$b_{2,m}$	0.13	0.11	0.13	0.13	0.12	0.11
	n_m	0.00	0.00	0.01	0.00	0.00	0.01
	$(nK)_m$	0.01	0.01	0.07	0.02	0.01	0.05
Bird Creek	c_m	0.00	0.00	0.01	0.00	0.00	0.01
	σ_m	0.01	0.00	0.01	0.01	0.00	0.01
	$b_{0,m}$	0.64	0.66	0.62	0.64	0.66	0.65
	$b_{2,m}$	0.00	0.00	0.00	0.00	0.00	0.00
	n_m	0.00	0.00	0.00	0.00	0.00	0.00
	$(nK)_m$	0.00	0.00	0.01	0.00	0.00	0.00
Wollombi Brook	c_m	0.00	0.00	0.01	0.00	0.00	0.01
	σ_m	0.01	0.00	0.01	0.01	0.00	0.01
	$b_{0,m}$	0.64	0.66	0.53	0.64	0.66	0.64
	$b_{2,m}$	0.00	0.00	0.02	0.00	0.00	0.02
	n_m	0.00	0.00	0.00	0.00	0.00	0.00
	$(nK)_m$	0.00	0.00	0.03	0.00	0.00	0.02

Table 7. Total effect sensitivity indices of the parameters of the fuzzy model $TSK_{1.5}$ in the measures of model performance R^2, REVF and REP.

Catchment	Parameter group	Calibration			Verification		
		R^2	REVF	REP	R^2	REVF	REP
Sunkosi-1	c_m	0.51	0.32	0.54	0.55	0.35	0.53
	σ_m	0.17	0.08	0.20	0.17	0.08	0.23
	$b_{0,m}$	0.94	0.98	0.91	0.94	0.98	0.91
	$b_{2,m}$	0.09	0.05	0.17	0.09	0.06	0.16
	n_m	0.12	0.07	0.10	0.13	0.07	0.10
	$(nK)_m$	0.08	0.07	0.10	0.08	0.07	0.11
Yanbian	c_m	0.62	0.44	0.49	0.61	0.43	0.45
	σ_m	0.16	0.08	0.28	0.16	0.08	0.26
	$b_{0,m}$	0.93	0.98	0.87	0.93	0.98	0.88
	$b_{2,m}$	0.09	0.06	0.11	0.09	0.06	0.12
	n_m	0.13	0.08	0.12	0.13	0.08	0.11
	$(nK)_m$	0.08	0.08	0.09	0.08	0.08	0.09
Shiquan-3	c_m	0.70	0.55	0.68	0.69	0.54	0.66
	σ_m	0.16	0.07	0.17	0.16	0.07	0.19
	$b_{0,m}$	0.93	0.98	0.91	0.93	0.98	0.89
	$b_{2,m}$	0.10	0.07	0.09	0.10	0.07	0.09
	n_m	0.14	0.09	0.12	0.13	0.09	0.13
	$(nK)_m$	0.09	0.08	0.08	0.09	0.08	0.09
Brosna	c_m	0.55	0.37	0.44	0.54	0.34	0.45
	σ_m	0.21	0.12	0.27	0.23	0.13	0.27
	$b_{0,m}$	0.89	0.98	0.87	0.88	0.97	0.88
	$b_{2,m}$	0.12	0.06	0.14	0.13	0.05	0.14
	n_m	0.12	0.06	0.11	0.12	0.06	0.12
	$(nK)_m$	0.09	0.07	0.11	0.09	0.07	0.11
Bird Creek	c_m	0.69	0.55	0.67	0.70	0.55	0.69
	σ_m	0.16	0.07	0.17	0.16	0.07	0.16
	$b_{0,m}$	0.94	0.98	0.90	0.94	0.98	0.91
	$b_{2,m}$	0.10	0.07	0.09	0.10	0.07	0.09
	n_m	0.13	0.09	0.12	0.14	0.09	0.12
	$(nK)_m$	0.09	0.08	0.08	0.09	0.08	0.08
Wollombi Brook	c_m	0.69	0.55	0.63	0.69	0.55	0.64
	σ_m	0.16	0.07	0.16	0.16	0.07	0.17
	$b_{0,m}$	0.94	0.98	0.91	0.94	0.98	0.91
	$b_{2,m}$	0.10	0.07	0.09	0.10	0.07	0.10
	n_m	0.13	0.09	0.12	0.13	0.09	0.12
	$(nK)_m$	0.09	0.08	0.09	0.09	0.08	0.10

Table 8. Total effect sensitivity indices of the parameters of the fuzzy model $TSK_{2.5}$ in the measures of model performance R^2, REVF and REP.

Catchment	Parameter group	Calibration			Verification		
		R^2	REVF	REP	R^2	REVF	REP
Sunkosi-1	c_m	0.07	0.03	0.42	0.10	0.03	0.38
	σ_m	0.29	0.23	0.37	0.29	0.23	0.33
	$b_{0,m}$	0.95	0.95	0.84	0.95	0.95	0.84
	$b_{2,m}$	0.10	0.08	0.59	0.09	0.07	0.48
	n_m	0.03	0.02	0.10	0.03	0.02	0.08
	$(nK)_m$	0.04	0.03	0.13	0.04	0.03	0.11
Yanbian	c_m	0.04	0.03	0.44	0.05	0.03	0.48
	σ_m	0.28	0.23	0.41	0.27	0.23	0.39
	$b_{0,m}$	0.96	0.98	0.84	0.96	0.97	0.84
	$b_{2,m}$	0.06	0.05	0.43	0.06	0.05	0.46
	n_m	0.03	0.02	0.10	0.03	0.02	0.12
	$(nK)_m$	0.03	0.03	0.18	0.03	0.03	0.18
Shiquan-3	c_m	0.03	0.03	0.12	0.03	0.03	0.18
	σ_m	0.26	0.23	0.24	0.26	0.23	0.25
	$b_{0,m}$	0.98	0.99	0.97	0.98	0.99	0.96
	$b_{2,m}$	0.03	0.03	0.05	0.04	0.03	0.06
	n_m	0.03	0.02	0.03	0.03	0.02	0.04
	$(nK)_m$	0.03	0.02	0.04	0.03	0.02	0.06
Brosna	c_m	0.05	0.03	0.31	0.07	0.03	0.34
	σ_m	0.27	0.23	0.43	0.28	0.23	0.47
	$b_{0,m}$	0.86	0.89	0.76	0.85	0.86	0.78
	$b_{2,m}$	0.23	0.16	0.52	0.26	0.19	0.54
	n_m	0.03	0.02	0.08	0.03	0.02	0.11
	$(nK)_m$	0.06	0.04	0.29	0.07	0.04	0.28
Bird Creek	c_m	0.03	0.03	0.08	0.03	0.03	0.06
	σ_m	0.26	0.23	0.24	0.26	0.24	0.23
	$b_{0,m}$	0.98	0.99	0.98	0.98	0.99	0.98
	$b_{2,m}$	0.03	0.03	0.04	0.03	0.03	0.04
	n_m	0.03	0.02	0.03	0.03	0.02	0.03
	$(nK)_m$	0.03	0.02	0.04	0.03	0.02	0.04
Wollombi Brook	c_m	0.03	0.03	0.14	0.03	0.03	0.06
	σ_m	0.26	0.23	0.25	0.26	0.23	0.24
	$b_{0,m}$	0.98	0.99	0.93	0.97	0.99	0.95
	$b_{2,m}$	0.04	0.03	0.10	0.04	0.03	0.07
	n_m	0.03	0.02	0.05	0.03	0.02	0.05
	$(nK)_m$	0.03	0.02	0.11	0.03	0.03	0.08

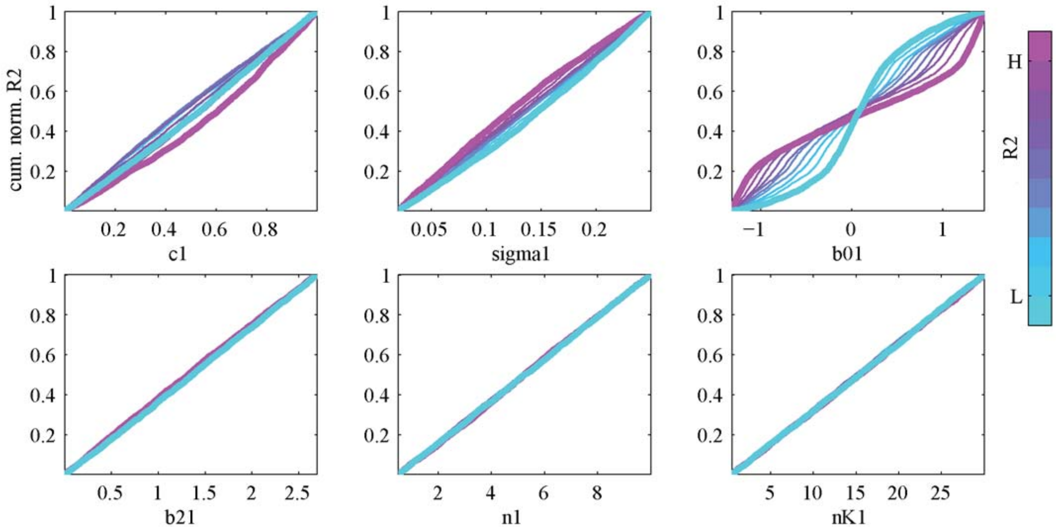

Fig. 1. Results of RSA method when applied to the fuzzy model $TSK_{1.5}$ in the Sunkosi-1 catchment, using the efficiency criterion R^2 as a measure model performance.