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Interactive comment on "A measure of watershed nonlinearity: interpreting a variable instantaneous unit hydrograph model on two vastly different sized –watersheds" *by* J. Y. Ding

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3. The Minshall unit hydrograph data for the Edwardsville catchment

Referee #2 questions what the author means by "the finished form" of the Minshall unit hydrographs. By this, he means that the tedious and uncertain rainfall-runoff data analysis, which involved extracting the rainfall excess from the storm rainfall was all performed by Minshall (1960). However, Minshall did not separate the baseflow from streamflow, and considered all flow past the watershed outlet as runoff, be it surface runoff, interflow or baseflow. From these, the unit hydrographs were derived and presented by him in both tabular and graphical forms. The Minshall family of unit hydrograph has been considered a classical case of watershed system nonlinearity (Beven, 1991; Lettermann, 1991; Blöschl and Sivapalan, 1995; and Dooge and O'Kane, 2003) and remains an enigma of the hydrologic system. Many hydrologists, including Amorocho (1961), Overton (1967) and Dooge (1973), have attempted over the decades to unlock its physical basis or law.

The author did some preliminary analysis of the Minshall data in preparation of his VUH paper (Ding, 1974) as noted in Sect. 1.3 above. Recently his curiosity was piqued by the Dooge (2005) grand synthesis of hydrologic processes across a vast scale, which, among others, reintroduces the Minshall unit hydrographs as a dimensionless plot by multiplying the discharge scale by a characteristic time, which in this case is the lag time to the centroid of each of the unit hydrographs.

Because of the continuing interest in, and the capability of the VIUH model to simulate nonlinear phenomenon as exemplified by, the classical Minshall family of unit hydrographs, the author thought it worthwhile to re-analyze the 45-year-old data and share his more definitive findings in the HESSD forum even after a passage of three decades.

Unfortunately, in Table 2b, because of one missing step in converting the rainfall excess intensity from mm h^{-1} in Column (9) to mm $(\Delta t)^{-1}$ as required for the calculation of the "internal" scale parameter c, it marred the presentation of his findings by including erroneous values of parameters c and C_h, and burdened all Referees with a review of an unfinished manuscript. Incidentally, this serves as a response to Referee #4's dissent on the inclusion as Appendix A of the spreadsheet template, which outlines the hydrograph synthesis step-by-step.

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3.1. Calibration of the Edwardsville catchment nonlinearity by the shape factor method

If one accepts the VIUH shape factor method as a valid one, as acknowledged by Referee Sivakumar, there should be little disagreement over the accuracy of calibrated N values as reported in the paper. Discussion should then move on to the interpretation of the shape parameter and, if necessary, the re-calibration for some unusual event by other means such as the standard method of convolution integral.

Revised Fig. 1 shows the variations of N and corrected C_h values with the rainfall excess intensity for all five events. It shows that the calibrated N values for four moderate storms are all very close and average 1.79, and the C_h values have a small scatter with an average of 0.74. In terms of the calibrated values, these four events can be considered a set and belong to the same population.

For the largest event, it has a lower N value of 1.47 and a higher C_h value of 1.30 than the rest as a set. Referee #3 rightly places a low confidence on the lower N value for its being for only one event, albeit the largest one. Referee Sivakumar notes a significantly large error in the regenerated hydrograph peak discharge, but a zero error in the regenerated time to the peak, using the standard method of convolution integral. The latter may be explained by the small size of the Edwardsville catchment (0.11 km^2) and the relatively large time-step size (i.e. storm duration) of 14 min as compared to the time to peak of 12 min. Because of these, the simulated time to peak will fall either at the end of the first time step, thus producing a perfect match in the peak time, or elsewhere, thus over-estimating by a multiple of the step size.

The incorrect and uncorrected C_h values as given in the paper also unfortunately misled Sivakumar as well as the author to the possible existence of an inverse relationship between parameter C_h and the storm duration, now not supported by the revised Fig. 1.

All of these plus the encouragement by Referee #2 prompt the author into taking a closer look at this largest event.

3.2. Re-calibration of the 27 May 1938 Storm

As explained in the paper, this largest event has an atypical unit hydrograph shape, in that the peak occurred before the storm ended. Its calibrated N and C_h values differ significantly from the rest as a set.

In Table 2b, the author has reported calibration results, warts and all, from using the special method of the VIUH shape factor. The results were then verified by the standard method of convolution integral as shown in Table 2c. For the largest event, the peak discharge is under-estimated by about 42%, as noted by most of the Referees. This clearly indicates the deficiency of the shape factor method in calibrating the N values for atypical unit hydrographs, such as the case with the largest storm event on the Edwardsville.

From an end user's point of view, on the Edwardsville catchment, we are endowed with a wealth of observational data and the newly derived analytical results as given in Table 3 (Revised 2) included in this response. The author offers the following observations on the re-calibration results obtained from the largest event:

a) Table 3 (Revised 2) shows the sensitivity of the peak characteristics to change in the computational time-step size for the largest as well as the second largest events. The sensitivity test by convolution is an excellent diagnostic tool to verify the calibrated N value obtained by the shape factor method. However, Referee #4 finds this part of the paper a distraction, which is caused undoubtedly by the erroneous simulation results reported in the manuscript.

b) For the largest event, Table 3 shows that the use of a single 14- min step under-estimates the observed peak discharge by about 42%. Reducing the size of the storm duration or time-step to anywhere between 7 and 1 min reduces the under-estimation to between 23 and 17%, respectively. These indicate that the 14-min storm need be divided into two 7-min storms to improve calibration. Since nonlinearity implies that the whole is greater than the sum of its parts, it is counter productive to divide the 14-min storm into more than two sub-storms.

c) By the VIUH theory, the calibrated parameter values for four moderate events should apply across the rainfall-excess-intensity scale to the largest storm event. This being the case, its re-calibrated N and C_h are set to the average values for the four moderate storms as a set, i.e. 1.79 and 0.74, respectively.

d) The hydrograph is then regenerated by convolution (Eqs. 20, 21 and 24) using the re-calibrated pair of N=1.79 and C_h=0.74, and the result is shown in the third to last entry of Table 3. The regenerated hydrograph has a peak flow of 50.60 mm h^{-1} and the peak time of 14 min. When compared with the observed hydrograph peak of 60.45 mm h^{-1}, this represents an under-estimation of about 16 %, which is quite acceptable for hydrologic design purposes. Increasing the C_h value by about 40 % to 1.03, as shown in the second to last entry, increases the regenerated peak to 54.75 mm h^{-1}, still an under-estimation of about 9%, but which represents the best estimate one can obtain with the average N value of 1.79.

e) When compared with the peak discharge regenerated for the 2 x 7 min storms by the pair of N=1.47 and C_h =1.30, which was originally calibrated by the shape factor method as discussed in item b) above, the revised peak discharge for the re-calibrated pair of N=1.79 and C_h=0.74 by convolution represents an improvement from –23% to –16% for the same 2 x 7 min steps.

f) By searching the parameter space, the optimum pair of N and C_h values to reproduce the observed peak characteristics is found to be 2.60 and 0.69 as shown in the last entry of Table 3. This is indicative of the capability of the VIUH model to fit an atypical hydrograph.

g) It may therefore be concluded that for the Edwardsville catchment, the N value of 1.47 determined by the shape factor method for the largest storm having an atypical hydrograph shape is incorrect, as hinted at by Referee #2, let alone being an optimum one. In comparison with the observed time to peak of 12 min, the use of a storm duration of 14 min is too long for hydrograph regeneration and it need be divided into two 7-min storms to improve the calibration. Its degree of nonlinearity N and scale parameter C_h should be reset to the average values 1.79 and 0.74, respectively, by transfer of calibrated results from the medium events.

4. The Naugatuck River and the implications of watershed nonlinearity for design flood estimation

Referee Sivakumar raises a number of questions regarding the accuracy of the calibrated degrees of nonlinearity for the larger Naugatuck River, which are higher than those of the much smaller Edwardsville catchment.

The author located a conference pre-print of the Childs (1958) paper in his employer's library a few years after the publication of his 1974 paper. The Childs data, in addition to the Minshall, lend support to the theory of the VIUH. Because of his professional interest in big flood estimation, he used both sets of data, but more from the Childs than the Minshall, to illustrate the pitfall of extrapolating a linear model, when calibrated to small- or medium-sized storms, to estimation of the design flood magnitude resulting from a generally larger storm.

In comparison with the Minshall (1960) paper, the lack of attention given the Childs

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(1958) work in open literature and standard texts in North American has puzzled the author. The calibration of the VIUH model to the Naugatuck River is less the proof or demonstration of the model's capability to apply upscale to medium- or large-sized watersheds than a recognition of one forgotten contribution in the emerging field of hydrologic science, a task Referee #2 acknowledges worthy of undertaking.

4.1. The variable IUH model in engineering practice

Regarding its use in the engineering practice as raised by Referee #4, the VIUH model was one of several hydrologic models acceptable for use in flood plain mapping projects in Ontario, Canada (Ontario Ministry of Natural Resources (OMNR), 1986). In his previous capacities with OMNR as a models specialist and later an engineering supervisor, and in connection with review of hydrologic design for major or contentious projects, he often would calibrate the VIUH model to observed flood events in the same manner as one would any linear models, and then apply it to generate the flood hydrograph produced by a much larger design storm. Since a nonlinear model always gives a higher flood estimate than those of linear ones, the VIUH model served as an order-of-magnitude check on flood estimates obtained by other approaches.

5. Interpretation of the variable IUH model

Referee #3 points out correctly that since $q = Av$, and $dq/dt = A(\delta v/\delta t) + v(\delta A/\delta t)$, the unit hydrograph ordinate (dq/dt) does not represent, contrary to what the author states, the flow acceleration, but is only related to it. The author may add, ($\delta A/\delta t$) L represents the rate of change in the storage, which is the term (ds/dt) on the left-hand side of the continuity equation (Eq. 1). Reinterpreting in this new light, the variable IUH not only reflects the acceleration of the flow on a watershed, but also becomes an alternative storage-based dynamic equation to the Saint Venant equations for unsteady flow. It is of interest to contrast the variable IUH in the physical sense as an "acceleration diagram" and the geomorphological IUH in the statistical sense as a probability density function (PDF), the latter point brought up by Referee #4.

5.1. Concepts of time invariance and superposition

Referee #4 asks how the basic IUH concepts or assumptions of "time invariance and superposition" are relaxed in the VIUH.

Both the linear and variable IUH models follow the same principle of time invariance and reflect the same physiographical characteristics of the watershed whenever the storm may occur (Chow, 1964). Mathematically, both also follow the same principle of linear superposition and use the same linear convolution integral (Eq. 7), except in the latter, the kernel or response function is allowed to vary with the causative rainfall excess intensity.

This relaxation of the principle of linear superposition produces some interesting results. Most important of all, the resultant VIUH model generates a family of unit hydrographs, each dependent on its causative rainfall excess intensity

In terms of mathematical operation, the principle of linear superposition still applies in the VIUH model. To synthesize a composite hydrograph from a complex storm, the ordinates of incremental hydrographs at time $j\Delta t$ generated by the storms hyetograph are linearly added at the same $j\Delta t$ to arrive at the ordinate of the composite hydrograph. However, the relaxation of linear superposition imposes instead a restriction on the size of the time step, thus the term "calibrated time-step size" as discussed below.

As shown in Eqs. (30) and (31) of the paper, the size of the computational time step, i.e. storm duration, has the greatest effect on the peak characteristics of a hydrograph resulting from a short-duration storm. This is supported by results from the sensitivity

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analysis of the peak characteristics due to change in the time-step size as shown in Table 3 (Revised 2) for the second largest and more typical storm event on 02 September 1941. The results show that once the duration of a storm is subdivided into smaller and smaller time-step sizes, the peak discharges drop by a steady rate of 5 to 6%, which is indicative of the model's robustness in the time domain. (Results for the largest and atypical event show similar robustness in that the under-estimation rates vary from 17 to 23%, except for the single time-step size which under-estimates the peak discharge by about 42%.) In spite of the model's robustness, the "calibrated" time-step size used in parameter optimization shall be used for hydrograph synthesis for other events.

5.2. Interpretation of the model parameters

The VIUH model is based on a nonlinear lumped storage-discharge relation expressed by: $q = c^N s^N$, and is therefore most applicable to zero- and first-order watersheds or streams. Parameter N represents the watershed's efficiency to convert the potential energy (s) to the kinetic energy (q). As mentioned in Sect. 1.1, the higher the N value, the more symmetrical the variable IUH shape and the higher the peak ordinate. Some physiographical characteristics, such as the watershed shape, will likely have effects on the N value, as suggested by Referee #4.

In a model calibration study in Ontario, Canada (Collins and Moon, 1981), results of which were summarized in a model user's manual (Ontario Ministry of Natural Resources, 1983). As mentioned in the paper, Collins and Moon fixed parameter N at 1.5 by Chezy friction, in a manner similar to what Singh (1975) did previously for his laboratory watershed data. In addition, they developed a "watershed topography factor" to help explain the variation of parameter C_h, a detailed description of which is beyond the scope of the response.

For turbulent overland flow on a rectangular plane, the Chezy friction law gives the

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following expressions for parameters N and C_h (Ontario Ministry of Natural Resources, 1983):

$$N = 1.5 \tag{38}$$

$$C_h = 0.235 C_z^{2/3} L^{-2/3} S_f^{1/3} \tag{39}$$

where L is the length of the plane in km, S_f is the slope (dimensionless), and C_z is the Chezy's coefficient in $\mathrm{m}^{1/2}\mathrm{s}^{-1}$.

Based on calibrated C_h values from 16 storm events on seven watersheds in southern Ontario, Collins and Moon (1981) conducted a regression analysis for parameter C_h using watershed area, slope and main channel length as predictors, and found that only the watershed area is significant as shown below:

$$C_h = 0.18 A^{-0.31} \tag{40}$$

where A is the area in km^2. In other words, the larger the watershed, the smaller the scale parameter C_h. In the present study, similar findings are obtained for the smaller Edwardsville and the largest Naugatuck. Referee Sivakumar rightly cautions against making such a sweeping interpretation based solely on results from only two watersheds, in absence of additional results from other studies such as the one now shown by Eq. (40).

When the model is applied upscale to second- and higher- order streams as is the case with the Naugatuck River, the stream network characteristics are expected to come into play in determining the N value, again as suggested by Referee #4. Since N is dimensionless, its value is expected to relate only to those dimensionless geomorphological factors, such as the bifurcation ratio (Strahler, 1964). To improve the simulation accuracy on complex watersheds, one may have to apply the variable IUH or kernel function model as a channel routing model as well as a catchment runoff one.

Finally, as the other half of the model parameters, the scale parameter c or C_h will have to represent the effects of the size of watershed and all other factors comprising the Chezy- or Manning-based friction law such as the stream slope and surface characteristics or roughness as noted by Referee Sivakumar and exemplified by Eq. (39).

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Table 3. (Revised 2). Sensitivity of the peak characteristics to the number and size of time steps using the standard method of convolution integral for the Edwardsville catchment.

Number and size of time steps	27 May 1938 (N=1.47, C_h=1.30)				02 Sept. 1941 (N=1.84, C_h=0.74)			
	Hydrograph peak		Time to peak		Hydrograph peak		Time to peak	
	Regen'd	Regen.	t_p		Regen'd	Regen.	t_p	
$m\times(\Delta t/m)$	$q(t_p)$	error			$q(t_p)$	error		
min	mm h^{-1}	%	Δt	min	mm h^{-1}	%	Δt	min
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*
Observed								
1×14	60.45			12				
1×12					9.65			18
Regenerated								
1×14	34.93	−42.2	1	14				
1×12					9.67	0.2	1	12
2×7	46.32	−23.4	2	14				
2×6					9.07	−6.0	3	18
3×5	46.69	−22.8	3	15				
3×4					9.16	−5.1	4	16
5×3	47.81	−20.9	5	15				
4×3					9.16	−5.1	6	18
7×2	50.33	−16.7	7	14				
6×2					9.21	−4.6	9	18
14×1	49.40	−18.3	14	14				
12×1					9.06	−6.1	18	18
2×7								
(N, C_h)								
(1.79*, 0.74#)	50.60	−16.3	2	14				
(1.79*, 1.03$)	54.75	−9.4	2	14				
(2.60+, 0.69+)	60.47	0.0	2	14				

* Average of parameter N values of the four moderate storms
\# Average of parameter C_h values of the four moderate storms
$ The optimum C_h value for the average N value
\+ The optimum pair of N and C_h values

Table 6: State of overland flow and indices of nonlinearity.

Overland flow state	Watershed nonlinearity N	IUH nonlinearity $(1 - 1/N)$	Hydrograph nonlinearity $(2 - 1/N)$
Linear storage	1	0	1
Turbulent			
Chezy	1.5	0.33	1.33
Manning	1.67	0.4	1.4
Mixed			
Horton	2	0.5	1.5
Laminar			
Izzard	3	0.67	1.67
"Transient" storage	∞	1	2

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Figure 1: (Revised). Variations of the VIUH model parameters with the rainfall excess intensity for the Edwardsville catchment. Parameter N values calibrated by the variable IUH shape factor method, and C_h by the unit peak flow equation. (Please see S1049: 'Acknowledgment of comments'.)

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