Hydrol. Earth Syst. Sci. Discuss., 2, S1256–S1270, 2005
www.copernicus.org/EGU/hess/hessd/2/S1256/
European Geosciences Union

Hydrology and Earth System Sciences Discussions

Interactive comment on "A measure of watershed nonlinearity: interpreting a variable instantaneous unit hydrograph model on two vastly different sized –watersheds" *by* J. Y. Ding

J. Y. Ding

Received and published: 13 January 2006

A consolidated response to all Referees:
With special emphasis on the variable IUH shape factor method and the re-calibration of the largest storm event on the Edwardsville catchment

The author appreciates the critique by four Referees of his paper, especially on those parts dealing with the application of the variable IUH shape factor method to the Minshall unit hydrograph data on the Edwardsville catchment. He would like to make use of this final response to clarify or expand on major points raised by them on this and related subjects.

He is grateful to Referee Sivakumar for his gracious offer to help edit a final manuscript for possible publication in the HESS. As the author reports earlier, the paper as originally published contains incorrect values of the scale parameter (c and C_h), thus some questionable findings and conclusions. As a result, the manuscript will have to be revised, the extent of which Sivakumar considers to be moderate. The extensive discussions on the use of adjustment factors to the hydrograph peak ordinates in both the time and space domains will be deleted, and several related conclusions will be redrawn to reflect corrected values. Cosmetic or minor changes such as the use of the same area unit of km^2 for the Edwardsville, the citation of Wu (1982) in the text to read Tsao (1981), and corrections of typographical errors suggested or noted by the Referees will be incorporated in the final manuscript.

By necessity, this rather lengthy response starts with a historical sketch to address the apparent lack of new theoretical contribution observed by Referee #3, provides justifications as asked for by Referee #2 for having in one paper, one review part and one application part, and ends with some thoughts on the interpretation of the model and its parameters as a follow-up to Referee #4's comment.

1. History of the theory of the 2-parameter variable unit hydrograph

Referee #3 notes, correctly, that there is no significant new contribution in the paper in terms of equations presented. In retrospect, the variable instantaneous unit hydrograph (VIUH or VUH) theory was figuratively "cast in stone" when the original paper of his was published three decades ago (Ding, 1974). The mathematical development of the model was substantially complete by then, and all the basic equations were presented.

Referee #2 remarks on the critical role of the antecedent moisture condition (AMC) in the conversion of storm rainfall into rainfall excess, which he states is both a "highly" nonlinear process and a greater source of watershed nonlinearity than that arises from the rainfall excess-direct runoff conversion process.

The determination of rainfall excess is a separate process by itself, but the central concept of storage or storage - complement (i.e. the moisture deficit in a storage element) embedded in the theory of the variable kernel or IUH remains a useful one to help unify the apparent dissimilarity between, say, the infiltration and overland flow processes. (Rather than the VIUH, Referee #4 suggests the use of the notion of the instantaneous response function (IRF), which means the same as the kernel function in the convolution integral.) For instance, Holtan et al. (1967) express the net infiltration capacity or recharge rate as a function of the moisture deficit, i.e. space available, in a soil profile raised to a power of N in the form of: $(f - f_c) = a(S - F)^N$. Based on soil infiltration data supplied by the same Minshall of the classical nonlinear phenomenon, they calculated an average value of 1.4 for exponent N. In other words, their infiltration process may be said to posses a nonlinearity of 1.4, according to the storage-based form of energy equation (Eq. 2). This is in contrast to and not higher than that of 1.5 by Chezy friction or of 1.67 by Manning for the overland flow process (Eq. 2).

Although not mentioned by Referees, the baseflow separation is another source of data uncertainty, and the baseflow follows its own nonlinear process having a degree of nonlinearity of 2 for unconfined aquifers (Ding, 1966). The lumping of these component processes, each having its own degree of nonlinearity and characteristic time, into a watershed runoff process will likely dampen the latter's response, i.e. decreasing its degree of nonlinearity. The conventional wisdom as noted by Referee #4 is that as the size of watershed increases, its nonlinearity decreases. However, this paper shows somewhat different results in that the larger Naugatuck displays a higher watershed nonlinearity than the smaller Edwardsville, as noted by Referees #4 and Sivakumar.

HESSD

2, S1256–S1270, 2005

1.1. Definitions of system nonlinearity

For the unit hydrograph shape generated by the VIUH model, Ding (1974) shows by way of an illustration that the higher the parameter N value is, the more symmetrical the variable IUH shape and the higher the peak ordinate both become. Referee #4 notes the similar fact that parameter N reflects the shape of a unit hydrograph, but disagrees with the proposition that it too be a measure of nonlinearity for a watershed. The author would like to clarify the context and the various definitions of nonlinearity used in the paper.

The *watershed*, or *watershed storage*, *nonlinearity* is the primary index used in the paper. It is defined by the exponent or power of the storage in a nonlinear storage – discharge relation (Eq. 2): $q = c^N s^N$. As described in Sect. 2 of the paper, for the storage element of overland flow, exponent Ndepends on its flow state and its values are listed in Tables 6 (at the end of this response) along with values for two limiting cases, i.e. the linear and the so-called "transient" storages (Ding, 1967a; Singh, 1988).

A secondary index of nonlinearity applies to the variable IUH, kernel or response function. Its nonlinearity is defined by the power (1–1/$N)$ of the input (i.e. rainfall excess intensity) in the variable IUH model (Eqs. 8 and 9): $u(t) \propto i^{1-1/N}(0)$ and $t \propto i^{-(1-1/N)}(0)$.

It is obvious that the nonlinearity of a variable IUH is derived from that of its watershed. As shown in Table 6, the watershed nonlinearity (N) varies from 1 to infinite. The corresponding variable IUH nonlinearity (1–1/$N)$ has a re-scaled range of 0 and 1, the former means the independency of the IUH from the causative input, i.e. the system is linear, and the latter implies the highest nonlinear system in which the IUH depends entirely on the input.

Both the concepts of watershed and variable IUH nonlinearities are useful in the mathematical formulation of watershed or runoff models, as is the case with the VIUH. For practical application, however, one has to express the direct runoff hydrograph in terms of the input variable and the model parameters characterizing the variable IUH shape, but not the watershed storage itself, as shown by a shorter form of the convolution integral (Eq. 20):

$$q(j) \propto Nc\Delta t \sum_{k=1}^{j} i^{2-1/N}(j-k+1) \tag{20A}$$

The power (2–1/N) of the rainfall excess intensity may be called the hydrograph non-linearity. As shown in Table 6, its value varies from 1 to 2. As can be seen from Eq. (20A), in the VIUH model, the effect on the direct runoff hydrograph, of the rainfall excess intensity ranges from a minimum of its own magnitude to a maximum of squaring it.

For the reader, the three definitions of nonlinearity as discussed above and used in the paper are to be understood all in the context of the rainfall excess – direct runoff conversion process.

1.2. Conceptual similarities among the overland flow, channel routing and catchment runoff processes: a missing link

Referee #2 observes this paper being a two-part one and asks for a clearer explanation on the link between the review of similarities among the overland flow, channel routing and catchment runoff processes, and the subsequent application of the VIUH model to the Edwardsville and Naugatuck watersheds.

If one considers the overland flow and channel routing the two major components of the catchment runoff process, the conceptual link between the first and second processes is established in Sect. 3, and that between the first and third in Sect. 4. What is missing is the link between the second, channel routing, and the third, catchment runoff, to connect the three processes. The following brief history of the model development will hopefully complete the missing link.

HESSD

2, S1256–S1270, 2005

Interactive Comment

1.3. Genesis of the variable IUH model

No major advance in science takes place in a vacuum and develops in a linear or quantum- jump fashion, and the development of the VIUH model is no exception. The author was and is indebted to all previous investigators who contributed generously to hydrology literature, especially those whose works were recorded in the proceedings of the (first) International Hydrology Symposium held 1967 in Fort Collins, Colorado.

The alternate form of a 3-parameter, nonlinear Muskingum model (Eq. 4 or 5): $q = c^N s^N - c_1(ds/dt)$ or $s = (1/c)[c_1 i + (1-c_1)q]^{1/N}$, was presented at the Fort Collins symposium (Ding, 1967b). It is of interest to note that papers on the solution of the Izzard overland flow by direct integration method using the Bakhmeteff function (Ding, 1967a) and the re-examination of the Minshall family of unit hydrographs (Overton, 1967), among others, were pre-printed in the first volume of the two-volume symposium proceedings. In other words, all the building blocks for the VIUH model were lying there and elsewhere just waiting to be assembled.

An analytical solution of the nonlinear Muskingum model using a similar direct integration method was first obtained by the author. However, the solution, being a step-wise one, could not be evaluated or extended analytically. While exploring the capability and limitations of the nonlinear Muskingum routing model, he came up the mathematical formulation of the variable IUH by making use of a classical concept relating the unit hydrograph and its summation hydrograph. As remarked briefly by the author (Ding, 1974), the family of unit hydrographs generated by the VIUH model for various rainfall excess intensities "displays a striking similarity to the Minshall (1960) unit hydrographs" observed on the Edwardsville catchment. In view of its historical development, the VIUH model was rooted more in the tradition of engineering hydrology than the advanced mathematics of systems theory.

The 1974 paper of his represents a modest synthesis by the author of the observational data, hydrologic concepts and analytical techniques available circa early 1970s

HESSD

2, S1256–S1270, 2005

in the area of rainfall excess – direct runoff process, a contribution which Referee #2 acknowledges as being “appreciable”.

1.4. Three parameters in the nonlinear Muskingum and variable IUH models

The original VIUH model has the same three parameters charactering the nonlinear Muskingum model (Ding, 1974). Only after the convolution integral having a nonlinear kernel (Eq. 7) was solved after some considerable effort, was the third parameter c_1 found to be zero, i.e. to vanish, in order to satisfy the initial condition that when the system is at rest, the initial IUH ordinate is zero.

Unfortunately, the author did not have the prescience of the redundancy of the third Muskingum model parameter, just as he did not of the later appearance of the geomorphological IUH model (Rodriguez - Iturbe and Valdes, 1979), an alternate approach to watershed nonlinearity raised by Referee #4. It is worth noting that the GIUH model applies to second- and higher-order basins, but not first- or zero-order ones where the geomorphological parameters such as the bifurcation ratio simply do not exist.

The present paper reintroduces the VIUH model starting from the 2- parameter overland flow model (Eq. 2). It will be both historically and technical correct to have started from the 3-parameter nonlinear Muskingum model (Eq. 4) as some others have followed (e.g. Singh, 1988), but then the reader would be asked too much to follow the author’s tortuous thought process.

1.5. Contents of the paper

As noted above, Referee #2 observes this paper being a two-part one, part review and part application. Actually there are two more parts, a short one on the tutorial including a spreadsheet template, and a very brief one on a survey of previous applications of

HESSD

2, S1256–S1270, 2005

the model. To provide further details as suggested by Referee #4 from the calibration studies cited in the References alone would lengthen considerably the paper. It is hoped this reintroduction by the developer of the 30-year-old model who is no longer in active practice, will encourage other investigators in China, Canada and elsewhere to share their experience in open literature with other hydrologists.

1.6. Presentation of the paper

In the presentation of the VIUH equations, Referee #2 suggests the author to rely still more on earlier work to avoid the risk that the long derivation may distract the reader from the central message of the paper that the variable IUH model of his is capable of simulating the classical Minshall family of unit hydrographs. To maintain consistency in the notation used between previous work and the present paper, Eqs. (26) to (32) will be recast in terms of the original or "internal" scale parameter c. Only near the end of the derivation, will parameter c be converted to the standard C_h by Eq. (24) for comparison purposes.

In the continuity equation (Eq. 1), Referee #2 suggests the use of a time unit instead of the variable dt or Δt for the inflow and outflow variables. However, both sets of the time units have their place in the discretization of the convolution integral for subsequent application. For a nonlinear system, the use of an explicit time unit of hours as in the paper is more instructive and easy to follow, but that of an implicit one as in the 1974 paper is more efficient for parameter optimization and routine operations. In either case, the time unit of scale parameter (c or C_h) must be consistent with the time-step unit used in the convolution.

Referee #4 finds the academic writing style of the paper satisfactory, but recommends the author use a simple language to make the paper accessible to a larger audience. In this connection, the Referee's rephrasing of the opening sentence in the Abstract

HESSD
2, S1256–S1270, 2005

will be added as a further explanation of the variable kernel concept.

To correct the lack of illustrations as noted by Referees #3 and 4,and to help the reader at large, the author will arrange to reprint or re-plot the forgotten plot by Childs (1958), and the classical one by Minshall (1960), both cited in the paper as examples of the watershed nonlinearity. However, the variations of the variable IUH shape with parameters N and c, and the rainfall excess intensity are readily available in the original paper (Ding, 1974) and a standard text (Singh, 1988), and will not be repeated.

2. The variable IUH shape factor method

Referees #2 and 4 have considerable misgivings about the use of shape factor method to determine the watershed nonlinearity. Since this is a fundamental issue with Referee #4, the author will address their concerns in some detail.

The variable IUH shape factor is defined in the paper as the product of the unit peak ordinate $u(t_p)$ in Eq. (15) and the lag time (to peak) t_L in Eq. (16). Although $u(t_p)$ varies linearly and t_L inversely with parameter c and the rainfall excess intensity $i^{1-1/N}(0)$, the product of the former two happens to cancel the latter two, leaving N as the remaining parameter. That the shape factor is a function of N only as shown in Eq. (19) is a logical consequence of the VIUH theory . This may also be argued heuristically as follows.

On the one hand, $u(t_p)$ has the time unit of h^{-1} and t_L that of h, the product of $u(t_p)$ and t_L , which happens to be the VIUH shape factor, is thus dimensionless. On the other hand, the shape parameter N is dimensionless, and the scale parameter c has some complicated units of (mm/Δt)$^{1/N}$/mm. From a dimension's point of view, the dimensionless shape factor is related to the dimensionless parameter N, and not c or both.

2.1. Method of moments

The shape factor, $u(t_p)\times t_L$, is a special case of the first moment of an IUH, in which only the peak characteristics, and not the whole IUH shape, are counted in the calculation of the moment.

For linear hydrologic systems, Nash (e.g. Ding, 1974) and Dooge (e.g. Dooge and O'Kane, 2003), among others, favour the method of moments for parameter estimation because it gives stable estimates of the parameter values. This method was followed by Ding (1974) for comparison of his nonlinear model with the Nash cascade of linear reservoir model which is a linear one, but both having two parameters. It was shown analytically that for a given IUH, the VIUH parameter N value is less than the Nash parameter n, i.e. the number of reservoirs. The latter also serves as an initial N value to start the iteration process of his nonlinear optimization scheme. It is of interest to note that both the Nash linear and the Ding nonlinear models become the kernels of the 2-dimensional variable IUH model (Chen and Singh, 1986).

2.2. Application of the shape factor method

The shape factor method gives equal weight to $u(t_p)$ and t_L, and is an objective method by moment matching of optimizing parameter N value for watersheds where and only where unit hydrograph data are available. Since N is a single-valued function of $u(t_p)\times t_L$ as shown in Table 1, the N value so calibrated is unique. When the N value has been determined, parameter C_h can be calculated directly from one of the unit peak characteristic equations (Eqs. 15 and 16, plus 24), if and only if the causative rainfall excess intensity is known.

For discussion purposes, the shape factor method will be referred to as a *special* method of parameter optimization, and the convolution integral the *standard* method

HESSD

2, S1256–S1270, 2005

for hydrograph generation. The latter is used in the paper for verification of calibrated parameter values obtained by the former.

2.3. Conceptual difference between linear and nonlinear models

In response to the query by Referee #2 about the dependency (or not) of the model parameters on the rainfall excess intensity, it is important to note the fundamental difference between the variable IUH model and the Nash or, for that matter, other linear models. The difference is in the presence or absence of a rainfall-excess variable in their respective equations for the *unit* hydrograph (emphasis added). To simulate the Minshall phenomenon, the VIUH model has an odd-looking intensity factor $i^{1-1/N}(0)$ in Eq. (8) for the hydrograph ordinate, and its reciprocal in Eq. (9) for the elapsed time, where N is the degree of the watershed nonlinearity.

While the VIUH model varies with the rainfall excess intensity, its parameters, N and c (or C_h), are in theory independent of the intensity, in answer to the query by Referee #2. Contrary to the impression Referee Sivakumar has from what the author may have inadvertently conveyed, he does not try to establish a direct relationship between the shape parameter N and the “magnitude” of flood events on either the Naugatuck or the Edwardsville, thus defeating the purpose of modelling in which the parameters are free of the input or output variable. The confusion may have been caused by the use in Eq. (7) of the technical term, an “input-dependent” kernel or response function, which happens to be one of the most important assumptions or concepts in the VIUH theory.

2.4. Degree of watershed nonlinearity

Referee #2 notes that the title (and indeed the abstract as well) of the paper deals with the watershed nonlinearity N, to the exclusion of all others. In a continuing drive for

HESSD
2, S1256–S1270, 2005

Interactive Comment

simplification of model construction in terms of reducing the number of model parameters, a trend and current focus in hydrologic modelling noted by Referee Sivakumar, the author has been attempting to reducing the 2-parameter VIUH model to a 1-parameter one to simplify calibration or application on watersheds where storm and streamflow data, if they exist at all, are often not of sufficient quality for calibration of complex hydrologic models with a large number of parameters.

There is the hydraulic reason, which Referee #4 notes approvingly, to choose parameter N over c as a default value, in that exponent N of a lumped storage element can be defined by the Manning or Chezy friction law, or by the Reynolds number for the flow state as shown in Table 6 (Ding, 1967a; Singh, 1988). In addition, there is a statistical reason in the calibration methodology for his preference of Nas discussed below.

2.4.1. Sensitivity of the unit peak ordinate

Eqs. (15) and (16) show that $u(t_p)$ varies linearly, and t_L inversely, with c, but they vary with N in more complicated manners. The latter is caused by the presence of N in the power of the rainfall-excess-intensity factor, $i^{1-1/N}(0)$.

Mathematically, the sensitivity of $u(t_p)$ to change in either N or c can be expressed by the partial derivatives of $u(t_p) = Eci^{1-1/N}(0)$ in Eq. (15) with respective to each of the parameters as given below:

$$\frac{\partial[u(t_p)]}{\partial N} = ci^{1-1/N}(0)\frac{\partial E}{\partial N} + \frac{Eci^{1-1/N}(0)}{N^2} = (\frac{1}{E}\frac{\partial E}{\partial N} + \frac{1}{N^2})u(t_p) \tag{33}$$

$$\frac{\partial[u(t_p)]}{\partial c} = Ei^{1-1/N}(0) = \frac{u(t_p)}{c} \tag{34}$$

where E is the peak ordinate function given previously by Eq. (17):

$$E = \frac{N^2(N-1)^{1-1/N}}{(2N-1)^{2-1/N}} \tag{17}$$

HESSD

2, S1256–S1270, 2005

Interactive Comment

The derivative of function E with respective to N as required by Eq. (33) is rather complicated, but can be simplified by making use of the expression for E:

$$\frac{\partial E}{\partial N} = \frac{N + \ln[(N-1)/(2N-1)]}{N^2} E \tag{35}$$

Eq. (33) can then be rewritten as follows:

$$\frac{\partial[u(t_p)]}{\partial N} = \frac{1 + N + \ln[(N-1)/(2N-1)]}{N^2} u(t_p) \tag{36}$$

Eq. (34) shows that the sensitivity of $u(t_p)$ to change in c is itself the ratio of $u(t_p)$ to c, i.e. $u(tp)$ varies linearly with c with a gradient of $Ei^{1-1/N}(0)$. However, Eq. (33) shows a much more complicated relation between $u(t_p)$ and N.

The relative sensitivity of $u(t_p)$ to changes in N and c depends on the relative magnitude of N and c. If one were to default parameter N to some constant, statistically $\delta[u(t_p)]/\delta N$ should be less than $\delta[u(t_p)]/\delta c$. Based on Eqs. (34) and (36), the following inequality condition has to be met:

$$c \le \frac{N^2}{1+N+\ln[(N-1)/(2N-1)]} \tag{37}$$

For a given degree of nonlinearity N, the right-hand side of Eq. (37) establishes the maximum c value below which $u(t_p)$ is more sensitive to change in c than N. In the final manuscript, the author will prepare a new Fig. 2 showing the equi-sensitivity line of $u(t_p)$ in the N and c plane. For the purpose of this response, it suffices to note that at N=1.67, the maximum c value reaches its lowest at 1.965, which is still quite a large number for almost all watersheds.

For example, on the Edwardsville catchment, for four moderate storms as shown in Table 2b and revised Fig. 1, the average N value is 1.79. For this N value, Eq. (37) yields a maximum c value of 1.994. For a storm duration of, say, 15 min, and from Eq. (24), the maximum C_h =1.994/(15/60)$^{1/1.79}$=4.327. This is much higher than the calibrated values of 0.74 to 1.03 as shown near the bottom of Table 3 (Revised 2). In other words, on the Edwardsville, $u(t_p)$ is much more sensitive to change in C_h than N.

One limiting case for N needs special attention. When $N \rightarrow$1, say, 1.001, Eq. (37) yields $c\leqq$-0.204. This contradicts the non-negative c value implied by the VIUH theory, and the sensitivity of $u(t_p)$ will have to be evaluated directly from Eqs. (34) and (36). From Eq. (36), $\delta[u(t_p)]/\delta N = -4.899u(t_p)$. This mean that in a linear system, $\delta[u(t_p)]/\delta N$ is less than $\delta[u(t_p)]/\delta c$ in Eq. (34) for any positive c value, thus $u(t_p)$ is more sensitive to change in c than N.

In the VIUH model, $u(t_p)$ depends on a combination of N and c values, but its sensitivity depends on their relative magnitude. As shown above, $u(t_p)$ is generally less sensitive to change in, as well as more complicatedly related to, N for the normal range of c or C_h values. If one were to reduce the number of the parameters by one, it would be statistically correct to fix the less sensitive N to some constant, and let the remaining parameter c or C_h fit the observed data, thus simplifying estimation procedure (e.g. Singh, 1988).

For transfer of calibration results to ungauged watersheds in which Referee #3 shows considerable interest, the lesser the number of parameters, the higher the explanatory or predictive power of the remaining parameter(s). This may be considered a corollary of the principle of parsimony in determining the worth of adding an additional parameter to a model (e.g. Dooge and O'Kane, 2003).

2.4.2. Sensitivity of the direct runoff hydrograph

The sensitivity of the whole hydrograph, instead of only the peak ordinate, to change in N or c, is given by a pair of equations in the Ding (1974) paper, one for parameter N having a formidable look (and it still will, even after possible streamlining in the

HESSD

2, S1256–S1270, 2005

notation), and one a simple one for c, as shown in Eqs. (43) and (44) of that paper.

(Correction: for the fourth term on the right-hand side of Eq. 43, the factor $v^{n0}/(1-v^{n0})$ should read $v^{n0}\ln v/(1-v^{n0})$. This was an obscure error, near the end of long mathematical derivation, brought to the author's attention in the early 1980s by a graduate student in Wuhan Institute of Hydraulic and Electrical Engineering, People's Republic of China, courtesy of Chen, the author of the extended VIUH paper (Chen, 1984; Chen and Singh, 1986). This means that the mathematical derivation of the VIUH model has been independently verified by others.)

End of part 1

Interactive comment on Hydrology and Earth System Sciences Discussions, 2, 2111, 2005.