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# SWRC fit – a nonlinear fitting program with a water retention curve for soils having unimodal and bimodal pore structure

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## Abstract

The soil hydraulic parameters for analyzing soil water movement can be determined by fitting a soil water retention curve to a certain function, i.e., a soil hydraulic model. For this purpose, the program "SWRC Fit," which performs nonlinear fitting of soil water retention curves to 5 models by Levenberg-Marquardt method, was developed. The five models are the Brooks and Corey model, the van Genuchten model, Kosugi's log-normal pore-size distribution model, Durner's bimodal pore-size distribution model, and a bimodal log-normal pore-size distribution model propose in this study. This program automatically determines all the necessary conditions for the nonlinear fitting, such as the initial estimate of the parameters, and, therefore, users can simply input the soil water retention data to obtain the necessary parameters. The program can be executed directly from a web page at http://purl.org/net/swrc/; a client version of the software written in numeric calculation language GNU Octave is included in the electronic supplement of this paper. The program was used for determining the soil hydraulic parameters of 420 soils in UNSODA database. After comparing the root mean square error of the unimodal models, the van Genuchten and Kosugi's models were better than the Brooks and Corey model. The bimodal log-normal pore-size distribution model had similar fitting performance to Durner's bimodal pore-size distribution model.

## 1 Introduction

Water movement in unsaturated soil is important in the analysis of hydrological processes. Unsaturated flow is described by Richard's equation, and the Richard's equation in one-dimensional vertical flow is written as

$$\frac{\partial\theta(h)}{\partial t} = \frac{\partial}{\partial z}\left[K(h)\frac{\partial h}{\partial z} - K(h)\right], \tag{1}$$

where $h$ is the suction head (the negative of matric potential), $\theta$ is the volumetric water content, and K is the unsaturated hydraulic conductivity. For solving Richard's equa-



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tion, it is critical to know the soil water retention function, $\theta$(h), and the unsaturated hydraulic conductivity, K(h). Various empirical and theoretical models have been proposed to represent $\theta$(h) in the functional form having specific numbers of parameters and estimating K(h) from the $\theta$(h) function (Brooks and Corey, 1964; Campbell, 1974; van Genuchten, 1980; Durner, 1994; Kosugi, 1996; Poulsen et al., 2002).

The soil water retention function, $\theta(h)$, is nonlinear, and, therefore, the determination of the soil water retention parameters requires nonlinear fitting of the equation. The method of nonlinear fitting is implemented in some statistics and graphic software, but the determination of the parameters with such software is sometimes not very easy because the user has to input an adequate estimate of the initial value of the parameters. In particular, when it is desired to use some complex functions, it is not always easy to obtain such an initial estimate. The RETC program (van Genuchten et al., 1991), which analyzes the soil water retention curve and unsaturated conductivity, conducts nonlinear fitting with the initial estimate of the parameters. It does not have the algorithm of automatically estimating initial parameters, and, therefore, it is the users' responsibility to give a good set of initial parameters to make the calculation.

Scientifically, it is sound to leave the responsibility of the initial estimate to the users, but practically, it would be more convenient if the program were to be responsibe for making the initial estimate from the retention data and users would not have to input the initial estimate. Therefore, a program was developed to perform the nonlinear fitting of the soil water retention curve, where no explicit input of the initial estimate is required. The program is called SWRC Fit and can be executed directly from a web page (http://purl.org/net/swrc/). It uses a calculation engine written in the numerical calculation language GNU Octave (Seki, 2007), to which a new model of the soil water retention curve proposed in this paper was also added.

There are two objectives in this paper. The first is to describe how SWRC Fit works and verify the ability of the program itself. The second is to compare the soil hydraulic models implemented in the SWRC Fit, in particular, the newly developed bimodal lognormal pore-size distribution model.

## 2 Theory

### 2.1 BC, VG and LN models

Table 1 summarizes the various soil hydraulic models. Among the five models in the table, the first four, BC (Brooks and Corey, 1964), VG (van Genuchten, 1980), LN (Kosugi, 1996), and DM (Durner, 1994), have been published before, while the last, the ML model, is proposed in this paper. In the Table 1, the equations are written in $S_e(h)$ functional form. $S_e$, the effective saturation, is defined as

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}, \tag{2}$$

where $\theta_s$ is the saturated water content and $\theta_r$ is the residual water content. From Eq. (2) and the $S_e(h)$ functions in Table 1, the soil water retention functions $\theta(h)$ for the models are obtained. The saturated water content is often considered to be identical to the porosity, but, in practice, it can be smaller than the porosity because, in the field-saturated condition, the pores are entrapped with air. The residual water content is conceptually defined as the water content that remains on soil particles even when high suction is imposed, and the value of $\theta_r$ is sometimes set to 0. In fact, when $\theta_r$ is set to 0, the BC model is identical to another famous model by Campbell (1974).

The determination of unsaturated hydraulic conductivity by laboratory experimentation is time-consuming, and, therefore, researchers have attempted to relate the soil water retention function with the unsaturated hydraulic conductivity. To date, the most widely used model is that proposed by Mualem (1976);

$$K_r = {S_e}^{\tau}\left[\frac{f(S_e)}{f(1)}\right]^2, f(S_e) = \int_0^{S_e} \frac{1}{h(x)}dx, \tag{3}$$

where h(x) is an inverse function of $S_e(h)$, $\tau$ is a pore connectivity parameter, and $K_r$ is a relative hydraulic conductivity, defined by $K_r{=}K/K_s$, where K is the unsaturated hydraulic conductivity and $K_s$ is the saturated hydraulic conductivity. Mualem (1976) estimated that $\tau$=0.5 on average. Using Eq. (3), the $K_r(h)$ function can be estimated as shown in the right-hand column of Table 1. In order to get a closed-form equation, the inverse function of $S_e(h)$ should be given as a closed-form equation. Therefore, the closed-form equation is available only for the BC, VG, and LN models. In the VG model, the closed-form equation is obtained by imposing the constraint of $m$=1−1/$n$.

Figure 1 shows the fitting of the BC, VG and LN models to a sand sample of the UNSODA database of the unsaturated soil hydraulic measurement (Nemes et al., 2001). The constraints of $m$=1−1/$n$ were imposed in the VG model. As seen from this figure, these three models can represent relatively similar patterns of the soil water retention curve, whereas the BC model is slightly different from the VG and LN models, having distinct air entry suction. This difference will be discussed in the Results section (Sect. 4).

While the BC and VG models were derived empirically, the LN model was derived theoretically by assuming a log-normal pore-size distribution. The derivation of the LN model by Kosugi (1996) is outlined as follows. The pore radius distribution function g(r), defined as

$$g(r) = d\theta/dr \tag{4}$$

is expressed in the LN model as

$$g(r) = \frac{\theta_s - \theta_r}{\sqrt{2\pi}\sigma r} \exp\left[-\frac{[\ln(r/r_m)]^2}{2\sigma^2}\right], \tag{5}$$

where $r$ obeys the log-nomal distribution, i.e., ln(r) obeys the normal distribution of $N[\ln(r_m), \sigma^2]$. Here the pore radius r is inversely proportional to the soil capillary pressure head h by the capillary function

$$h = A/r, A = 2\gamma \cos\beta/\rho_w g, \tag{6}$$

where $\gamma$ is the surface tension between the water and air, $\beta$ is the contact angle, $\rho_w$ is the density of water, and g is the acceleration of gravity. A constant value of

A=0.149 cm$^2$ (Brutsaert, 1966) is often used. The distribution of the pore radius, g(r), can be transformed into the distribution of suction head, $f(h)=-d\theta/dh$, by the following equation,

$$f(h) = -g(r)dr/dh. \tag{7}$$

Substituting Eq. (5) and $dr/dh=-A/h^2$ (from Eq. 6) into Eq. (7) yields

$$f(h) = -\frac{A}{h^2}\frac{\theta_s - \theta_r}{\sqrt{2\pi}\sigma r}\exp\left[-\frac{[\ln(r/r_m)]^2}{2\sigma^2}\right]. \tag{8}$$

Defining $h_m=A/r_m$, we get $r/r_m=h_m/h$, and hence $\ln(r/r_m)=-\ln(h/h_m)$. Substituting this and $r=A/h$ into Eq. (8), we get

$$f(h) = \frac{\theta_s - \theta_r}{\sqrt{2\pi}\sigma h}\exp\left[-\frac{[\ln(h/h_m)]^2}{2\sigma^2}\right]. \tag{9}$$

This equation shows that h also obeys a log-normal distribution, i.e., ln(h) obeys the normal distribution of $N[\ln(h_m), \sigma^2]$. In fact, this could have been expected from the relationship of $\ln(h)=\ln(A)-\ln(r)$.

The relationship between $S_e$ and $h$ is

$$Se(h) = \frac{1}{\theta_s - \theta_r}\int_h^\infty f(h)dh. \tag{10}$$

Here, f(h) is a function whose definite integral from 0 to ∞ equals $\theta_s-\theta_r$ and not a probability density function itself. Substituting Eq. (9) into Eq. (10), we get

$$S_e = Q\left[\frac{\ln(h/h_m)}{\sigma}\right] \tag{11}$$

as shown in Table 1. Here, Q(x) is the complementary cumulative normal distribution function, defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-t^2/2)dt \tag{12}$$

## 2.2 DM, DB, and DT model

The BC, VG and LN models can represent many soil water retention curves for soils having a homogeneous pore structure, but, in some soils that have a heterogeneous distribution of pore size, these models do not represent the measured soil water retention curve very well. Figure 2 is such an example, showing the fitting of the BC, VG, and LN models to a silty-loam sample. The deviation of the fitting curves to the measured values at the suction head of around 1000 cm, as shown in this figure, is typically observed with loamy soil with the aggregated structure. The aggregated soil usually has two peaks of pore-size distribution, where the large pore is that between the aggregates and the small pore is that inside the aggregate, and two such peaks cannot be expressed by models with a unimodal pore-size distribution.

For soils with a heterogeneous pore structure, Durner (1994) developed a multimodal retention function, constructed by a linear superposition of subcurves of the VG model, as shown in the DM model in Table 1. In Table 1, k is the number of "subsystems" that form the pore-size distribution, and $w_i$ is the weighing factor for the subcurves. Similar conditions to those of the VG model also apply; $\alpha_i>0, m_i>0, n_i>0$. The special case of k=2 is called Durner's bimodal model (DB model), and the case of k=3 is Durner's trimodal model (DT model).

## 2.3 ML, BL, and TL models

The basic idea of Durner's method is that the soil water retention curve of soils having a heterogeneous pore structure can be expressed as a superposition of the curves of a homogeneous pore structure. Any homogeneous pore structure model can be used for the base model, for which Durner selected VG model. After Durner published the DM model, Kosugi developed the LN model, deriving the soil water retention curve from the log-normal pore-size distribution. Therefore, it would be reasonable to make another model superimposing the log-normal pore-size distribution, deriving a multimodal log-normal pore-size distribution model. However, such an approach has not been taken.

Here, the multimodal pore-size distribution model is introduced, combining the ideas of Durner and Kosugi.

The Pore-size distribution function of the LN model, Eq. (5), can be extended to the multimodal log-normal distribution function (ML model) by a linear superposition,

$$g(r) = \sum_{i=1}^{k} w_i \left( \frac{\theta_s - \theta_r}{\sqrt{2\pi}\sigma_i r} \exp\left[ -\frac{[\ln(r/r_{mi})]^2}{2\sigma_i{}^2} \right] \right) \tag{13}$$

where $0<w_i<1$ and $\Sigma w_i=1$. This means that the pore size r obeys the multimodal log-normal distrubition, i.e., ln(r) obeys multimodal normal distribution, each normal distribution being expressed by $N[\ln(r_{mi}), \sigma^2]$, where $r_{mi}$ is calculated from $h_{mi}$ with Eq. (6). In the same way as we have derived Eq. (8) from Eq. (5), the f(h) function for the ML model can be derived as

$$f(h) = \sum_{i=1}^{k} w_i \left( \frac{\theta_s - \theta_r}{\sqrt{2\pi}\sigma_i h} \exp\left[ -\frac{[\ln(h/h_{mi})]^2}{2\sigma_i{}^2} \right] \right) \tag{14}$$

Substituting Eq. (14) into Eq. (10), we get

$$S_e = \sum_{i=1}^{k} w_i Q\left[ \frac{\ln(h/h_{mi})}{\sigma_i} \right], \tag{15}$$

as shown in Table 1.

The special case of k=2 can be termed as a bimodal log-normal pore-size distribution (BL model), and the case of k=3 is a trimodal log-normal pore-size distribution model (TL).

Figure 3 shows the fitting of the DB and BL models to the silty-loam sample, similarly to Fig. 2 with constraints of $\theta_r$=0. It was shown that both models could fit the soil water retention curve of this soil very well. There are as many as 7 parameters in the bimodal models, and by decreasing one fitting parameter by setting $\theta_r$=0, a reasonably good fit was obtained.

# 3 Nonlinear fitting software – SWRC Fit

## 3.1 Structure of the software

The nonlinear fitting software, SWRC Fit, consists of two compartments; the calculation engine and the Web interface program. When a user inputs soil water retention data in the Web interface, it performs the nonlinear fitting by using the calculation engine. The calculation engine can also be used as independent software. In this way, users can attempt the calculation using the Web interface, and, after that, when they need to use it frequently or calculate massive data, they can download the calculation engine and perform the calculation with their own computers.

## 3.2 Calculation engine

The nonlinear fitting program is included in the electronic supplement to this paper at http://www.hydrol-earth-syst-sci-discuss.net/4/407/2007/hessd-4-407-2007-supplement.zip. By extracting the supplementary zip file, three files are obtained, as summarized in Table 2. The program is based on the code developed by Seki (2007), where the BC, VG, LN and DB models can be fitted. The bimodal program, bimodal.m, is modified to include the BL model developed in this paper.

The two types of software, swrc.m and bimodal.m, are written in GNU Octave, and, therefore, GNU Octave should be installed in the system. GNU Octave is available for downloading from the GNU Octave Website (http://www.gnu.org/software/octave/), and the installation instructions are given in the Website. It works on various operating systems including Windows, Mac OS X, Linux and OS/2. After installing GNU Octave, the octave-forge package, which contains some necessary packages for SWRC Fit (leasqr.m, dfdp.m and normcdf.m), is to be installed. Octave-forge is available as a contributed package of GNU Octave.

The input data, i.e., the soil water retention curve, should be prepared as a text file with two columns, using the file name swrc.txt. The first column is the suction head and the second column is the volumetric water content, where space is used as a delimiter. Any unit can be used as the input data, and the calculated data depends on the unit used as the input data. In some cases, users may want to set $\theta_r$=0. In this case, the following part of the program, swrc.m,

```
#qrin = 0; cqr = 0;
```

is to be commented out, i.e., the first "#" mark is to be deleted by a text editor. If users want to set restriction of $\theta_r \geq 0$, the following part of the program,

```
pqr = 0;
```

is to be changed to pqr=1.

The program (swrc.m and bimodal.m) and data (swrc.txt) should be placed in the same directory (folder). In that directory, "swrc.m" should be typed to run the fitting of unimodal (BC, VG, and LN) models, and "bimodal.m" should be typed to run the fitting of the DB and BL models. In the UNIX system "./swrc.m" and "./bimodal.m" is preferred, and the executable file mode should be set. The result is shown as

```
=== BC model ===

qs = 0.3831634
qr = 0.04763780
hb = 41.70445
lambda = 7.010440
R2 = 0.9927859
=== VG model ===
qs = 0.3865553
qr = 0.05518984
alpha = 0.02154675
n = 15.92316
```



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```
R2 = 0.9924632
=== LN model ===
qs = 0.3862481
qr = 0.05634534
hm = 46.63087
sigma = 0.1081815
R2 = 0.9916682
```

Using the Microsoft Excel worksheets in the electronic supplement, swrc.xls, the fitted curves can be checked by copying and pasting the result onto the spreadsheet.

The software performs the nonlinear fitting of the soil water retention data using the Levenberg-Marquardt method (Marquardt, 1963) with the leasqr function of GNU Octave. In SWRC Fit, the initial estimate of the parameters is automatically determined by the program. The key algorithm for the determination of each initial parameter is as follows. First, the parameters of the BC model are estimated by the slope of the logarithmic plot of two points in the soil water retention curve. After calculating the BC parameters by nonlinear fitting, the initial estimate of the VG model is calculated from the BC parameters. In this estimation, the relationship of $n=\lambda+1$ is used, because the VG model is approximated to the BC model, where $\lambda=n-1$ when $\alpha h \gg 1$. After calculating the VG model, the initial estimate of the LN model is given by the fitted VG parameters with the relationship of $\sigma=1.2(n-1)^{-0.8}$, which was determined by fitting many VG curves to LN curves.

The algorithm for determining the initial estimate of DB parameters is as follows. The estimate is first performed by fitting certain numbers of the data points in the low suction region with VG equation to calculate the effect of large pore ($\alpha_1, n_1$). By subtracting the VG curve from all data plots and fitting the subtracted curve to the VG equation, the effect of the small pore ($\alpha_2, n_2$) is calculated. The numbers of the data points used for the first fitting can be changed, and among the possible fitting curves, the one that gives the best $R^2$ value is taken as the first estimate of the set of parameters. The 7 parameters in the DB model are estimated after the initial estimate is obtained. When

$w_1<0.01$ or $w_1>0.99$, the DB equation can be approximated to the VG equation by either neglecting the term of i=1 or i=2, and therefore only the result of "Not bimodal" is displayed.

As for the BL model, the initial parameters are obtained from the fitted values of the DB model. Two VG equations in the DB equation are fitted by the LN equation to obtain the initial estimate of the BL equation.

For each model, fine-tuning of the algorithm of the initial parameters and the incremental vector in the parameter estimation were performed by trial and error. As a result, the software can automatically determine the initial estimate and give a reasonably good fitting to most of the data sets in the UNSODA database, where the result is shown in the Results section.

### 3.3 Web interface of the SWRC Fit

The Web interface of the SWRC Fit is written in the program language perl and works as a cgi program. The screenshot of the user interface is shown in Fig. 4. Users can copy soil water retention data in the textbox or they can select from the sample soil water retention data in the UNSODA database (Nemes et al., 2001). In other textboxes, the description of the soil sample, soil texture, and name can be written. The description written here appears in the results screen. The calculation options of $\theta_r=0$ can be set by checking appropriate boxes. By default, only unimodal models are used, and when the users select the "Bimodal models" checkbox, bimodal models will also be used. After that, the calculation starts by pressing the "Calculate" button.

The result of the nonlinear fit is shown as Fig. 5. The models, equations, parameters, and $R^2$ values are shown in tabular form, and the fitting curves with measured data points are also shown in a graph. If the bimodal model is selected, the results of the bimodal models are shown separately. By looking at the results, the accuracy of the fit with different models can be compared in both $R^2$ values and fitting curves. The description of the soil sample and the original data is also displayed in the results screen so that the users can print out and store all the necessary information.

## 4 Results

### 4.1 Unimodal (BC, VG and LN) models

To verify how this program works, datasets from the UNSODA database (Nemes et al., 2001) were used for the sample data. Seki (2007) also made similar verification, and, in this paper, a more quantitative discussion will be conducted regarding the different characteristics of each model. Soil water retention curves in the drying process measured in the laboratory having more than 9 data points were selected from the UNSODA database. The total number of the datasets is 420.

Figure 6 shows the relationship between the RMSE (Root Mean Square Error) of the model fitting and data for the BC and VG models. Figure 7 shows a similar relationship for the BC and LN models, and Fig. 8 shows a similar relationship for the VG and LN models. From these figures, it appears that the RMSE of the VG and LN models is smaller than that of the BC model on average and that the VG and LN models are not very different. The average values of the log (RMSE) for the BC, VG, and LN models are −2.114, −2.229, and −2.223, respectively. The paired t-test was performed to test the difference between the means for each pair (BC and VG, BC and LN and VG and LN) of the log (RMSE). According to the t-test, the log (RMSE) for VG and LN are significantly smaller than the log (RMSE) for BC ($P<0.01$), while the log (RMSE) for VG and LN are not significantly different. Therefore, the VG and LN models give better fitting than the BC model on average, while the VG and LN models are not different regarding the precision of the fitting.

It is noted that this discussion is on the average accuracy of the fit, and, for some soil, the BC model can provide a better fit than the VG and LN models, especially for cases in which air entry suction or bubbling pressure appears sharply. In such cases, the BC model might be preferable.

Sometimes, air entry suction is introduced in the VG model. In the VG model, the discrepancy in the soil water retention curve near saturation is very problematic in estimating the K(h) function by Mualem's equation because it is extremely sensitive in the nearly saturated region for fine textured soil having a small value of n. The discrepancy in the K(h) function leads to instability in the numerical simulation. Therefore Vogel et al. (2000) introduced the minimum capillary height, $h_s$, in the VG model and made a modified van Genuchten equation:

$$S_e = \begin{cases} \left[\frac{1}{1+(\alpha h)^n}\right]^m & (\ h > h_s\ ) \\ 1 & (\ h \le h_s\ ) \end{cases} \tag{16}$$

$$K_r = \begin{cases} \left[{S_e}^{\tau}\left[\frac{1-F(S_e)}{1-F(1)}\right]^2\right. & (\ h > h_s\ ) \\ 1 & (\ h \le h_s\ ) \end{cases} \tag{17}$$

where

$$F(S_e) = (1 - {S_e^*}^{1/m})^m, S_e^* = \frac{\theta_s - \theta_r}{\theta_m - \theta_r} S_e. \tag{18}$$

Vogel et al. (2000) showed that, by introducing a $h_s$ value of 2 cm, the predictability of the conductivity function was improved. In the BC model, the air entry suction is usually not as small as 2 cm, but in the modified VG model, a minimum capillary height of 1 to 2 cm is often used.

In the same way, introducing a minimum capillary height or air entry suction into the LN model might improve the predictability of the conductivity function. In fact, the original version of the LN model (Kosugi, 1994) included the parameter expressing the air entry suction, $h_s$, as follows,

$$S_e = \begin{cases} Q\left[\frac{\ln\left(\frac{h_s-h}{h_s-h_0}\right)}{\sigma} - \sigma\right] & (\ h > h_s\ ) \\ 1 & (\ h \le h_s\ ), \end{cases} \tag{19}$$

and $h_s$ was later eliminated by setting $h_s$=0 (Kosugi, 1996). Therefore, in some cases, when a numerical calculation is conducted for fine textured soil and the sensitivity of K(h) in the nearly saturated region is high, introducing the air entry suction by using Eq. (19) might improve the predictability of the hydraulic conductivity function.

The BC, VG and LN models all have their own merits. The BC model is simple, and the mathematical calculation based on the BC model is easier than that based on either the VG or the LN model. Sometimes, when the soil has distinct air entry suction, the BC model represents the soil water retention curves better than the VG and LN models. The VG model can fit most unimodal soil water retention curves very well. It is widely used, and therefore, a comparison of the parameters to other soils and a discussion of the parameters with soil scientists would be easier. Many people use the VG model for the simulation of water movement in soil, and numerous results have been published in this regard. The LN model can also fit most unimodal soil water retention curves very well, and the accuracy of the fit is similar to that of the VG model. The LN model has merit in that it is a theoretically derived model, and, therefore, the physical meaning of each parameter is clearly defined. Therefore, when discussion of the pore-size distribution is desired, the LN model is preferable to the VG model.

### 4.2 Bimodal (DB and BL) models

As for the multimodal models, only bimodal models (DB and BL) are implemented in the SWRC Fit. When the soil water retention curve can be fitted with unimodal models, there is no need to use bimodal models. Therefore, among the 420 datasets used in the unimodal fit, 98 datasets were selected such that $R^2<0.99$ for BC, VG and LN. For these 98 datasets, the DB and BL models were used for fitting. Of the 98 datasets, 8 gave the result of "Not bimodal", i.e., the value of $w_1$ or $w_2$ became less than 0.01 and the program determined that it is not to be expressed by bimodal models. Therefore 90 datasets were selected for the analysis of the DB and BL models.

Figure 9 shows the relationship between the RMSE of unimodal and bimodal models. In this calculation, the RMSE of unimodal models is the minimum RMSE of the BC, VG and LN fitting, and the RMSE of bimodal models is the minimum RMSE of the DB and BL models. As the number of parameters increases in the bimodal model, the precision is expected to improve in the bimodal model, but in some datasets, the improvement is slight, and the RMSE of the unimodal and bimodal models is only slightly different.

For such soils, using the unimodal model is prefereble to using the bimodal model. For many datasets, the RMSE of bimodal models is smaller than the RMSE of unimodal models, indicating that these samples are better expressed in bimodal models, as exemplified in Figs. 2 and 3. By drawing the soil water retention curves of such soils, it was observed that most of the bimodal curves represented the typical pore structure of aggregated loams, as shown in Figs. 2 and 3. Another type of soils fitted better in a bimodal model is shown in Fig. 10. In Fig. 10, the soil water retention curve would have been fitted well by the LN model if there were no point at the suction of 0 cm. Both bimodal models, DB and BL, depict the 0 cm suction with the first mode and the rest of the curve with the second mode, thereby increasing the accuracy of the fit in the whole region. Durner (1994) originally intended to solve the discrepancy of the soil water retention curve to the measured data in the nearly saturated region. However, as seen in Fig. 10, the curve in the range of 0.01 cm to 4 cm differs significantly in the DB and BL models. There are no data points in these points, and, therefore, trying to determine which model better represents the "real" soil pore structure is not possible. Technically, it is not easy to obtain precise measurements of the retention curve in this nearly saturated region.

Figure 11 shows the relationship between the RMSE of the DB and BL models. According to the paired t-test, no significant difference was found between the means for the log (RMSE) of the DB and BL models. Therefore, the DB and BL models have similar capability of fitting the soil water retention curve, similarly to the relationship between the VG and LN models. This is a reasonable result, because DB model is the superposition of the VG curves and the BL model is the superposition of the LN curves. While the DB and BL models have similar fitting performance, both have their own merits. The merit of the DB model over the BL model is that it has already been used for 10 years and it is implemented in such software as Hydrus-1D (http://www.pc-progress.cz/). On the other hand, the advantage of the BL model over the DB model is that the parameters in the BL models directly represent the pore-size distribution, which obeys the bimodal log-normal distribution.

The soil water retention curve of many soil samples in the UNSODA database was expressed by the LN model for the unimodal case and the BL model for the bimodal case. The LN and BL models are, in fact, a special case of an ML model, where k=1 and =2, respectively. This suggests that soil water retention can be derived by expressing the pore-size distribution in a multimodal log-normal distribution.

## 5 Conclusions

SWRC Fit, a program for a nonlinear fitting of the soil water retention curve of three unimodal (BC, VG and LN) and two bimodal (DB and BL) models, was developed. By applying this program to the soil water retention data in the UNSODA database, a comparison of the accuracy of the fit for the unimodal and bimodal models was attempted. In unimodal models, the VG and LN models gave better fitting performance than the BC models, and the VG and LN models gave similar fitting precision on average. For bimodal soils with a bimodal pore structure, the bimodal log-normal distribution model (BL) introduced in this paper is shown to have a similar fitting performance to that of the Durner's bimodal model (DB). SWRC Fit can simultaneously calculate the fitting parameters of these models with $R^2$ values and draw the fitting curves. Therefore by comparing the result of the models users can choose the model to be used for further analysis. With the easy-to-use Web interface, SWRC Fit helps determine the soil hydraulic parameters for research, investigation and educational purposes.

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**Table 1.** Three unimodal models: the Brooks and Corey (BC), van Genuchten (VG), and Kosugi's log-normal pore-size distribution (LN); and two multimodal models: Durner's multimodal model (DM) and the proposed multimodal log-normal pore-size distribution (ML). In the SWRC Fit, the bimodal version (k=2) of the DM and ML models (DB and BL models) are implemented.

| Model | Equation | $K_r$ estimation by Mualem model |
|---|---|---|
| BC | $S_e=\begin{cases}\left(\frac{h}{h_b}\right)^{-\lambda} & (h>h_b)\\ 1 & (h\le h_b)\end{cases}$ | $K_r=\begin{cases}\left(\frac{h}{h_b}\right)^{-(2+\tau\lambda+2\lambda)} & (h>h_b)\\ 1 & (h\le h_b)\end{cases}$ |
| VG | $S_e=\left[\frac{1}{1+(\alpha h)^n}\right]^m (\alpha>0, n>1, 0<m<1)$ | $K_r=S_e^{\tau}\left[1-(1-S_e^{1/m})^m\right]^2$ |
| LN | $S_e=Q\left[\frac{\ln(h/h_m)}{\sigma}\right]$ | $K_r=S_e^{\tau}\left(Q\left[\frac{\ln(h/h_m)}{\sigma}+\sigma\right]\right)^2$ |
| DM | $S_e=\sum_{i=1}^{k} w_i\left[\frac{1}{1+(\alpha_i h)^{n_i}}\right]^{m_i} (0<w_i<1, \Sigma w_i=1)$ | (Closed-form equation not given) |
| ML | $S_e=\sum_{i=1}^{k} w_i Q\left[\frac{\ln(h/h_{mi})}{\sigma_i}\right] (0<w_i<1, \Sigma w_i=1)$ | (Closed-form equation not given) |

**Table 2.** Electronic supplement of this paper.

| Filename | Description |
|---|---|
| swrc.m | Fitting of unimodal models (BC, VG and LN). |
| bimodal.m | Fitting of bimodal models (DB and BL). |
| swrc.xls | Microsoft Excel worksheets for checking the result. |

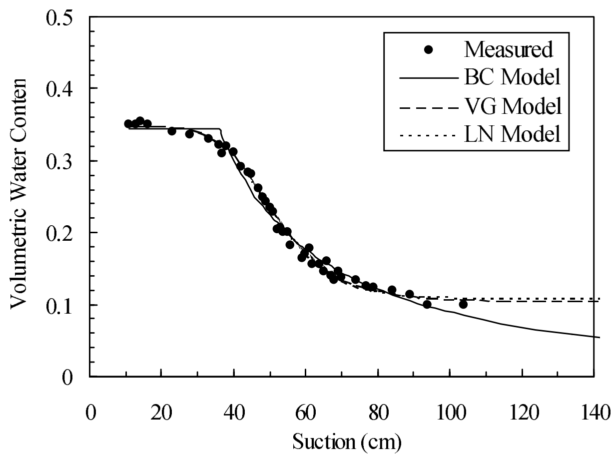



**Fig. 1.** Fitting of the BC, VG and LN models for sand (UNSODA ID=4440).

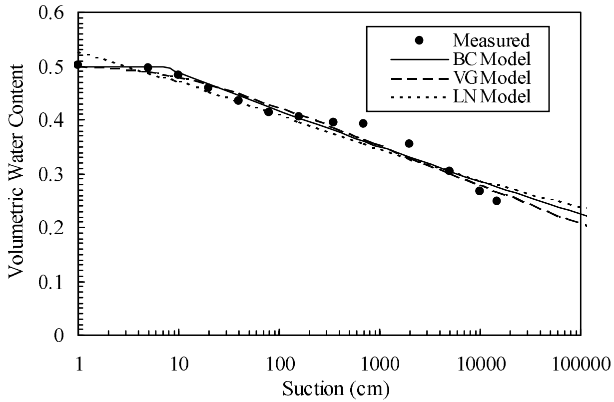



**Fig. 2.** Fitting of BC, VG and LN models for silty loam.

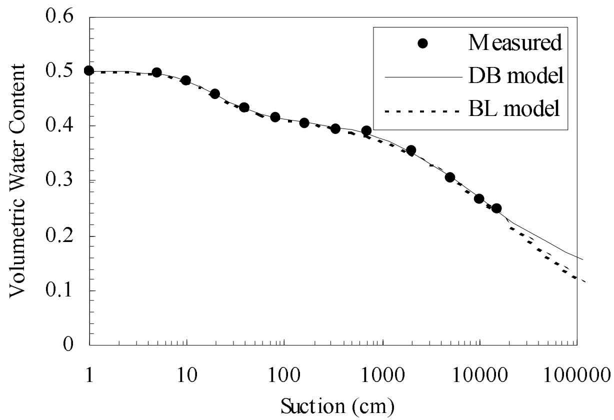



**Fig. 3.** Fitting of the DB and BL models for silty loam fitted by constraints of $\theta_r$=0.

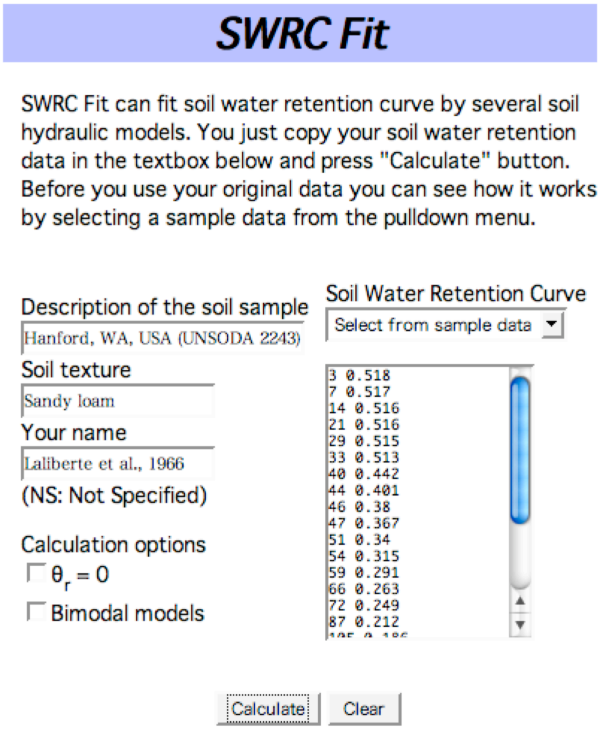
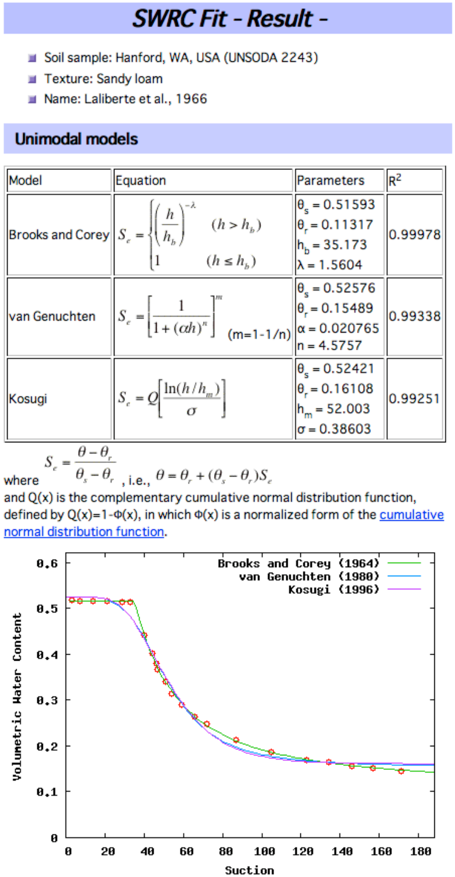



**Fig. 4.** Screenshot of the input display of the SWRC Fit (http://purl.org/net/swrc/).



## SWRC Fit - Result -

- Soil sample: Hanford, WA, USA (UNSODA 2243)
- Texture: Sandy loam
- Name: Laliberte et al., 1966

### Unimodal models

| Model | Equation | Parameters | $R^2$ |
|---|---|---|---|
| Brooks and Corey | $S_e = \begin{cases} \left(\dfrac{h}{h_b}\right)^{-\lambda} & (h > h_b) \\ 1 & (h \le h_b) \end{cases}$ | $\theta_s = 0.51593$<br>$\theta_r = 0.11317$<br>$h_b = 35.173$<br>$\lambda = 1.5604$ | 0.99978 |
| van Genuchten | $S_e = \left[\dfrac{1}{1+(\alpha h)^n}\right]^m$ (m=1-1/n) | $\theta_s = 0.52576$<br>$\theta_r = 0.15489$<br>$\alpha = 0.020765$<br>$n = 4.5757$ | 0.99338 |
| Kosugi | $S_e = Q\left[\dfrac{\ln(h/h_m)}{\sigma}\right]$ | $\theta_s = 0.52421$<br>$\theta_r = 0.16108$<br>$h_m = 52.003$<br>$\sigma = 0.38603$ | 0.99251 |

where $S_e = \dfrac{\theta - \theta_r}{\theta_s - \theta_r}$, i.e., $\theta = \theta_r + (\theta_s - \theta_r) S_e$
and Q(x) is the complementary cumulative normal distribution function, defined by Q(x)=1-Φ(x), in which Φ(x) is a normalized form of the cumulative normal distribution function.



**Fig. 5.** Screenshot of the results display of the SWRC Fit (http://purl.org/net/swrc/).

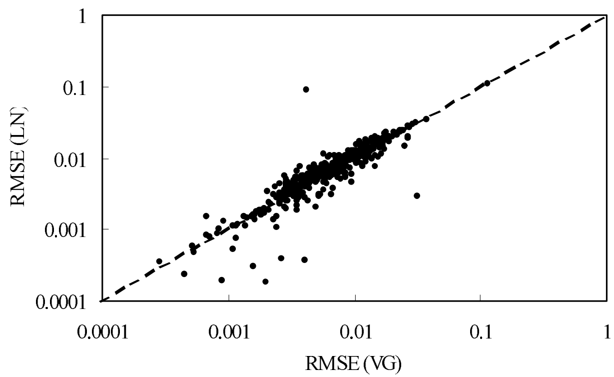



**Fig. 6.** Relationships between RMSE for the fitting of the BC and VG models.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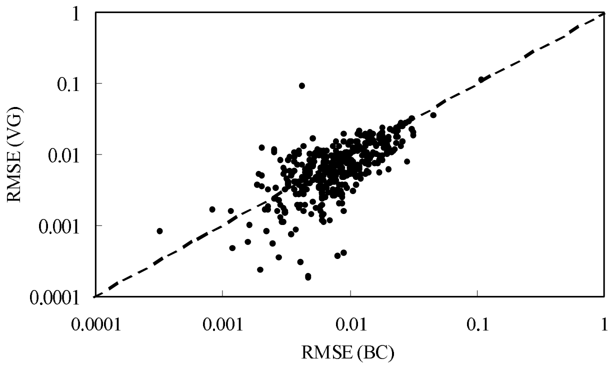


**Fig. 7.** Relationships between RMSE for the fitting of the BC and LN models.

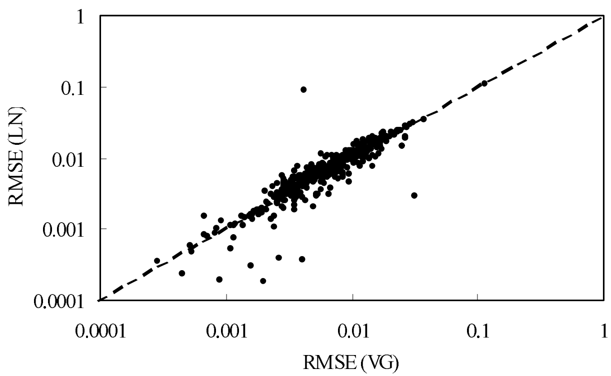


**Fig. 8.** Relationships between RMSE for the fitting of the VG and LN models.

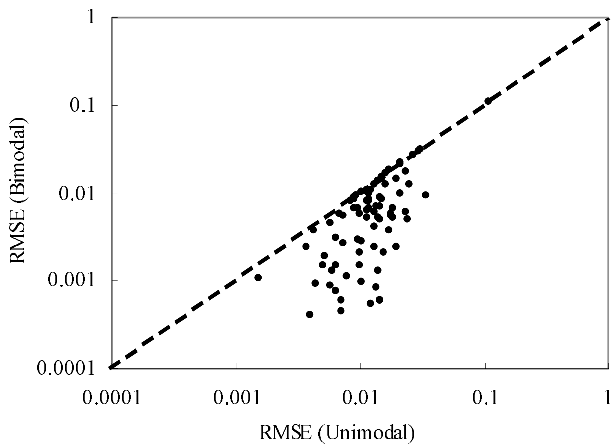


**Fig. 9.** Relationships between minimum RMSE for the fitting of unimodal (BC, VG, and LN) and bimodal (DB and BL) models.

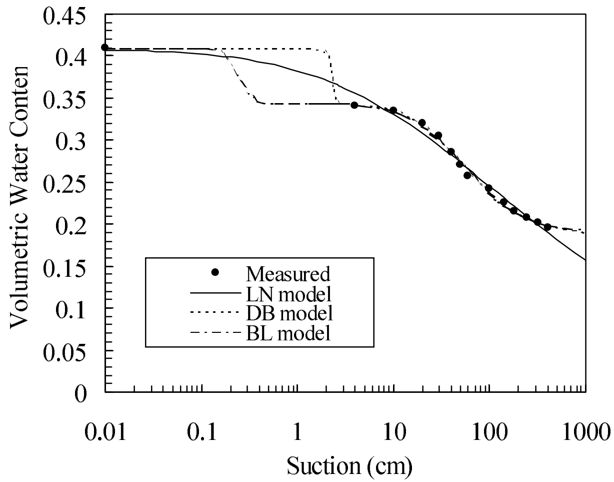


**Fig. 10.** Fitting of LN, DB and BL models for sandy loam (UNSODA ID=2130). The original data at the suction of 0 cm cannot be represented in the logarithmic scale, and therefore it is plotted at the suction of 0.01 cm.



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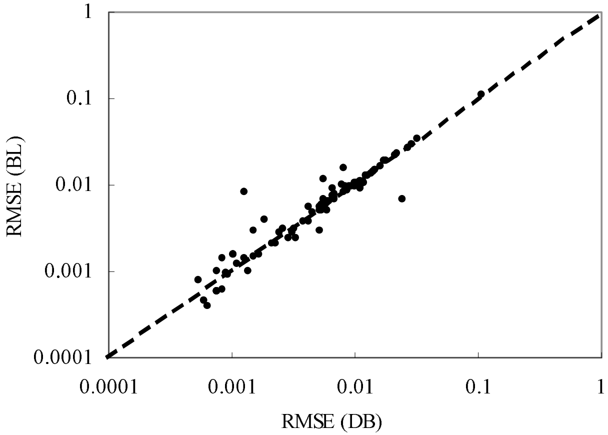


**Fig. 11.** Relationships between RMSE for the fitting of the DB and BL models.