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# Comparison of different multi-objective calibration criteria of a conceptual rainfall-runoff model of flood events

N. Chahinian<sup>1,\*</sup> and R. Moussa<sup>2</sup>

<sup>1</sup>Agrocampus Rennes, Laboratoire Physique des Surfaces Naturelles et Génie Rural, 65 Rue de Saint Briec, 35042 Rennes, France

<sup>2</sup>INRA, Laboratoire d'étude des Interactions entre Sol – Agrosystème – Hydrosystème UMR LISAH AgroM-INRA- IRD, 2 Place Pierre Viala, 34060 Montpellier, France

\* now at: HydroSciences Montpellier, Université Montpellier 2, Case courrier MSE Place Eugène Bataillon, 34095 Montpellier Cedex 5, France

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Correspondence to: N. Chahinian (chahinia@msem.univ-montp2.fr)

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## Abstract

A conceptual lumped rainfall-runoff flood event model was developed and applied on the Gardon catchment located in southern France and various mono-objective and multi-objective functions were used for its calibration. The model was calibrated on 15 events and validated on 14 others. The results of both the calibration and validation phases are compared on the basis of their performance with regards to six criteria, three global criteria and three relative criteria representing volume, peakflow, and the root mean square error. The first type of criteria gives more weight to strong events whereas the second considers all events to be of equal weight. The results show that the calibrated parameter values are dependent on the type of criteria used. Significant trade-offs are observed between the different objectives: no unique set of parameter is able to satisfy all objectives simultaneously. Instead, the solution to the calibration problem is given by a set of Pareto optimal solutions. From this set of optimal solutions, a balanced aggregated objective function is proposed, as a compromise between up to three objective functions. The mono-objective and multi-objective calibration strategies are compared both in terms of parameter variation bounds and simulation quality. The results of this study indicate that two well chosen and non-redundant objective functions are sufficient to calibrate the model and that the use of three objective functions does not necessarily yield different results. The problems of non-uniqueness in model calibration, and the choice of the adequate objective functions for flood event models, emphasise the importance of the modeller's intervention. The recent advances in automatic optimisation techniques do not minimise the user's responsibility, who has to chose multiple criteria based on the aims of the study, his appreciation on the errors induced by data and model structure and his knowledge of the catchment's hydrology.

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## 1 Introduction

It is common for hydrologists to model individual storm events at the catchment scale (e.g. Bates and Ganeshanandam 1990; Zariello, 1998; Moussa et al., 2002; Jain and Indurthy, 2003), for flood forecasting, spillway design or flood protection schemes. The first important challenge that awaits the modeller in this task is to choose a rainfall-runoff model, and to calibrate a set of parameters, that can accurately simulate a number of flood events and related hydrographs shapes. Most of the models used currently for flood forecasting in France are lumped conceptual models (Garçon, 1996; Yang and Michel, 2000; Paquet, 2004) i.e. they have parameters which cannot, in general, be obtained directly from measurable catchment characteristics, and hence model calibration is needed. Various calibration algorithms and procedures have been presented in the literature extensively (Rosenbrock, 1960; Nelder and Mead 1965; Duan et al., 1992; Gan and Biftu, 1996; Yapo et al., 1998; Vrugt et al. 2003a and 2003b; see a review in Gupta et al., 2003). Although they differ in the ways they seek the optimal value, they all aim at minimising or maximising an objective function. It is important to note that, in general, trade-offs exist between the different objective functions. For instance, when using the bias as an objective function, one may find a set of parameters that provides a very good simulation of volume, but a poor simulation of the hydrograph shape or peak flow, and vice versa. Conventional objective functions such as the root mean square error, the Nash and Sutcliffe (1970) efficiency coefficient, or the index of agreement (Willmott, 1981) tend to emphasize the high flows, and consequently, are oversensitive to extreme values and outliers (Legates and McCabe, 1999). On the opposite, the mean absolute percent error tends to emphasize the low flows (Yu and Yang, 2000).

However, in most real world applications, models are used to reproduce the entire shape of the hydrograph, sometimes even to simulate more than one flow component in the catchment i.e. groundwater and surface water, water and nutrient fluxes etc. In these occurrences, the use of a single objective function may be questionable and it

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would be advisable to take into account various objective functions by considering the calibration problem in a multi-objective framework (Yapo et al., 1998; Madsen, 2000). Most of the studies related to multi-objective calibration have investigated the use of two objective functions and few ones have looked into the use of three or more functions (Schoops et al., 2005, Parajka et al., 2007). In this study we will extend the use of the balanced aggregative function suggested by Madsen (2000) to three objective functions. In addition, while most of these studies have dealt with continuous simulations, we will investigate multi-objective calibration issues related to event based modelling.

Hence, the objective of this paper is to develop an event based lumped conceptual rainfall-runoff model and then to compare mono-objective and multi-objective calibration approaches. The emphasis is put on the impact of the selected objective functions on the actual hydrograph shapes and simulation errors rather than on the calibration algorithm as several studies have already investigated this issue (Yu and Yang, 2000; Johnsen et al., 2005; Tang et al., 2006). The Gardon catchment located in southern France is used in the applications because of the recurrent flooding problems its inhabitants encounter yearly. The objective functions used in this study can be divided in two broad categories: global and relative. Given the diversity of the flood events to be modelled such an approach was deemed necessary as the first type of criteria gives more weight to strong events whereas the second considers all events to be of equal weight. For each category, three different objective functions are considered: volume conservation which is important for gauging problems, peakflow reproduction which is essential for flood applications and the root mean square error as a measure of the global agreement between the simulated and observed curves. The paper is organised in four sections: i) model presentation; ii) formulation of the calibration criteria; iii) presentation of the study zone, and iv) model performance analysis based on mono-objective and multi-objective calibration.

## 2 The lumped conceptual rainfall-runoff model

Since the 1960s, lumped, conceptual rainfall-runoff models have been used in hydrology (e.g. Crawford and Linsley, 1966; Cormary and Guilbot, 1969; Duan et al., 1992; Bergström, 1995; Donigan et al., 1995; Havnø et al., 1995). These models consider the catchment as an undivided entity, and use lumped values of input variables and parameters. For the most part (for a review, see Fleming, 1975; Singh, 1995), they have a conceptual structure based on the interaction between storage elements representing the different processes with mathematical functions to describe the fluxes between the stores.

The modelling approach followed herein will be lumped and the catchment will be considered as a single entity. A two-reservoir-layer model has been developed to represent the catchment on the basis of the CREC model (Cormary and Guilbot, 1969) and the Diskin and Nazimov (1995) production function (Fig. 1). Evaporation is not represented since the purpose of the model is to simulate individual flood events during which evapotranspiration is negligible. The first layer, noted “soil-reservoir”, represents the upper soil layer and controls surface runoff, infiltration, interflow and percolation. The second layer, noted “aquifer-reservoir”, represents the aquifer where mainly base flow occurs. A unit hydrograph transfer function is used to route flows to the outlet. The output of the model will be a simulated hydrograph which will be compared to the original measured hydrograph to assess model performance. A general description of each procedure is given below.

### 2.1 The production function

A regulating element  $f$  separates the precipitation  $P$  into surface runoff  $R$ , and infiltration  $I$ . The soil-reservoir element has one input, the infiltration  $I$ , and two outputs, the lateral interflow  $q$ , and the vertical flow  $g$  which represents the percolation from the upper soil layer to deeper layers. The state variable of the regulating element, denoted  $f$ , is determined by the magnitude of the reservoir state variable  $S$  according to the

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Diskin and Nazimov (1995) relationship

$$f = f_0 - (f_0 - f_c) \frac{S}{S_m} \quad (1)$$

where  $f_0$  [ $LT^{-1}$ ] is the maximum infiltration capacity,  $f_c$  [ $LT^{-1}$ ] the minimum infiltration capacity, and  $S_m$  [L] the maximum storage in the soil-reservoir layer. The value of  $f_c$  characterises the soil's infiltration capacity at saturation, and the term  $(S/S_m)$  characterises the soil moisture. The two outputs  $I$  and  $R$  of the regulating element depend on the value of the state variable  $f$  and on the value of the input  $P$ , at the same instant according to the following equations

$$\text{If } P < f \text{ then } I = P \text{ and } R = 0 \quad (2)$$

$$\text{If } P > f \text{ then } I = f \text{ and } R = P - f \quad (3)$$

The two outputs of the soil-reservoir,  $q$  and  $g$ , are calculated function of a parameter  $b$  (with  $0 \leq b \leq 1$ ) and the term  $(S/S_m)$

$$q = f_c b \frac{S}{S_m} \text{ and } g = f_c (1 - b) \frac{S}{S_m} \quad (4)$$

It should be noted that the sum  $q + g = f_c \frac{S}{S_m}$  is independent of the parameter  $b$  and verifies the soil-reservoir output of the Diskin and Nazimov (1995) model. As the storage  $S$  approaches the threshold value  $S_m$ , both the infiltration capacity  $f$  and the sum  $q + g$  tend to the same value  $f_c$ .

The aquifer-reservoir has one input, the percolation  $g$ , and one output, the base flow  $B$  [ $LT^{-1}$ ], which is calculated function of the aquifer-reservoir level  $S_b$  using a linear relation

$$B = kS_b \quad (5)$$

where  $k$  [ $T^{-1}$ ] is a constant characterizing the recession curve of the aquifer. In order to reduce the number of parameters, the aquifer-reservoir hasn't a maximum storage

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depth. The value of the state variable  $S_b$  of the aquifer-reservoir is obtained using the continuity equation

$$\frac{dS_b}{dt} = g(t) - B(t) \quad (6)$$

## 2.2 The transfer function

5 A transfer function is used to route the rainfall excess to the catchment outlet. A unit hydrograph linear model, based on a Hayami (1951) kernel function, which is a resolution of the diffusive wave equation, was used to simulate the transfer of the sum of  $(R + q + B)$  to the outlet (Moussa and Bocquillon, 1996). Let  $I(t)$  [ $L^3T^{-1}$ ] be the input hydrograph

$$10 \quad I(t) = (R + q + B) \cdot A \quad (7)$$

where  $A$  [ $L^2$ ] is the catchment area. Let  $O(t)$  be the routed hydrograph at the outlet

$$15 \quad O(t) = \int_0^t I(\tau) \cdot H(t - \tau) \cdot d\tau \quad (8)$$

with  $H(t)$  the Hayami kernel function defined as

$$H(t) = \left(\frac{w \cdot z}{\pi}\right)^{1/2} \cdot \frac{\exp^{z(2 - \frac{t}{w} - \frac{w}{t})}}{(t)^{3/2}} \quad \text{with} \quad \int_0^\infty H(t) \cdot dt = 1 \quad (9)$$

15 where  $w$  [ $T$ ] is a time parameter that represents the centre of gravity of the unit hydrograph,  $z$  [dimensionless] a form parameter,  $\pi=3.1416$  and  $t$  the time [ $T$ ]. For low values of  $z$  (i.e.  $z=1, 2$  or  $5$ ), the unit hydrograph represents both translation and diffusivity (resolution of the diffusive wave equation), while for high values of  $z$  (i.e.  $z=20, 50$  or  $100$ ), the unit hydrograph tends to represent only a translation equal to  $w$  (resolution of the kinematic wave equation).

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### 2.3 Model properties and parameters

The input rainfall  $P$  is usually given as a function of time in the form of a histogram using a fixed time interval. Consequently, the other variables are also presented as functions of time, and the computations are carried out for the same fixed time interval. The regulating element  $f$  and the soil-reservoir element are linked by a feedback path transmitting information about the state of the storage element to the regulating element. The regulating element is related to the soil-reservoir element by the fact that one of its outputs is the input of the soil-reservoir element. It is also related to the transfer function by the fact that its output  $R$  is one input of the transfer function.

Computations start at an instant adopted as zero time  $t=0$ , with a known, or an assumed, initial value of the soil-reservoir  $S_0$  and the aquifer-reservoir  $S_{b0}$  at the start of that time interval. For each flood event, the value of  $S_0$  is defined according to the 5-day antecedent rainfall  $R_{5d}$  corresponding to three classes (Soil Conservation Service, 1972)

$$\text{If } R_{5d} < 10 \text{ mm then } S_0 = 0.25S_m \quad (10)$$

$$\text{If } 10 < R_{5d} < 30 \text{ mm then } S_0 = 0.50S_m \quad (11)$$

$$\text{If } R_{5d} > 30 \text{ mm then } S_0 = 0.75S_m \quad (12)$$

At the beginning of the rainfall event, the measured discharge  $Q_0(0)$  at  $t=0$  is the sum of the lateral interflow  $q$  and the base flow  $B$ . Using Eqs. (4) and (5), and substituting  $S$  by  $S_0$ , the initial value of the aquifer-reservoir  $S_{b0}$  is calculated as

$$S_{b0} = \frac{1}{k} \left( \frac{Q_0(0)}{A} - f_c b \frac{S_0}{S_m} \right) \quad (13)$$

For each time interval, the three state variables  $f(t)$  which separates rainfall into surface runoff and infiltration, the level  $S(t)$  in the soil-reservoir and the level  $S_b(t)$  in the aquifer-reservoir are calculated from the known values of the variables at the beginning of the

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time interval and the rainfall input to the model during the interval. The values of the other variables at the end of the computation time interval are derived from the value of the three state variables by using the equations above.

The model needs : i) five parameters for the production function, the minimum value of the infiltration capacity  $f_c$ , a coefficient “a” such as maximum value of the infiltration capacity  $f_0 = af_c$  (with  $a > 1$ ), the maximum level of the soil-reservoir  $S_m$ , the parameter  $b$  of the lateral interflow, and the parameter  $k$  of the aquifer-reservoir’s recession curve, ii) two parameters for the transfer function, the lag time  $w$  and the shape parameter  $z$  and iii) two initial conditions  $S_0$  and  $S_{b0}$  calculated function of the 5-day antecedent rainfall  $R_{5d}$  and the measured discharge  $Q_0(t)$  at  $t = 0$ .

### 3 Formulation of calibration criteria

The objective of model calibration is to select parameter values so that the model simulates the measured hydrograph as closely as possible. Our aim is to consider multiple objectives that measure different aspects of the hydrological response (Madsen, 2000): i) a good agreement between the average simulated and observed runoff volume (i.e. a good water balance); ii) a good agreement of the peak flows; iii) a good overall agreement of the hydrograph shape.

When a calibration procedure is used, the quality of the final model parameters will depend on the structure of the model, the power of the optimisation algorithm, the quality of the input data, and the estimation criteria or objective functions used in the optimisation procedure (Gan and Biftu, 1996). It is beyond the scope of this paper to address all the above factors. We will focus on the last one only and discuss the definition of objective functions and multi-objective calibration procedures.

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### 3.1 The objective functions

The objective functions used in this study include both relative and absolute error measures as suggested by Legates and McCabe (1999). The selected criteria can be divided in two broad categories: “global” which gives more weight to strong flood events and “relative” which considers all events to be of equal weight. For each category, three different objective functions were considered: volume conservation, peakflow prediction, and the root mean square error (RMSE) which to a certain extent is comparable to the widely used Nash and Sutcliffe (1970) efficiency measure:

1. The global volume error  $V_g$  and the relative volume error  $V_r$

$$V_g = \left| \frac{\sum_{i=1}^N (Ls_i - Lo_i)}{\sum_{i=1}^N n_i} \right| \text{ and } V_r = \frac{1}{N} \sum_{i=1}^N \left| \frac{Ls_i - Lo_i}{Lo_i} \right| \quad (14)$$

where  $N$  is the total number of flood events used for calibration,  $i$  an index representing a flood event ( $1 \leq i \leq N$ ), and for each flood event  $i$ :  $n_i$  the number of time steps,  $Lo_i$  the observed runoff depth and  $Ls_i$  the simulated runoff depth.

2. The global root mean square error  $RMSE_g$  and the relative root mean square error  $RMSE_r$

$$RMSE_g = \left[ \frac{\sum_{i=1}^N \sum_{j=1}^{n_i} (Qo_{ij} - Qs_{ij})^2}{\sum_{i=1}^N n_i} \right]^{\frac{1}{2}} \text{ and } RMSE_r = \frac{1}{N} \sum_{i=1}^N \left[ \frac{1}{n_i} \sum_{j=1}^{n_i} (Qo_{ij} - Qs_{ij})^2 \right]^{\frac{1}{2}} \quad (15)$$

where  $n_i$  is the number of time steps in the flood event  $i$ ,  $j$  is an index representing the time step in a flood event  $i$  ( $1 \leq j \leq n_i$ ),  $Qo_{ij}$  the observed discharge at time  $j$  in the flood event  $i$  and  $Qs_{ij}$  the simulated discharge at time  $j$  on the flood event  $i$ .

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3. The global peakflow  $P_g$  and the relative peakflow  $P_r$

$$P_g = \frac{1}{N} \sum_{i=1}^N |(Qxs_i - Qxo_i)| \text{ and } P_r = \frac{1}{N} \sum_{i=1}^N \left| \frac{Qxs_i - Qxo_i}{Qxo_i} \right| \quad (16)$$

where  $Qxo_i$  is the observed peak flow of discharge in the flood event  $i$  and  $Qxs_i$  is the simulated peak flow of discharge in the flood event  $i$ .

5 The six objective functions,  $V_g$ ,  $V_r$ ,  $RMSE_g$ ,  $RMSE_r$ ,  $P_g$  and  $P_r$ , are positive functions, and the optimum value of the parameters corresponds to the minimum value of each “0”. A mono-objective calibration procedure was undertaken separately with each of the six criteria. Most model calibration procedures suffer from the same problems, namely the existence of multiple optima and the presence of high interaction or correlation between subsets of fitted model parameters. In order to avoid these, no automatic calibration procedure was undertaken, instead a grid-based calibration procedure was carried out to locate the optimum, and over 50000 simulations were run to calibrate the model using a progressively finer grid.

3.2 Multi-objective calibration procedures

15 When using multiple objectives, the calibration problem can be stated as follows (Madsen, 2000)

$$\min \{F_1(\theta), F_2(\theta), \dots, F_m(\theta)\} \text{ with } \theta \in \Theta \quad (17)$$

20 where  $F_i(\theta)$  ( $i = 1, 2 \dots, m$ ) are the different objective functions. The optimisation problem is constrained because  $\theta$  is restricted to the feasible parameter space  $\Theta$ . The parameter space is usually defined as a hypercube by specifying lower and upper limits on each parameter. These limits are chosen according to physical and mathematical constraints, information about physical characteristics of the system, and from modelling experiences (Kuczera, 1997; Madsen, 2000, 2003; Madsen et al., 2002).

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The solution of Eq. (17) will not, in general, be a single unique set of parameters but will consist of the so-called Pareto set of solutions according to different trade-offs between the different objectives (Gupta et al., 2003). The parameter space can be divided into “good” (Pareto optimal) and “bad” solutions, and none of the “good” solutions can be said to be “better” than any of the other “good” solutions (Madsen, 2000). A member of the Pareto set will be better than any other member with respect to some of the objectives, but because of the trade-off between the different objectives it will not be better with respect to other objectives. In practical applications, the entire Pareto set may be too expensive to calculate, and one is only interested in part of the Pareto optimal solutions.

When dealing with the multi-objective calibration, the problem is usually transformed into a single-objective optimisation problem by defining a scalar that aggregates the various objective functions (Madsen, 2000; Parakja et al., 2007) such as the Euclidean distance

$$F_{agg}(\theta) = \left[ (F_1(\theta) + A_1)^2 + (F_2(\theta) + A_2)^2 + \dots + (F_p(\theta) + A_p)^2 \right]^{1/2} \quad (18)$$

where  $A_i$  are transformation constants, reflecting the priorities assigned to the different objective functions. Herein, the balanced aggregated objective function suggested by Madsen (2000) was applied. In this case, the transformation constants in Eq. (18) are automatically calculated so that all  $(F_i(\theta) + A_i)$  have about the same distance to the origin near the optimum.

The multi-objective calibration procedure was first undertaken for each of the couples within the same function type i.e.  $(V_g$  and  $V_r$ ),  $(RMSE_g$  and  $RMSE_r$ ) and  $(P_g$  and  $P_r$ ). Then, we crossed two “global” criteria i.e.  $(V_g$  and  $RMSE_g$ ),  $(V_g$  and  $P_g$ ),  $(RMSE_g$  and  $P_g$ ), and two “relative” criteria  $(V_r$  and  $RMSE_r$ ),  $(V_r$  and  $P_r$ ) and  $(RMSE_r$  and  $P_r$ ). In the last step, the calibration was carried out function of the triples  $(V_g, RMSE_g$  and  $P_g)$  and  $(V_r, RMSE_r$  and  $P_r)$ .

## 4 The study site

### 4.1 Catchment description

The Gardon d'Anduze is a 543 km<sup>2</sup> Mediterranean catchment located in Southern France. It has a highly marked topography consisting of high mountain peaks, narrow valleys, steep hillslopes and a herring-bone shaped channel network. The highest point is the Mont Aigoual at 1567 m a.s.l. and the outlet is located at Anduze at 123 m a.s.l. The catchment's soils developed essentially on metamorphic (64% of the catchment area) and granitic terrains. The substrate is made of shale and crystalline rocks overlain by silty clay loams (83% of the catchment area) and sandy loam top soil. The vegetation is dense and composed mainly of beech and chestnut trees, holm oaks and garrigue, conifers, moor, pasture and cultivated lands. These vegetation classes are typical of Mediterranean forests.

Rainfall data for the 1977–1984 period were obtained on paper medium from the “Direction Départementale de l'Équipement du Gard” of the French Ministry of Equipment on seven rain gauge stations. The “Direction Départementale de l'Équipement du Gard” provided also analogue streamflow hydrographs at the outlet at Anduze (Moussa, 1991). Mean rainfall was calculated as the arithmetic mean of the seven rain gauges. The Gardon region is characterized by the highest rainfall intensities recorded in France e.g. a maximum daily rainfall of 608 mm was recorded on the Mont Aigoual during 24 h on 30–31 October 1963. The analysis of long rainfall time series shows that a daily rainfall of 70 mm has a return period of 1 year and daily rainfall of 170 mm has a return period of 100 years. The conjunction of high intensity rainfall, shallow soils and steep slopes produce very devastating floods in autumn.

### 4.2 Characteristics of the studied flood events

Flood events from the 1977–1984 period were selected based on a continuous rainfall greater than 50 mm. All corresponding hyetographs and hydrographs were digitised

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at an hourly time step (Moussa, 1991). In total, 29 events were retained: the event durations range between 24 h and 108 h, the total rainfall between 50 mm and 300 mm, the total runoff between 9 and 166 mm, the runoff coefficients between 15% and 67%, and the initial discharge between 3 and 92 m<sup>3</sup>s<sup>-1</sup>. Figure 2 shows the relations between the total rainfall, the total runoff depth, runoff coefficient, peakflow, and the initial discharge. No clear correlation can be seen between them i.e. the most important rainfall events in terms of precipitation volume aren't necessarily those that have the highest runoff coefficients or peakflow. The initial discharge value which represents the catchment's moisture condition doesn't seem to yield linear trends either. This finding is typical of Mediterranean climatic conditions where short duration and high intensity rainfall events are often the cause of the most important runoff events in terms of both runoff depth and peakflow.

Fifteen events, corresponding to the 1977–1979 period, were chosen for calibration and the remaining fourteen, corresponding to the 1980–1984 period, were used for validation (Fig. 2). Both data sets are representative of the various hydrological behaviours observed on the catchment. They cover all climatic seasons and display a large spectrum of rainfall intensity, peakflow and runoff coefficient values.

## 5 Mono- and multi-objective calibration and validation results

For the fifteen events of the calibration period, a number of tests were carried out in order to optimise the parameters first using mono-objective functions, and then to estimate the Pareto front and analyse the trade-offs between the different objectives when using two- or three-objective approaches. We defined a lower and an upper variation bound for each parameter. For all numerical tests, the hypercube search space shown in Table 1a was used.

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## 5.1 Results of the mono-objective calibration procedure

The calibrated parameter values for each of the six objective functions  $V_g$ ,  $V_r$ ,  $RMSE_g$ ,  $RMSE_r$ ,  $P_g$  and  $P_r$ , are presented in Table 1b. Results show that the parameters vary considerably depending on the objective function used.

5 For the production function, the soil-reservoir maximum capacity  $S_m$  ranges between 9 and 71 mm; this appears to be a small value if it is supposed to represent the storage in the root zone. The parameter  $f_c$ , which represents the soil's infiltration capacity at natural saturation, varies between  $0.29 \times 10^{-5}$  and  $2.6 \times 10^{-5} \text{ ms}^{-1}$  and compares well with the values estimated when using Rawls and Brakensiek's (1989) pedotransfer function. It is interesting to note that the two empirical parameters  $a$  and  $b$  have the widest span, probably because these parameters are used to compensate for the errors made on the remaining parameters of the production function. Indeed, even when using conceptual models, modellers tend to be less permissive with parameters that can evoke physical characteristics or be assimilated to such. As a direct consequence, such parameters, even in an un-constrained calibration procedure, will be allowed to vary in a tighter interval.

The transfer function has two parameters  $w$  and  $z$ . The travel time values of  $w$  obtained through the use of  $V_g$  and  $V_r$  ( $w=21-23$  h) are clearly overestimated both in comparison with the observation data (the basin response time ranges between 3 and 9 h) and the values obtained through  $RMSE_g$ ,  $RMSE_r$ ,  $P_g$  and  $P_r$  ( $w=3-4$  h). This is because the two volume criteria  $V_g$  and  $V_r$  are less sensitive to the hydrograph's global shape and consequently to the parameters of the transfer function. The second parameter of the transfer function  $z$  ranges between 3 and 17. These values highlight the importance of the diffusive factor.

25 For most parameters the use of  $RMSE_g$  or  $RMSE_r$  yields close results. However, when using  $P_g$  or  $P_r$ , the calibrated parameters differ from those obtained with  $V_g$ ,  $V_r$ ,  $RMSE_g$  or  $RMSE_r$ . These results highlight the differences between the various criteria:  $V_g$  or  $V_r$  describe the mean value of the discharge,  $RMSE_g$  or  $RMSE_r$  describe

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the whole shape of the hydrograph, while  $P_g$  and  $P_r$  refer to a single point representing peakflow.

No unique solution to the mono-objective problem is found and an “equifinality” of parameter sets can be seen as stated by Beven and Binley (1992) and Beven (1992 and 1993) i.e. many different parameter combinations give acceptable solutions.

## 5.2 Results of the multi-objective calibration procedure when crossing two objective functions

The results of the multi-objective calibration obtained when crossing two objective functions are shown in Figs. 3, 4 and 5. In Fig. 3, the calibration is based on two objective functions, the global volume error  $V_g$  and the relative volume error  $V_r$  (Fig. 3a), the global RMSE $_g$  and the relative RMSE $_r$  (Fig. 3c) and the global peakflow  $P_g$  and the relative peakflow  $P_r$  (Fig. 3e). Figures 3a, c and e show the objective function values corresponding to the evaluated parameter sets for two different objective functions. The Pareto optimal front (indicated by a star “\*” on Figs. 3a, c and e) is identified, and finally the balanced aggregated objective function (indicated by an arrow on Figs. 3a, c and e) is calculated. The mono-objective optimisation provides the tails of the Pareto front, and the optimisation based on the balanced aggregated measure approximates the balanced central part of the Pareto front.

The estimated Pareto front for the calibration of  $V_g$  and  $V_r$  (Fig. 3a) presents a trade-off. A very good calibration of  $V_g$  (corresponding to  $V_g=0$ ) provides a bad calibration of  $V_r$  ( $V_r=16.7\%$ ), and vice-versa ( $V_g=0.166\times 10^{-4} \text{ m}^3$  for  $V_r=8.7\%$ ). The same comment can be made about  $P_g$  and  $P_r$ :  $P_g=40.3 \text{ m}^3 \text{ s}^{-1}$  when  $P_r=18.3\%$  and  $P_g=58.0 \text{ m}^3 \text{ s}^{-1}$  when  $P_r=13.3\%$ . This result is not surprising as peakflow refers to instantaneous values that are both difficult to determine and simulate. The Pareto front of RMSE $_g$  and RMSE $_r$  shows a high correlation. This can be explained by the fact that the majority of flood events have similar duration (48 to 96 h) and consequently the two objective functions presented in Eq. (15) tend to be similar.

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The variation of the optimum model parameter sets along the Pareto front is shown in grey in Figs. 3b, d and e for the multi-objective calibration of ( $V_g$  and  $V_r$ ), ( $RMSE_g$  and  $RMSE_r$ ) and ( $P_g$  and  $P_r$ ) respectively. The parameter values are normalised with respect to the upper and lower limits given in Table 1a so that the range of all normalised parameters is between 0 and 1. For the calibration of  $V_g$  and  $V_r$  (Fig. 3b), a remarkably large span is observed in the parameter values when moving along the Pareto front. The range is larger than 50% for the main parameters  $S_m$ ,  $a$ ,  $b$ ,  $w$  and  $z$ . The compromised solution using the balanced aggregated function (Eq. 18) is shown in bold on Figs. 3b, d and e, and the corresponding values of the calibrated parameters are given in Table 1c1. The calibrated parameters of the compromised solution of ( $V_g$  and  $V_r$ ) in Table 1c1 range within the interval delimited by the calibrated parameters of  $V_g$  and  $V_r$  separately (Table 1b). Similar results are observed for ( $RMSE_g$  and  $RMSE_r$ ) and for ( $P_g$  and  $P_r$ ).

Figure 4 shows results when crossing two “global” criteria ( $V_g$  and  $RMSE_g$ ; Figs. 4a, b), ( $V_g$  and  $P_g$ ; Figs. 4c, d) and ( $RMSE_g$  and  $P_g$ ; Figs. 4e, f) and Fig. 5 shows results when crossing two “relative” criteria ( $V_r$  and  $RMSE_r$ ; Figs. 5a, b), ( $V_r$  and  $P_r$ ; Figs. 5c, d) and ( $RMSE_r$  and  $P_r$ ; Figs. 5e, f). Again, we observe significant trade-offs for the three cases of Fig. 4 :

- $V_g=0$  when  $RMSE_g=65.5 \text{ m}^3 \text{ s}^{-1}$  and  $V_g=5.0 \times 10^{-5} \text{ m}^3$  when  $RMSE_g=31.0 \text{ m}^3 \text{ s}^{-1}$  (Fig. 4a);
- $V_g=0$  when  $P_g=92.4 \text{ m}^3 \text{ s}^{-1}$  and  $V_g=23.8 \times 10^{-5} \text{ m}^3$  when  $P_g=40.3 \text{ m}^3 \text{ s}^{-1}$  (Fig. 4c);
- $RMSE_g=31.0 \text{ m}^3 \text{ s}^{-1}$  when  $P_g=61.6 \text{ m}^3 \text{ s}^{-1}$  and  $RMSE_g=50.9 \text{ m}^3 \text{ s}^{-1}$  when  $P_g=40.3 \text{ m}^3 \text{ s}^{-1}$  (Fig. 4e).

Comparable results are obtained with the relative criteria in Figs. 5a, c and e. The variation of the optimum model parameter sets along the Pareto front for the multi-objective calibration of ( $V_g$  and  $RMSE_g$ ; Fig. 4b), ( $V_g$  and  $P_g$ ; Fig. 4d), ( $V_r$  and  $RMSE_r$ ; Fig. 5b), ( $V_r$  and  $P_r$ ; Fig. 5d) are similar and sometimes higher than those observed for

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( $RMSE_g$  and  $P_g$ ; Fig. 4f) and ( $RMSE_r$  and  $P_r$ ; Fig. 5f). Figures 3 to 5 indicate that the global volume criteria ( $V_g$ ) is the most insensitive i.e. volume conservation can be easily respected with a number of septuplet combinations. This translates into a somehow flat Pareto front (Figs. 3a, 4a, b). In comparison RMSE and peakflow produce sharper fronts; RMSE seems to be the most restrictive of the tested criteria. The relative criteria seem to yield sharper fronts than the global criteria which seem to be controlled by the extreme events.

In comparison with the mono-objective calibration, the use of a multi-objective calibration technique seems to yield tighter variation intervals. These findings are in accordance with those of Engeland et al. (2006). The variation ranges we obtained for the multi-objective calibration are smaller than those reported in Schoops et al. (2005) and Madsen (2000) who had respectively 3 and 2 additional parameters to calibrate. However, the number of free parameters cannot be the only explanation behind the wider variation spans as Gupta et al. (2003) obtained tighter intervals when calibrating the 13 parameters of the SAC-SAM model.

### 5.3 Results of the multi-objective calibration procedure when crossing three objective functions

The same methodology was extended in order to combine three objective functions. Figure 6 shows the results obtained when crossing the three global objective functions ( $V_g$ ,  $RMSE_g$  and  $P_g$ ; Fig. 6a), and the three relative objective functions ( $V_r$ ,  $RMSE_r$  and  $P_r$ ; Fig. 6b). Results show comparable parameter variation ranges for both ( $V_g$ ,  $RMSE_g$  and  $P_g$ ) and ( $V_r$ ,  $RMSE_r$  and  $P_r$ ).

For the optimal solution (Table 1d), the soil-reservoir maximum capacity  $S_m$  is equal to 172 mm when crossing ( $V_g$ ,  $RMSE_g$  and  $P_g$ ) and 225 mm when crossing ( $V_r$ ,  $RMSE_r$  and  $P_r$ ). This value is more representative of the storage in the root zone. For both combinations ( $V_g$ ,  $RMSE_g$  and  $P_g$ ) and ( $V_r$ ,  $RMSE_r$  and  $P_r$ ), the optimal parameter  $f_c$  ranges between  $5.5 \times 10^{-5}$  and  $7.4 \times 10^{-5} \text{ ms}^{-1}$ , and compares well with pedotransfer

functions. It is interesting to note that the two empirical parameters  $a$  and  $b$  still have the widest span. The travel time values of  $w$  obtained range between 3 and 4 h and correspond to the approximate lag time of the basin during intense flood events, while the dimensionless shape parameter  $z$ , ranges between 14 and 28.

The parameter variation ranges are comparable to those obtained when using only two functions. The parameters that vary mostly between the two combinations are  $b$ ,  $k$  and  $z$ . The parameters  $b$  and  $k$  are clearly linked to each other as they both control the “outward” fluxes and can be used to correct possible flow over-estimation by increasing percolation and interflow. Data on water levels both in the unsaturated and saturated zones could have been useful in constraining these parameters further. Unfortunately such data were unavailable for the studied period.

#### 5.4 Validation and uncertainty analysis

As the “relative” and “global” criteria gave comparable results and interpretation, we choose to validate the results of the “relative” criteria ( $V_r$ ,  $RMSE_r$  and  $P_r$ ) which are less sensitive to extreme events.

To compare the calibration procedures, we computed the value of each objective function (Table 2). It is not surprising that the values of  $V_r$ ,  $RMSE_r$  and  $P_r$  are minimal when mono-objective calibration are conducted function of each of these criteria:  $V_r=8.7\%$ ,  $RMSE_r=26.0\text{ m}^3\text{ s}^{-1}$  and  $P_r=13.4\%$ . The maximal value of  $V_r=19.1\%$  is obtained when using a mono-objective calibration minimising  $P_r$ ; the maximal value of  $RMSE_r=43.2\text{ m}^3\text{ s}^{-1}$  is obtained when using a mono-objective calibration minimising  $V_r$ ; and the maximal value of  $P_r=27.8\%$  is also obtained when using a mono-objective calibration minimising  $V_r$ . Once again the use of a volume based criterion is restrictive while the use of RMSE can yield relatively acceptable results for peakflow and vice versa. The sensitivity of the RMSE criterion to peakflow has already been reported by many authors (Parada et al., 2003; Yapo et al., 1998) and our findings are similar to theirs.

When using two- or three-objective functions, the error values fall within the intervals

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indicated above. However, it is interesting to note that for all three criteria, the maximal errors values obtained with the multi-objective methods are always lower than those obtained when using a mono-objective calibration in which the given criterion is not considered. Finally the combination of ( $V_r$ ,  $RMSE_r$  and  $P_r$ ) gives a reasonable compromise between the three criteria with  $V_r=12.6\%$ ,  $RMSE_r=33.3\text{ m}^3\text{ s}^{-1}$  and  $P_r=21.4\%$ . These values are quite comparable to those obtained when using just two objective functions i.e.  $V_r$ ,  $RMSE_r$ .

To further compare the performance of the calibration procedures, we computed the relative error on both runoff depth and peakflow: for a given event  $i$ , the error on runoff depth and peakflow are defined respectively by  $\varepsilon_{vi}=(Ls_i-Lo_i) / Lo_i$  and  $\varepsilon_{Qi}=(Qxs_i-Qxo_i)/Qxo_i$ . Let  $\overline{\varepsilon_V}$  and  $\overline{\varepsilon_Q}$  be the mean of  $\varepsilon_{vi}$  and  $\varepsilon_{Qi}$  respectively. The quantities  $\overline{\varepsilon_V}$  and  $\overline{\varepsilon_Q}$  represent the bias of runoff depth and peakflow predictions. Table 3 illustrates the findings when using either mono and multi-objective method (i.e. two or three objective functions) and shows that  $\overline{\varepsilon_V}$  and  $\overline{\varepsilon_Q}$  fall within similar ranges for two- or three-objective functions. It is worthy to note that the greatest errors on runoff depth are obtained through the use of a peakflow criterion while the greatest errors on peakflow are caused by the use of a volume criterion. Apart from two cases, the error on runoff depth is acceptable (<5%) while, not surprisingly, the error on peakflow is far higher (>10%) but less than 27%. In addition, the model seems to have a tendency to overestimate runoff depth and to underestimate peakflow.

The best compromise between both errors is reached by using the parameters obtained through the multi-objective calibration with three functions ( $V_r$ - $RMSE_r$ - $P_r$ ) but the combination of  $V_r$  and  $RMSE_r$  gives once again comparably good results. In this instance the use of two “well” chosen and complementary objective functions seems to be sufficient for runoff simulation. However, the lack of soil-moisture and groundwater data prevents us from extending our results to the other vertical components of the model.

Figure 7 compares the measured and simulated runoff depths (Fig. 7a) and the measured and simulated peakflows (Fig. 7b) obtained when using the parameters of

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the balanced aggregated objective function ( $V_r$ ,  $RMSE_r$  and  $P_r$ ) given in Table 1d. It can be seen that the model gives similar results both for the calibration and validation periods. An illustration of the equifinality problem is shown in Fig. 8 where simulated hydrographs are obtained using the parameters corresponding to the entire Pareto front of ( $V_r$ ,  $RMSE_r$  and  $P_r$ ). The results indicate that the hydrographs are also well simulated for high (Fig. 8a) and low intensity events (Fig. 8b). The figure shows clearly that many different parameter sets, may produce equally good simulations according to the three objective functions.

## 6 Discussion and conclusion

A conceptual lumped event based rainfall-runoff model, coupling a production and a transfer function, was developed and applied on the Gardon catchment in southern France. The model has seven free parameters which need to be calibrated. The model was calibrated on 15 events and validated on 14 others. The results of both the calibration and validation phases were compared on the basis of their performance with regards to six objective functions, three global and three relative, representing volume, peakflow, and the root mean square error.

The results showed that the calibrated parameter values were dependent on the type of the objective function used. Trade-offs are observed between the different objectives and no single set of parameter was able to optimise all objectives simultaneously. Thus a set of Pareto optimal solutions and a balanced aggregated objective function were calculated with two and three objective functions.

The comparison of the mono and multi-objective calibration results, using two or three functions, illustrates the “non-uniqueness problem” (Beven and Binley, 1992) since many different parameter combinations gave acceptable solutions according to a given objective. However, the volume conservation criterion seemed to be the most permissive whereas the RMSE and the peakflow prediction criteria yielded sharper Pareto fronts. The model was then validated with the set of optimised parameters ob-

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tained using the “combined relative” criteria ( $V_r$ ,  $RMSE_r$  and  $P_r$ ). The use of a triple objective function does not seem to be justified in our case. Indeed given the impact peakflow values have on the RMSE, there seems to be a redundancy in their use, hence a combination of either ( $V_r$  and  $P_r$ ) or ( $V_r$  and  $RMSE_r$ ) can yield equally acceptable results.

Differences between measured and simulated hydrographs were assessed by calculating the bias of the simulated runoff depth and peakflow. These errors can be due to the use of non-optimal parameter values but also to errors inherent to the model structure and the meteorological input data. In the model calibration herein, only the error due to the parameter values is minimised. However, the calibration of model parameters can also compensate the other error sources. In our case the best results in terms of bias were obtained through multiple calibration with a volume and an RMSE criterion

The choice of an adequate objective function when modelling separate flood events, emphasise the importance of the modeller’s intervention for tailoring the model calibration to a specific application. Attempts have already been made to include this knowledge objectively in the model calibration procedure (Boyle et al., 2000). Our results highlight the importance of the modeller’s professional judgement as often the criteria values and error estimates are within close bounds and may not be significantly different from a statistical point of view. It is therefore important to plot the hydrographs and assess the graphical differences in the simulated hydrograph’s shape.

A sound hydrological knowledge is required to evaluate data and model errors. In most real world application, especially in an operational framework and for real-time predictions, data quality checks could be too time consuming and hence difficult to carry out. Thus a robust calibration procedure becomes even more essential.

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**Table 1.** Parameter ranges applied for the different automatic calibration together with the mono-objective calibration and the balanced multi-objective Pareto optimum solution.

Objective Function	$S_m$ (m)	$a$	$f_c (\times 10^{-5})$ ( $\text{ms}^{-1}$ )	$b$	$k (\times 10^{-5})$ ( $\text{s}^{-1}$ )	$w$ (h)	$z$
<b>a. Parameter ranges</b>							
	[0–1.0]	[1–70]	[0–30]	[0–1]	[0–30]	[0–72]	[0.01–100]
<b>b. Mono-objective calibration</b>							
$V_g$	0.015	1.39	2.634	0.809	0.361	23.03	13.71
$V_r$	0.009	19.92	0.112	0.024	0.249	21.29	11.03
$\text{RMSE}_g$	0.051	12.94	0.289	0.260	0.103	5.68	3.48
$\text{RMSE}_r$	0.053	12.52	0.295	0.267	0.111	5.79	3.24
$P_g$	0.029	1.70	1.313	0.462	0.789	3.06	17.16
$P_r$	0.071	17.84	0.737	0.064	0.342	4.32	4.20
<b>c. Pareto optimum with two objective functions</b>							
<i>c.1. Crossing one “global” and one “relative” objective function</i>							
$(V_g)-(V_r)$	0.076	1.44	5.695	0.010	0.278	20.83	4.93
$(\text{RMSE}_g)-(\text{RMSE}_r)$	0.051	12.94	0.289	0.260	0.103	5.68	3.48
$(P_g)-(P_r)$	0.158	5.89	5.206	0.010	3.592	3.16	15.78
<i>c.2. Crossing two “global” objective functions</i>							
$(V_g)-(\text{RMSE}_g)$	0.315	5.10	0.051	0.474	9.867	7.82	11.43
$(V_g)-(P_g)$	0.120	24.25	0.011	0.700	5.620	5.23	0.22
$(\text{RMSE}_g)-(P_g)$	0.040	9.68	0.129	0.634	0.112	11.65	0.42
<i>c.3. Crossing two “relative” objective functions</i>							
$(V_r)-(\text{RMSE}_r)$	0.212	27.07	0.036	0.955	0.820	6.08	15.01
$(V_r)-(P_r)$	0.223	13.69	0.041	0.792	0.602	8.53	13.38
$(\text{RMSE}_r)-(P_r)$	0.075	16.80	0.637	0.065	0.382	4.62	5.10
<b>d. Pareto optimum with three objective functions</b>							
$(V_g)-(\text{RMSE}_g)-(P_g)$	0.172	10.549	5.547	0.220	0.2375	4.36	28.2
$(V_r)-(\text{RMSE}_r)-(P_r)$	0.225	4.289	7.372	0.055	0.367	3.03	14.1

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**Table 2.** Values of the three objective functions  $V_r$ ,  $RMSE_r$  and  $P_r$  when using the calibrated parameters with the mono-objective calibration and the balanced two-objective or three-objective Pareto optimum solution.

Objective function used for calibration	Criteria value		
	$V_r[-]$	$RMSE_r(m^3s^{-1})$	$P_r[-]$
Mono-objective ( $V_r$ )	0.087	43.2	0.278
Mono-objective ( $RMSE_r$ )	0.158	26.0	0.159
Mono-objective ( $P_r$ )	0.191	32.4	0.134
Two-objective ( $V_r$ )–( $RMSE_r$ )	0.117	33.4	0.216
Two-objective ( $V_r$ )–( $P_r$ )	0.116	34.4	0.197
Two-objective ( $RMSE_r$ )–( $P_r$ )	0.180	29.4	0.145
Three-objective ( $V_r$ )–( $RMSE_r$ )–( $P_r$ )	0.126	33.3	0.214

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**Table 3.** Means  $\overline{\varepsilon_V}$  and  $\overline{\varepsilon_Q}$  of the relative prediction error on runoff depth and peakflow of the calibration and validation events when using the calibrated parameters with the mono-objective calibration and the balanced two-objective or three-objective Pareto optimum solution.

Objective function used for calibration	Runoff depth $\overline{\varepsilon_V}$	Peakflow $\overline{\varepsilon_Q}$
Mono-objective ( $V_r$ )	-0.049	-0.269
Mono-objective (RMSE <sub>r</sub> )	0.047	0.129
Mono-objective ( $P_r$ )	0.172	-0.107
Two-objective ( $V_r$ )-(RMSE <sub>r</sub> )	0.021	-0.161
Two-objective ( $V_r$ )-( $P_r$ )	0.026	-0.190
Two-objective (RMSE <sub>r</sub> )-( $P_r$ )	0.169	-0.112
Three-objective ( $V_r$ )-(RMSE <sub>r</sub> )-( $P_r$ )	0.014	-0.183

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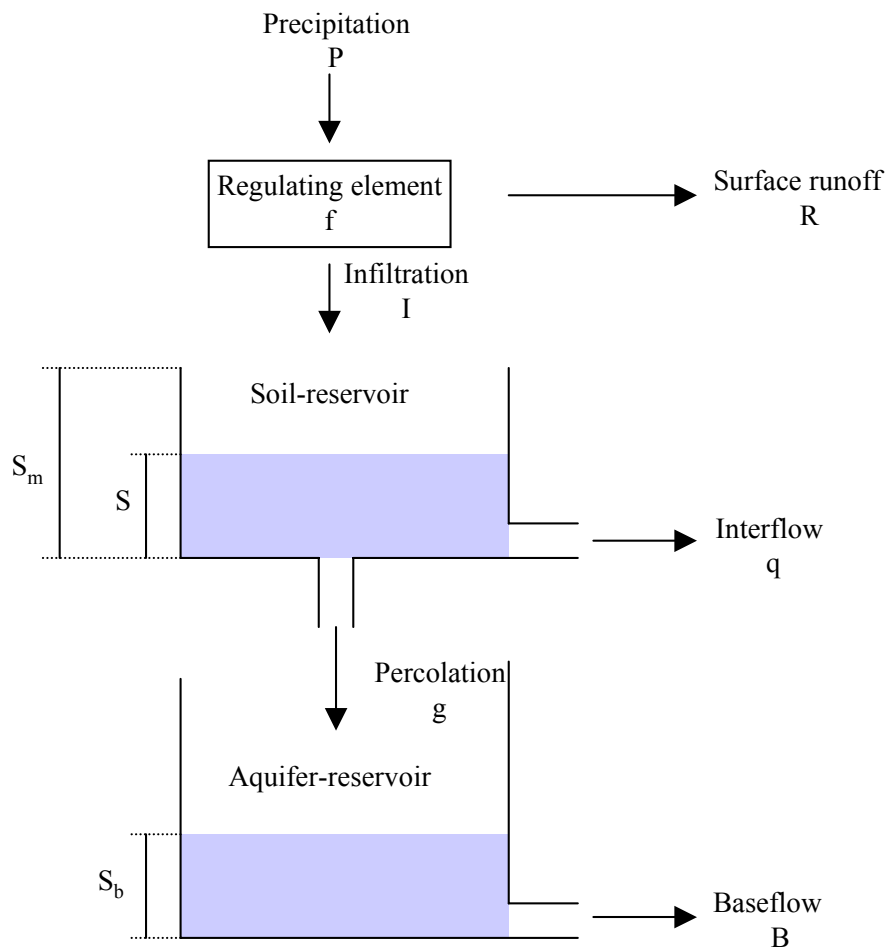


Fig. 1. Schematic of the model production function.

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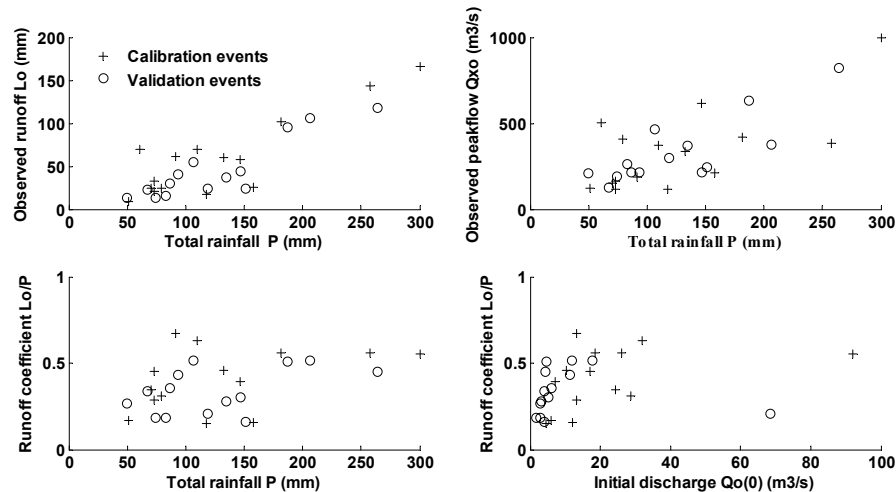
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**Fig. 2.** Relationships between total rainfall, total runoff and runoff coefficient for the calibration (+) and validation (o) events.

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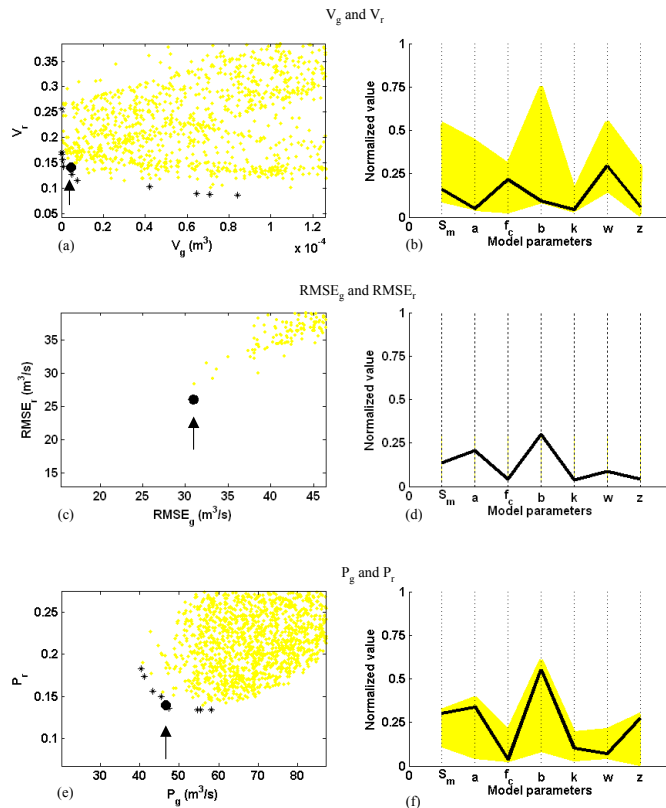
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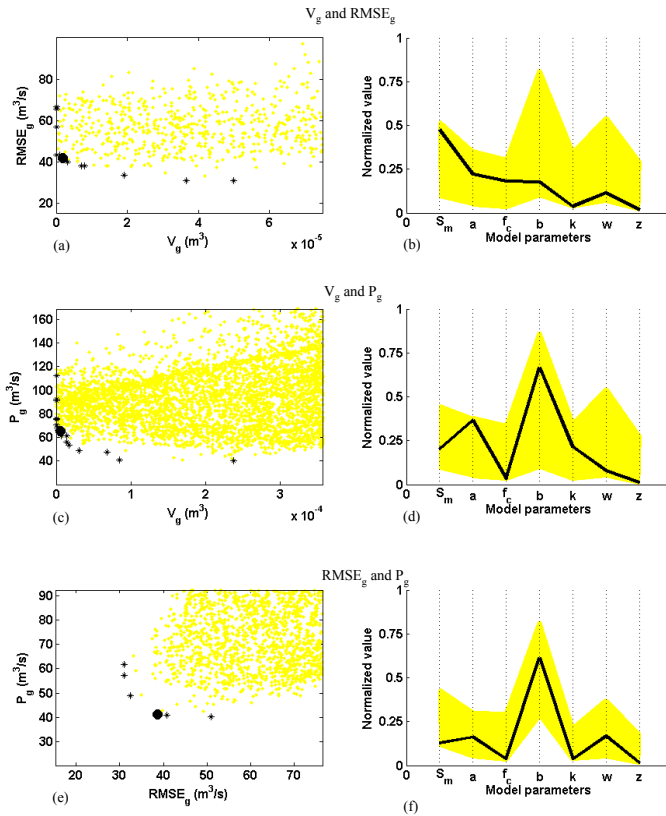


**Fig. 3.** Multi-objective calibration using one “global” and one “relative” objective function. **(a)**, **(c)**, **(e)** Objective function values during the calibration process; the “\*” indicates the Pareto front and the bold point indicates the balanced aggregated objective function. **(b)**, **(d)**, **(f)** Normalized range of parameter values along the Pareto front shown in **(a)**, **(c)** and **(e)** respectively; full bold line indicate the parameter set corresponding to the Pareto balanced aggregated objective function.



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**Fig. 4.** Multi-objective calibration using two “global” objective functions. **(a), (c), (e)** Objective function values during the calibration process; the “\*” indicates the Pareto front and the bold point indicates the balanced aggregated objective function. **(b), (d), (f)** Normalized range of parameter values along the Pareto front shown in **(a), (c)** and **(e)** respectively; full bold line indicate the parameter set corresponding to the Pareto balanced aggregated objective function.

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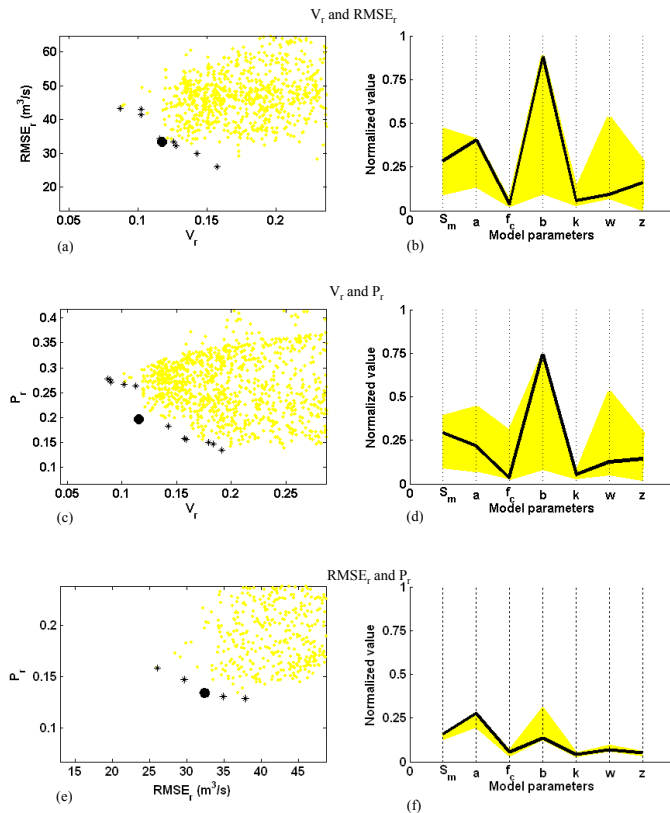
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**Fig. 5.** Multi-objective calibration using two “relative” objective functions. **(a), (c), (e)** Objective function values during the calibration process; the “\*” indicates the Pareto front and the bold point indicates the balanced aggregated objective function. **(b), (d), (f)** Normalized range of parameter values along the Pareto front shown in (a), (c) and (e) respectively; full bold line indicate the parameter set corresponding to the Pareto balanced aggregated objective function.

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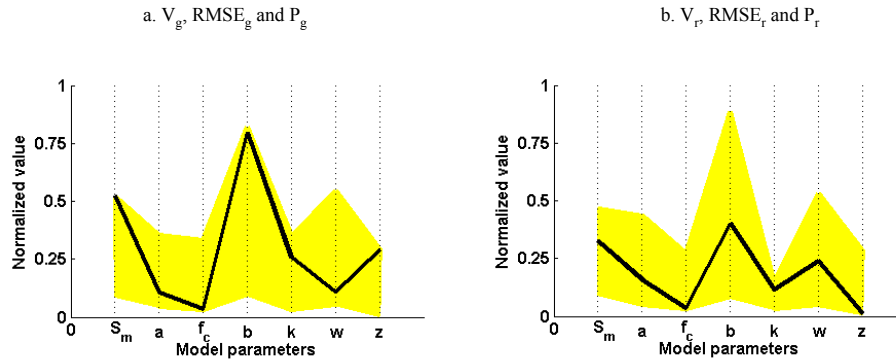
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**Fig. 6.** Normalized range of parameter values along the Pareto front shown for three “global” (a) and three “relative” (b) objective functions; full bold line indicate the parameter set corresponding to the Pareto balanced aggregated objective function.

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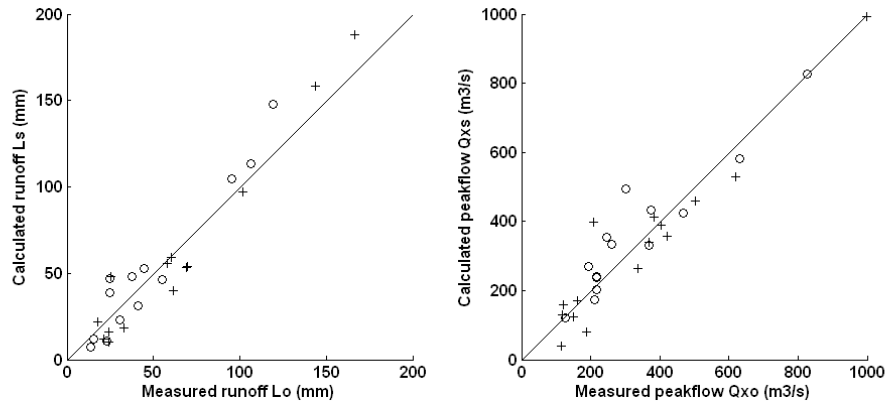
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**Fig. 7.** Comparison between measured and simulated runoff depths **(a)** and peakflows **(b)** using the set of parameters from the multi-objective  $V_r$ -RMSE $_r$ - $P_r$  procedure.

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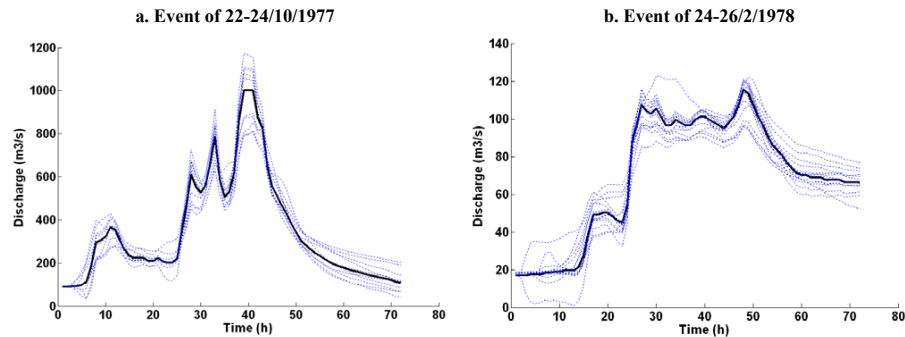
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**Fig. 8.** Example of simulated hydrographs of high event **(a)** and low event **(b)** using the set of parameters from the multi-objective  $V_r$ -RMSE $_r$ - $P_r$  procedure.

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