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Calibration of hydrological model parameters for ungauged catchments

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HESSD

3, 1105–1124, 2006

**Calibration of
hydrological model
parameters and PUB**

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

EGU

Abstract

The parameters of hydrological models for catchments with few or no discharge records can only be estimated using regional information. One can assume that catchments with similar characteristics show a similar hydrological behaviour and thus can be modeled using similar model parameters. Therefore a regionalisation of the hydrological model parameters on the basis of catchment characteristics is plausible. However, due to the non-uniqueness of the rainfall-runoff model parameters (equifinality), a workflow of regional parameter estimation by model calibration and a subsequent fit of a regional function is not appropriate. In this paper a different approach for the transfer of entire parameter sets from one catchment to another is discussed. Transferable parameter sets are identified using regional statistics: means and variances of annual discharges estimated from catchment properties and annual climate statistics.

1 Introduction

Hydrological modelling of water balances or extremes (floods and droughts) is important for planning and water management. Unfortunately the small number (or even the lack) of observations of key variables that influence hydrological processes limits the applicability of rainfall runoff models; discharge is only measured at a few locations, precipitation measurements are taken at some selected points. Thus modelling is an important tool to estimate the elements of the water cycle in areas of interest. In principle, if the models are based on the basic principles of physics (mass and energy conservation), the estimation of model parameters should be a straightforward task. Unfortunately, the extreme heterogeneity of the influencing parameters, such as soil properties or the unresolved spatial and temporal variability of meteorological variables (mainly rainfall), limits the applicability of physically based models to mainly process studies on small well observed experimental catchments. On meso-scale catchments with observed discharge series, conceptual or partly conceptual models can be used

Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

well if they are calibrated on observed events. These types of models are often used for flood forecasting or design purposes. The application of these models on ungauged catchments is very limited as the model parameters are estimated using calibration. The model parameters could be transferred using regionalisation methods (Abdulla and Lettenmeier, 1997). However the transfer of parameters is difficult as:

- optimal parameter sets depend on the models and the objective functions used to measure their performance (Gupta et al., 1998; Madsen, 2001)
- parameters are themselves uncertain (Kuczera and Mroczkowski, 1998)
- parameters are not unique – a diverse set of possible parameter values can lead to similar model performances (equifinality) (Beven and Freer, 2001).

As model calibration can lead to non-unique sets of parameters, it is difficult to associate the parameters estimated through calibration with the characteristics of the catchment and to transfer them to ungauged locations. In Hundecha and Bardossy (2004), model parameters were regionalized through simultaneous calibration of the same hydrological model on different catchments. This procedure however assumes parameters whose dependence can be described using an a priori defined function. Further, only one set of parameters is obtained for the ungauged catchment. It would be desirable if – for the triple: a catchment, a model and a parameter set – one could assign a quality metric to help decide how well the model performs on the given catchment by using the selected parameter set. This task however is not realistic.

The purpose of this paper is to develop a methodology to transfer hydrological model parameter sets to other catchments as sets, entities or vectors. The idea of using parameter vectors instead of individual parameters is explained with a simple example in the second section of this paper. The properties of those parameter vectors leading to good model performances are investigated in the third section. In the fourth section, the transfer methodology is outlined and applied to selected subcatchments of the German part of the Rhine catchment. The paper ends with a summary and conclusions.

2 Interdependence of model parameters

Hydrological models describe the natural processes of the water cycle. Due to the large complexity of the corresponding natural phenomena, these models contain substantial simplifications. They consist of basic equations, often loosely based on physical premises, whose parameters are specific for the selected catchment and problem under study. For partly or fully conceptual models, some parameters cannot be considered as physically measured (or measurable) quantities and thus have to be estimated on the basis of the available data and information. Due to the fact that in the range of possible (or already observed) input data, different model parameters lead to a similar performance, the identification of a unique dataset is practically impossible (Beven and Freer, 2001). However those model parameter sets which lead to a good model performance might have interesting internal structures. This fact is illustrated with an extremely simple two parameter unit hydrograph model. For a selected flood event in May 1999 on the River Kocher at Abstgmünd, the hydrograph of the direct runoff was modelled using a Nash cascade (Nash, 1960) as a unit hydrograph model. Precipitation and discharge were both observed with a 1 h temporal resolution. The instantaneous unit hydrograph (impulse response function) of the Nash cascade is:

$$u(t, k, n) = \frac{1}{k\Gamma(n)} \left(\frac{t}{k}\right)^{n-1} e^{-\frac{t}{k}} \quad (1)$$

The Nash cascade is described by two parameters n (number of reservoirs) and k (storage constant). A large number of different parameter combinations were generated, and the performance of the model was estimated using the Nash-Sutcliffe efficiency (NS) (Nash and Sutcliffe, 1970). Figure 1 shows the observed and some of the fitted hydrographs. The hydrographs calculated correspond to different parameter vectors (n, k) , but all have nearly the same performance (NS-value). A large number of independent pairs (n, k) were randomly generated and the corresponding NS values were calculated. Figures 2 and 3 respectively show the model performance for a large number of possible values for the first parameter (n) and for the second parameter (k).

Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

These two figures show that there is a large range of values for both parameters, such that for each parameter k (or n) one can select a second parameter n (or k) and the model's performance will still be close to optimal. This figure indicates a high uncertainty for both of the parameters. They can be taken from a wide interval of possible values which might lead to good model performance. However if one investigates the set of parameters as pairs, one obtains a well structured set. Figure 4 shows the pairs of parameters which perform better than 95% of the NS recorded for the best set. The pairs with good performance all lie along a (hyperbolic) curve indicating that the uncertainty is mainly due to a compensation – for a large range of parameters k one can find a more or less unique parameter n such that the model performs well, the product nk being the collective lag of the cascade. In the case of more complicated models one can assume similar behaviour, however the identification of higher dimensional hyper surfaces of parameters whose points lead to nearly equal performance is much more complicated.

The (usually unknown) non-linear relationships between the parameters make their transfer to ungauged catchments extremely difficult. Imagine one would have two locations with “good” model parameters on the same line denoted by points A and B on Fig. 4. If one would interpolate between these points using a linear scheme, point C would be obtained. Despite both model parameter vectors performing well, the interpolated C is far from the line and would lead to a bad model performance. This simple example shows that the transfer of model parameters of more complex models to ungauged catchments is an extremely difficult task, and that model parameters cannot be treated as independent individual values but instead as complimentary parameter vectors.

3 Transfer of hydrological model parameters

In this section, the transfer of model parameters of a simple conceptual rainfall-runoff model is considered. A distributed hydrological model based on the HBV (Bergström,

1995) concept was used to explain the methodology. The model consists of different elements describing the relevant hydrological processes. The spatially distributed processes calculated for each zone are:

- snow accumulation and melt, modeled by a degree-day method;
- the proportion of rain or snowmelt that produces runoff as a function of the soil moisture deficit;
- evapotranspiration calculated on the basis of the long term monthly mean values of potential evapotranspiration. This value is adjusted on a daily basis for temperature anomalies using a monthly factor.

The runoff response routine calculates the transformation of the excess water into discharge at the outlet of the subcatchment according to the soil moisture situation. The routine consists of one upper reservoir with two lateral and a vertical outlet and a lower, linear reservoir. The reservoirs are linear and each outlet has a different recession constant. For further details, refer to Hudecha and Bardossy (2004).

This model was applied to selected meso-scale subcatchments of the German part of the Rhine catchment. The selected catchment sizes vary between 700 and 2000 km². Daily discharge data from 100 gauging stations, as well as daily temperature data from 150 stations and precipitation data from 601 stations within and around the study area were obtained for the period 1985–2000.

Meteorological input for the hydrological model was interpolated from observations with External Drift Kriging (Ahmed and De Marsily, 1987) using topographical elevation considered as external drift. Model parameters related to soil and evapotranspiration were estimated directly for each subcatchment.

A set of 13 subcatchments representing different land use and topographical conditions was selected as a basis for transferable parameter sets. A large number of different parameter vectors was considered for all of these catchments. The model's 5 conceptual parameters describing the runoff concentration using two reservoirs were

Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

selected for possible parameter vector transfer from one catchment to another. The other parameters, related to runoff formation in a spatially distributed manner, were estimated based on the soil land use and topographical information using transfer functions according to the regionalization procedure described in Hundecha and Bardossy (2004). For each of the five selected model parameters, a range was fixed and a uniform distribution within the range was assumed to generate the candidate parameter vectors. No explicit dependence between the parameters was assumed. A large number of parameter vectors was generated according to these assumptions. Following this, the hydrological model was applied and the model performance was measured using an appropriate “quality” function. A typical performance measure is the Nash-Sutcliffe efficiency (NS) between observed and modelled daily values. The model performance was calculated for each realisation of random parameter vectors. The set of good parameter vectors was defined as the subset of the generated vectors for which the NS exceeded 90% of the maximal NS value of the set for catchment i . This set, denoted as $D(i)$, was subsequently investigated in the parameter space. In order to identify possible linear parameter dependencies, a principal component analysis was carried out. This analysis showed that the linear dimension of $D(i)$ is dependent on the catchment and varies between 3 and 4, not 5. Subsequently, the Hausdorff dimension of the set $D(i)$ was estimated. The Hausdorff dimension is defined as the limit (Falconer 1985):

$$d_H(i) = \lim_{\varepsilon \rightarrow 0^+} \frac{\ln N(\varepsilon)}{\ln \varepsilon} \quad (2)$$

with $N(\varepsilon)$ being the minimum number of open spheres of diameter ε required to cover the set $D(i)$. Due to the high dimension of the parameter vector (5 parameters were considered) the available sample did not allow an accurate estimation of the Hausdorff dimension. However, a comparison of the $N(\varepsilon)$ -s for a random set of the same cardinality and the set $D(i)$ indicated that the statement that the dimension of $D(i)$ is much lower than the linear dimension identified using PCA. The calculated values suggest that the Hausdorff dimension of the good parameter set is between 1.5 and 3 and de-

Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

pendes on the catchment. A highly non-linear compensation of the model parameters is responsible for this low dimension. Note that for the example of the Unit hydrograph (Nash cascade with two parameters), the Hausdorff dimension of the good parameter set slightly exceeds 1 (as the points are on one curve). Unfortunately there are no tools available to identify the analytical form of the lower dimensional manifold which contains the points of the set $D(i)$ for the HBV model. Thus a transformation of the parameters with a functional relationship remains very uncertain. Instead, all elements of the set $D(i)$ can be considered as candidates for the transfer of parameters from i to another catchment, j . They all have the property of representing a reasonable compensation of the individual parameters. Unfortunately not all of the elements of $D(i)$ deliver reasonable hydrographs for catchment j .

In order to identify those elements of $D(i)$ which could be reasonably used for catchment j , a regionalisation of the discharge statistics was first carried out. For this purpose, the methodology described in Samaniego and Bardossy (2005) was used. The mean annual discharges, $M_Q(j)$, and the standard deviation, $S_Q(j)$, of the daily discharges were estimated as a function of the catchment characteristics and statistics of the meteorological input; the estimator of the mean being of the form

$$M_Q(j) = a_0 \prod_{k=1}^K x_k(j)^{a_k} + \varepsilon_M(j) \tag{3}$$

and for the standard deviation

$$S_Q(j) = b_0 \prod_{k=1}^K x_k(j)^{b_k} + \varepsilon_S(j) \tag{4}$$

with $x_k(j)$ as the k -th characteristics of catchment j and with ε_M (or ε_S) being the corresponding estimation errors. The characteristics were selected in a stepwise manner from a large number of catchment descriptors listed in Samaniego and Bardossy (2005). The descriptors consist of time dependent parameters (for example annual/seasonal precipitation, temperature) slowly changing parameters (for example

Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

land use) and time invariant parameters (for example soil and topography). Elements of the good parameter set for catchment i ($D(i)$) are taken to obtain a reasonable parameter set for catchment j . For each parameter vector θ the model is applied with the meteorological input of catchment j . The mean discharge and the standard deviation are calculated from the simulated discharge series. These means and standard deviations are compared to regionalisation results for catchment j obtained from Eqs. (3) and (4). For a given parameter vector θ if the mean and the standard deviation calculated from the modeled discharges are sufficiently close to the mean and the standard deviations obtained via regionalisation then the model parameter vector is judged to lead to a reasonable water balance and reasonable dynamics of the discharges on catchment j . Thus this parameter vector θ can be transferred from catchment i to catchment j . Formally, if

$$|M_Q(j) - \bar{Q}_j(\theta)| < k \times s_M(j) \tag{5}$$

and

$$|S_Q(j) - s_j(\theta)| < k \times s_S(j) \tag{6}$$

then parameter vector θ is considered as a reasonable candidate for catchment j ; $s_M(j)$ and $s_S(j)$ being the standard deviations of the estimation errors in Eqs. (3) and (4). The parameter k is selected such that a portion ρ (typically 90%) of the estimated errors are below the limit. This way the number of transferable parameter sets from catchment i to catchment j becomes a sensible subset of $D(i)$. According to this procedure, only those parameter vectors that provide a reasonable water balance and daily variability of discharge for the target ungauged catchment are considered for transfer. For the case study the mean discharge was of secondary importance as the runoff formation parameters were not transferred directly from one catchment to the other. The reason for considering it is to avoid cases where water balances are obtained by filling up storages, and leading to non-stationary conditions.

4 Results

A large set of possible candidates for 5 dimensional parameter vectors were generated at random. Uniform distributions were assumed for each parameter. The individual parameters were generated independently, without assuming any kind of parameter dependence. The HBV model was subsequently applied to all selected catchments using all possible candidate vectors. The model performance was expressed using the Nash-Sutcliffe coefficients. The regionalisation of the discharge statistics (annual mean flow and variance) was carried out using the approach described in Samaniego and Bárdossy (2005). To check the transferability of a parameter vector from catchment i to j the following steps were performed:

1. The Nash-Sutcliffe coefficient corresponding to the model for catchment i was calculated for the selected parameter set.
2. If the parameter set performed well on catchment i (exceeding 90% of the performance of the best parameter vector) then it was applied for catchment j .
3. The annual statistics of the discharge series corresponding to catchment j were calculated using the HBV model.
4. If the annual discharge statistics did not differ much from the regionalisation (condition of Eqs. 5 and 6), the parameter vector was considered as a possible parameter vector for the modelling of catchment j .
5. In order to compare the results with observations for all retained candidate parameter vectors the model performance on catchment j was calculated.

To evaluate the performance of the method, the NS values of for catchment i and j were plotted as a scatter plot for all parameter vectors where their transfer was reasonable according to the annual statistics (step 4). Figure 5 shows an example for the transfer from catchment 1 to 11, without considering the condition described in step 4.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page	
Abstract	Introduction
Conclusions	References
Tables	Figures
◀	▶
◀	▶
Back	Close
Full Screen / Esc	
Printer-friendly Version	
Interactive Discussion	

Figure 6 shows the subset for the parameter vectors for which the mean and the standard deviation were estimated correctly. As one can see in the second case, the selected parameter vectors yielded a good model performance on the catchment to which the parameter vector was transferred. The number of possible transferable candidate vectors was different depending on which pair of catchments was considered. The reason for the difference in number of transferable parameter vectors depends on the catchment characteristics. For some pairs of catchments it happened that none of the parameter vectors fulfilled the conditions of transferability. This fact limits the applicability of the method in a reasonable way – parameter vectors should not be transferred from a catchment to another if their characteristics are very different. Figure 7 shows the observed and the modelled time series for catchment 11 using one of the set of the parameters transferred from catchment 1. As one can see, the performance of the transferred model is very good.

5 Conclusions

Parameters of hydrological models cannot be identified as unique sets of values. This is mainly due to the fact that changes of one parameter can be compensated for by changes of one or more others due to their interdependence. The non-linearity of the models results in interpolation between parameter values possibly leading to unreasonable results. Thus parameters should not be considered as individual values, but instead as parameter vectors “teams”. Parameter vectors corresponding to a selected group of hydrological processes can be transferred from one catchment to another without any modification. This transfer is reasonable if the model obtained for the unobserved catchment gives good water balances and reproduces the variability of the daily discharges. The annual mean discharge and the variability can be regionalized using catchment characteristics and non linear regression models.

The presented approach is a kind of trial and error procedure. For some catchments it delivers good results, while for others, no parameter vectors can be found. In order

to have a general methodology based on these ideas, further research on the dependence of parameters and catchment properties is required. The main lesson to be learned from this study is that regionalisation should not focus on relating individual parameter values to catchment properties but on relating them to compatible parameter sets, or vectors.

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Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

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HESSD

3, 1105–1124, 2006

**Calibration of
hydrological model
parameters and PUB**

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

EGU

Calibration of hydrological model parameters and PUB

A. Bárdossy

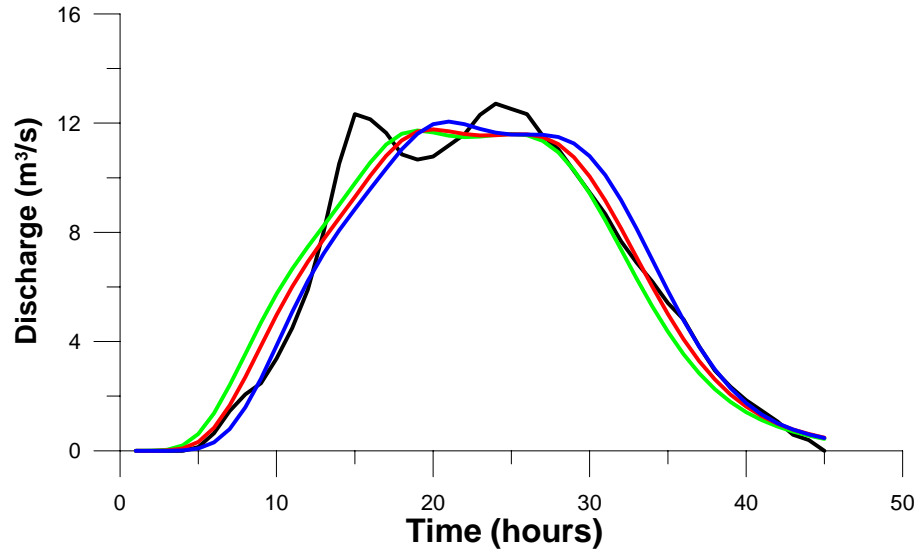


Fig. 1. Observed (solid black) and modelled (red, green and blue lines) direct discharges for the Kocher at Abtsgmünd, May 1999.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Calibration of hydrological model parameters and PUB

A. Bárdossy

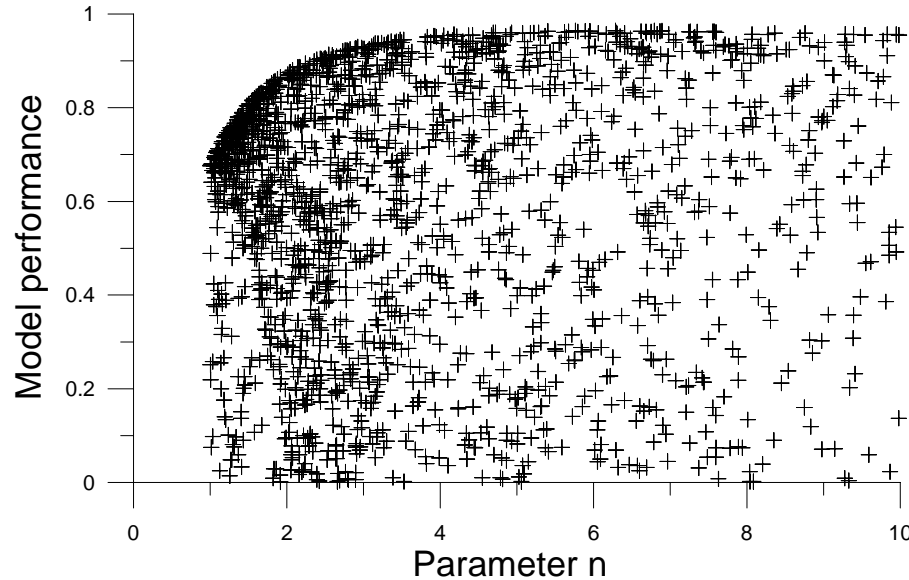


Fig. 2. Model performance (Nash-Sutcliffe) for random parameter sets – as a function of the parameter n .

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Calibration of hydrological model parameters and PUB

A. Bárdossy

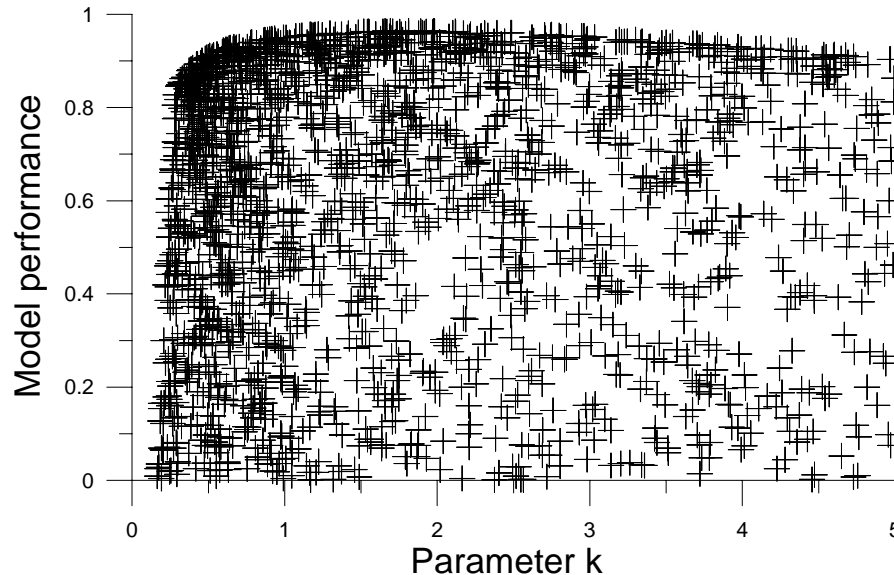


Fig. 3. Model performance (Nash-Sutcliffe) for random parameter sets – as a function of the parameter k .

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Calibration of hydrological model parameters and PUB

A. Bárdossy

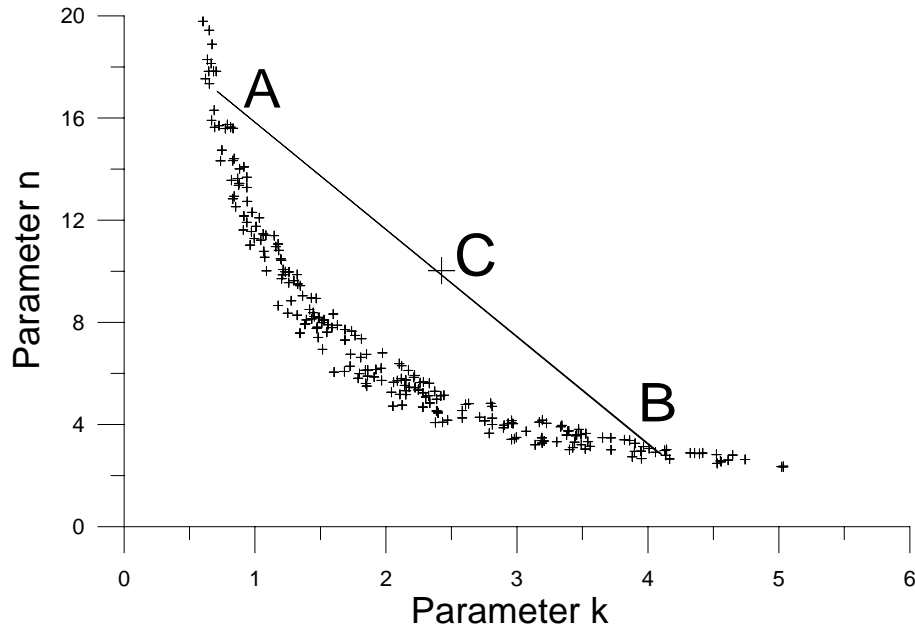


Fig. 4. Model parameters k and n with good performance (NS >95% of the maximum NS).

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

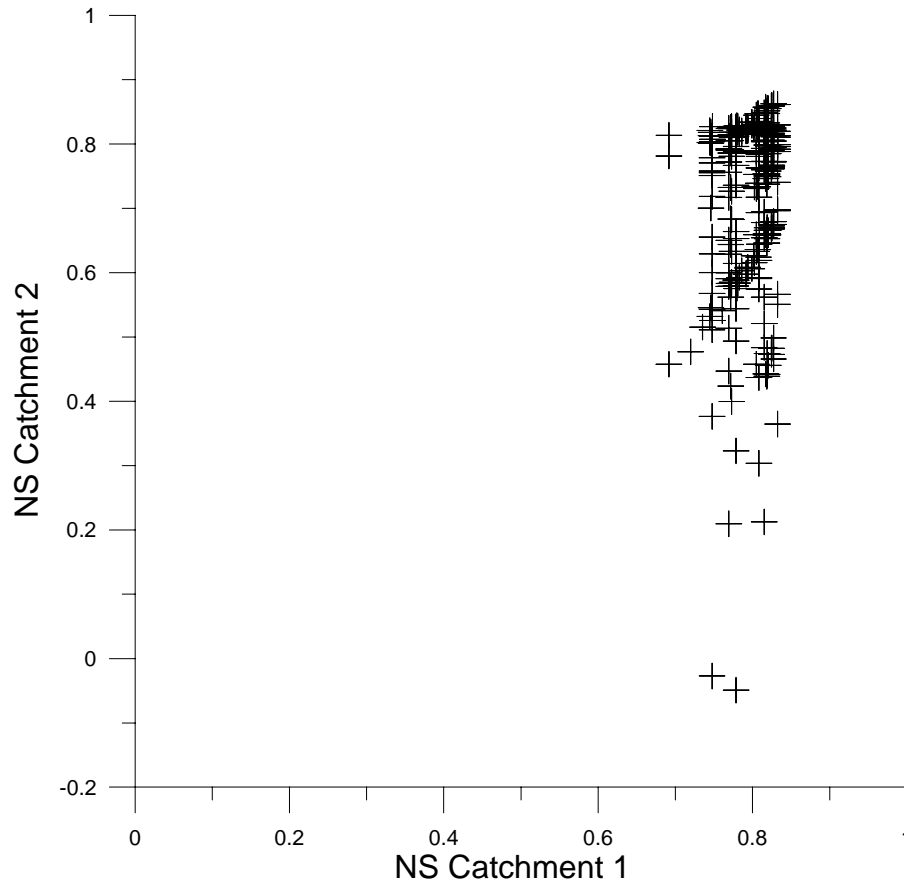


Fig. 5. Quality of parameter vectors for catchment 1 and catchment 11 for the transfer without considering the quality of the mean and the standard deviation.

Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

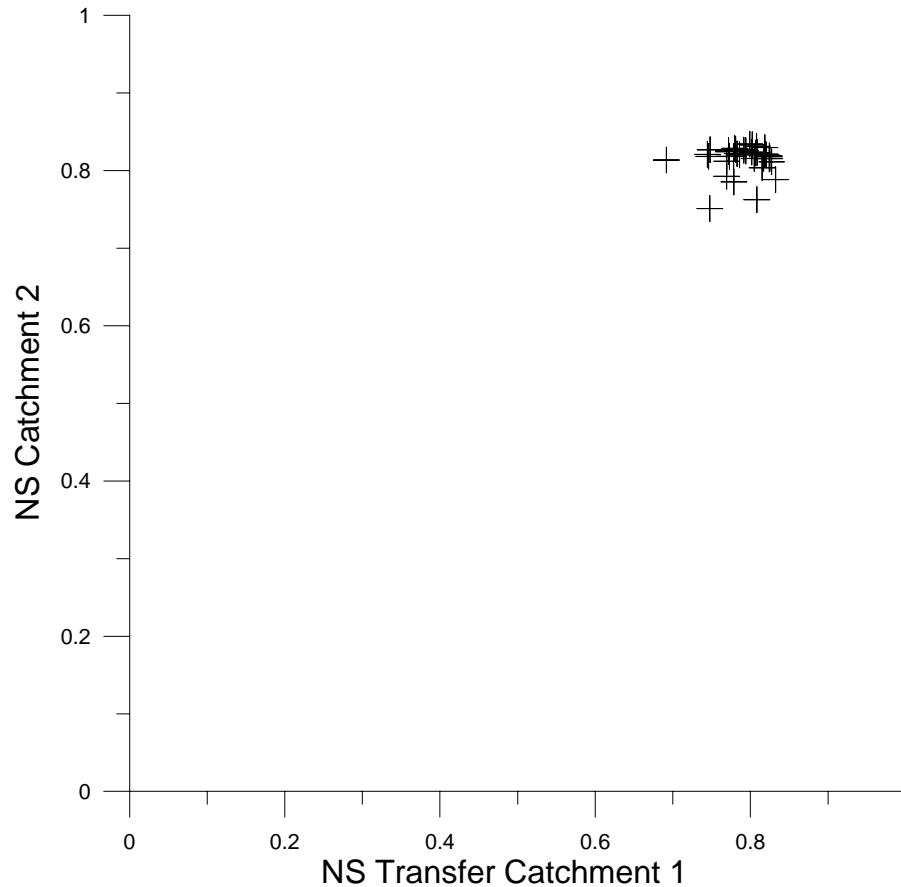


Fig. 6. Model performances for the transfer of parameter vectors from catchment 1 to catchment 11 for parameters fulfilling the conditions of the mean and the standard deviation.

Calibration of hydrological model parameters and PUB

A. Bárdossy

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Calibration of hydrological model parameters and PUB

A. Bárdossy

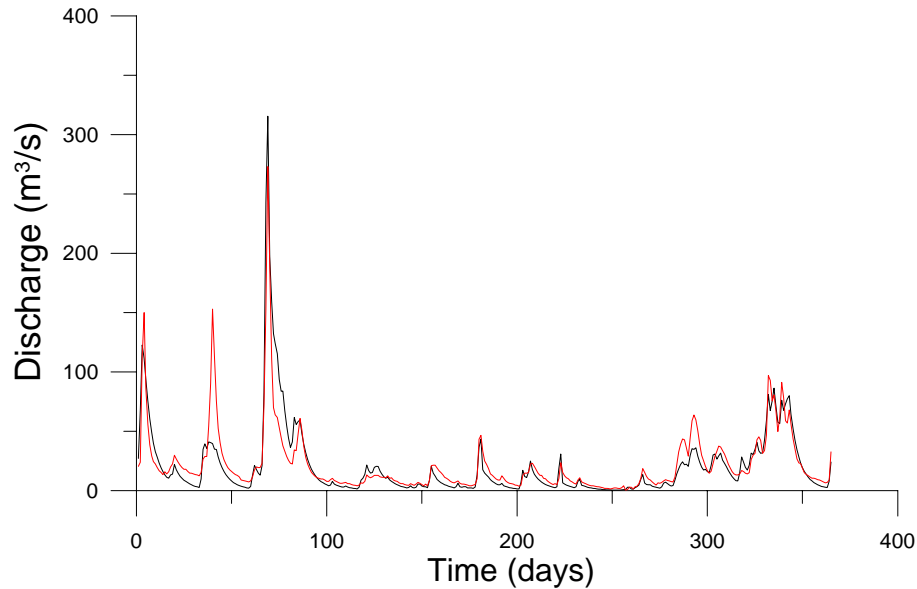


Fig. 7. Simulated (black) and observed discharges using model parameters transferred from catchment 1 to catchment 11.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion