Hydrol. Earth Sys. Sci. Discuss., 2, 2111–2151, 2005
www.copernicus.org/EGU/hess/hessd/2/2111/
SRef-ID: 1812-2116/hessd/2005-2-2111
European Geosciences Union

Hydrology and Earth System Sciences Discussions

Papers published in *Hydrology and Earth System Sciences Discussions* are under open-access review for the journal *Hydrology and Earth System Sciences*

A measure of watershed nonlinearity: interpreting a variable instantaneous unit hydrograph model on two vastly different sized – watersheds

J. Y. Ding

Retired Engineer, 134 Heatherside Drive, Toronto, Ontario, M1W 1T9, Canada

Received: 30 August 2005 – Accepted: 13 September 2005 – Published: 27 September 2005

Correspondence to: J. Y. Ding (johnding_toronto@yahoo.com)

Abstract

This paper reviews the use of an input-dependent kernel in a linear convolution integral as a quasi-nonlinear approach to unify nonlinear overland flow, channel routing and catchment runoff processes. The conceptual model of a variable kernel or instantaneous unit hydrograph (IUH) is characterized by a nonlinear storage-discharge relation, $q=c^N s^N$, where the storage exponent N is an index or degree of watershed nonlinearity. When the causative rainfall excess intensity of a unit hydrograph is known, parameters N and c can be determined directly from its shape factor, the product of the unit peak ordinate and the time to peak. The model is calibrated by the shape factor and verified by convolution integral on two watersheds of vastly different sizes, each having a family of four or five unit hydrographs, data of which were published by Childs in 1958 for the Naugatuck River and by Minshall in 1960 for the Edwardsville catchment. For an 11-hectare catchment near Edwardsville in southern Illinois, the US, four moderate storms show an average N value of 1.79, which is 7% higher than the theoretical value of 1.67 by Manning friction law, while the heaviest storm, which is three to six times larger than the next two events in terms of the peak discharge and runoff volume, follows the Chezy law of 1.5. At the other end of scale, for the Naugatuck River at Thomaston in Connecticut, the US, having a drainage area of 186.2 km^2, the average N value of 2.28 varies from 1.92 for a minor flood to 2.68 for a hurricane-induced flood, all of which lie between the theoretical value of 1.67 for turbulent overland flow and that of 3.0 for laminar overland flow. Short examples and a spreadsheet template are given to illustrate key steps in generating the direct runoff hydrograph by convolution integral with the 2-parameter variable IUH model.

1. Introduction

In a comprehensive survey of similarities and contrasts between analyses of hydrologic elements and processes over a very large range of scales, Dooge (2005) makes a convincing case that progress in analysis has been made through simplification of these complex processes. He advocates a strategy based on a *rigorous analysis of simplified equations of motion* (emphasis added). According to him, a wide range of forms of simplification has been used in hydrology, including: reducing the number of independent and dependent variables, and of parameters, such as by the dimensional analysis; and simplifying the basic equations. He cites previous studies on, among others, overland flow, flood routing in channels, and catchment runoff processes. Specifically, he reviews the work of Amorocho and Orlob (1961) on laboratory experiments of overland flow, and of Minshall (1960) on unit hydrographs on a small experimental watershed.

The purpose of this paper is to present an additional approach of simplification or approximation that the author has found useful, over his professional life of some 30 years, in unifying concepts behind these and other nonlinear processes. In essence, this involves the use of an input-dependent or nonlinear kernel in a linear convolution integral, a relaxation of the principle of superposition in linear systems. The concept of variable kernel or instantaneous unit hydrograph (IUH) will be reviewed, and the parameters reinterpreted. The classical example of the Minshall (1960) nonlinear unit hydrograph data on a small watershed in southern Illinois, the United States, will be analyzed using the variable IUH model to determine the degree of nonlinearity and scale parameter. Another set of unit hydrograph data from an earlier study by Childs (1958) on a large Naugatuck River in Connecticut, the United States, will be re-examined to determine its nonlinearity.

It is hoped this fresh look at two sets of 40-plus-year-old unit hydrograph data from a nonlinear perspective will help identify areas for research by the younger generations. Although the concept of nonlinear systems is not much difficult to grasp than that of linear ones, it is much harder to carry out numerical analysis for even a simple nonlinear system, such as the 2-parameter variable IUH model, characterized by a nonlinear storage-discharge relation, $q=c^N s^N$. Because of the presence of the exponent N, it is rather confusing, even to the author, to convert variables and parameters from one set of units to another, short examples including a spreadsheet template will be given to illustrate key calculations.

2. Basic equations and assumptions for the overland flow

For flow over a plane subjected to a constant rate of rainfall excess, the continuity equation is expressed by:

$$\frac{ds}{dt} = i - q \qquad (1)$$

where i is the inflow rate in mm/dt, q is the outflow rate in mm/dt, s is the storage in mm, and t is time in h.

The equation of motion is approximated by a nonlinear storage-discharge relation:

$$q = c^N s^N \qquad (2)$$

where N is the storage exponent (dimensionless) known as a shape parameter, and c is the unadjusted discharge coefficient in $(\text{mm}/dt)^{1/N}$/mm known as a scale parameter. For flow on a wide rectangular channel, $N = 1.5$ by Chezy friction law, and 1.67 by Manning (Horton, 1938; Ding, 1967a; Dooge, 2005). In the case of laminar overland flow, N=3.0 (Horton, 1938; Izzard, 1946; Ding, 1967a). Note that Horton used the depth of flow instead of the volume of water in Eq. (2). The volume or storage is approximated by depth times the surface area. Parameter N has been proposed by Ding (1974) as an index or degree of nonlinearity for storage elements.

Equation (2) is known as a kinematic wave approximation to the equation of motion (Dooge, 2005). In the author's view, Eq. (2) may be looked at more appropriately as a simplification of the Bernoulli energy equation, as it converts the potential energy (s) of a storage element into a kinetic energy (q) without loss. Therefore, some other form of the equation of motion will have to be specified to account for the flow acceleration. An alternative based on a lumped storage concept will be discussed in Section 4 below.

In a review of overland flow data from laboratory experiments by Amorocho and Orlob (1961), Dooge (2005) observes that if the laboratory system represents a wide rectangular channel with Manning friction, then the characteristic time should be inversely proportional to the characteristic discharge to a power of 0.4. His analysis of their experimental data shows a power of 0.3997, which is very close to the theoretical value. More on this will be discussed in Sect. 7.1 below.

For a laboratory watershed having a converging surface towards the outlet, Singh (1975), like Horton (1938) before him, used the local depth of flow in Eq. (2):

$$q = ah^N \tag{3}$$

where h is the depth of flow at the outlet, and a is a constant.

Based on data from 210 experimental runs for 50 geometric configurations having varying physical characteristics collapsed into seven groups of similar surface characteristics, Singh (1975) found that parameter N is relatively stable, and parameter a is extremely sensitive to rainfall input characteristics and surface composition, and there exhibits a high correlation between the two. He fixed the N value at 1.5 by Chezy friction, which also led to a smaller variance of parameter a. For the 1-parameter kinematic wave model, he found the prediction error based on the hydrograph peak to be well below 25%.

3. Similarity between channel routing and overland flow

The movement of a flood wave down a channel reach typically exhibits a looped storage-discharge relation, a characteristic the well-known Muskingum model is capable of simulating.

The kinematic wave approximation, Eq. (2), can be modified to simulate the hysteretic phenomenon by adding a term reflecting the rate of change in storage:

$$q = c^N s^N - c_1 \frac{ds}{dt} \tag{4}$$

HESSD

2, 2111–2151, 2005

where c_1 is a constant. Substituting *ds/dt* in Eq. (1) into Eq. (4):

$$s = \frac{1}{c}[c_1 i + (1 - c_1)q]^{1/N} \quad (5)$$

When N=1, this reduces to the form of Muskingum model (Ding, 1967b, 1974).

The 3-parameter, nonlinear form of Muskingum model was evaluated by Gill (1978), Tung (1985) and Singh and Scarlatos (1987). Gill (1978) used a segmented-curve method to determine the three parameters on one test example and found an optimal N value of 1/2.347. Tung (1985) used four parameter optimization methods on the same test example and found the N values varying from 1/1.7012 to 1/2.3470. Note these fractional exponents are contrary to that of greater than unity as defined in connection with Eq. (2).

Singh and Scarlatos (1987) pre-set a moderately high N value of 2.0, and found that the model's accuracy depends mainly on the scale parameter c, and unlike the linear case, the weighting factor c_1 is much less significant. They found that the use of a lower N of 1.33 would improve the performance of the nonlinear model. A comparison by them with the linear case using four sets of inflow-outflow data shows that the nonlinear method is less accurate than its linear counterpart.

The Singh and Scarlatos (1987) findings are indicative of the stability problem associated with nonlinear analysis in which the impact of the inflow rate is amplified by the degree of system nonlinearity. It is noted that assessment on the accuracy of linear or nonlinear form of Muskingum model is complicated by the presence of local inflow along the river reach, which affects the accuracy of the outflow data used for calibration. The somewhat contradictory findings regarding the degree of nonlinearity by these investigators point to the need for verification by flume tests, similar to those for overland flow in Sect. 2 above, in a hydraulic laboratory where the effects of local inflow can be eliminated or controlled.

Besides the looped storage-discharge relation, another characteristic of the Muskingum model is the occurrence of negative outflow rates at the beginning of the outflow hydrograph (e.g. Chang et al., 1983). This problem can be fixed by imposing in Eq. (4)

a non-negative condition for q, which, depending on the ratio of the storage to its rate of change, will define the size of computational time steps.

In passing, the variable IUH model, which was originally developed by Ding (1974) to simulate catchment runoff process as discussed in Sect. 6 below, has been extended by Wu (1982), as well as suggested by Kundzewicz (1984), for use as a flood routing model as well.

4. Similarity between catchment runoff and overland flow

The transformation of rainfall into runoff on small catchments, a building block of watershed models, is probably the most difficult problem to tackle in hydrology. A distinct feature of the process is the existence of a time lag observed on most watersheds between a short, intense storm and the resultant hydrograph peak. The pair of continuity equation and the kinematic wave approximation (Eqs. 1 and 2) on their own, however, fails to model this characteristic time.

From a review of the Horton (1938) and Izzard (1946) experiments, Ding (1974) realized that the rising limbs of their overland flow hydrographs are essentially a summation or S-hydrograph. This fact, apparently having been overlooked by previous investigators, provides a conceptual link to the catchment runoff process via a classical concept, which states that the ordinate of an instantaneous unit hydrograph (IUH) is the first derivative of an S-hydrograph normalized by the rainfall excess intensity. Mathematically, the relation between the two is expressed as follows:

$$u(t) = \frac{1}{i(0)} \frac{dq(t)}{dt} \quad (6)$$

where $u(t)$ is the IUH ordinate in h^{-1}. Lesser known is the fact that the variable $u(t)$, reflecting the time rate of change in discharge, represents the flow acceleration. Because of this, the IUH or, more precisely, the variable IUH which retains the rainfall excess intensity term, may be considered as alternate and simplified form of the equation of motion.

5. Catchment runoff process

For a special case of constant rainfall excess intensity over an indefinite period of time, i.e. $i(t)=i(0)>0$, Eq. (6) is a differential form of the linear convolution integral with an input-dependent or variable kernel:

$$q(t) = \int_{\tau=0}^{t} i(t-\tau)u[i(t-\tau);\tau]d\tau \tag{7}$$

where $u[i(0);t]$ is a nonlinear kernel associated with the causative rainfall excess intensity $i(0)$. For convenience, $u[i(0);t]$ will be abbreviated as $u(t)$, on the understanding that the IUH ordinate depends on the causative rainfall excess intensity as well as the elapsed time.

Also note the difference between two related terms being used in this paper. The kernel or IUH *ordinate* has the time unit of h^{-1}, and the unit hydrograph peak *rate* or *discharge* produced by one unit of rainfall excess, i.e. 1 mm in this paper, has the volumetric units of mmh^{-1} or $m^3 s^{-1}$.

The use of an input-dependent kernel in the linear convolution integral was proposed by Amorocho (1967) to simulate the systematic variation of the unit hydrographs observed by Minshall (1960). The latter showed that on a 27.2-acre (11-hectare) experimental watershed near Edwardsville in southern Illinois, there exists not a single unit hydrograph, but a family of five, each dependent on its causative rainfall intensity (this watershed will be referred to as the Edwardsville catchment).

Similar phenomenon has been reported for medium-sized watersheds as well. For example, two years prior to Minshall's work, Childs (1958) presented an illuminating example of nonlinear runoff response for the 71.9 sq. mi. (186 km^2) Naugatuck River at Thomaston in Connecticut. He showed a family of four 3-hour unit hydrographs derived from flood records, in which as the flood peak discharge increases from a low of 3200 c.f.s. ($91\,m^3\,s^{-1}$) to a high of 41 600 c.f.s ($1178\,m^3\,s^{-1}$), the latter caused by Hurricane Diane in August 1955, the unit hydrograph peak rate increases from approximately 3000 c.f.s ($85\,m^3\,s^{-1}$) to 7400 c.f.s ($211\,m^3\,s^{-1}$), and the peak time shortens from 9 h to 6 (see Table 5a below).

The work of Minshall (1960) has been cited by many studies as a classical case of nonlinear watershed response, some of which were cited by Ding (1974). Since then, other studies citing Minshall's work include Overton and Meadows (1976), Chen and Singh (1986), Singh (1988), Robinson et al. (1995), Lee and Yen (2000), Cranmer et al. (2001), Sivapalan et al. (2002), Kokkonen et al. (2004), and Paik and Kumar (2004). In contrast, the work of Childs (1958) has rarely been cited, Ashfag and Webster (2000) being a notable exception.

6. Variable instantaneous unit hydrograph in catchment runoff process

Equation (7) is a 1-dimensional convolution integral having a variable kernel. It is of interest to note that a 2-dimensional extension having an additional variable kernel was proposed by Chen and Singh (1986). In keeping with the Dooge (2005) strategy of simplification, only the original 1-dimensional variable IUH model is reviewed in this paper. Detailed derivation of the model and its properties can be found in the Ding (1974) paper, and only those results required for this review are summarized below.

6.1. Derivation of the variable IUH

The solution of Eqs. (1), (2) and (7) for a constant $i(t)$ is a pair of parametric equations having a dummy variable v:

$$u(t) = Ncv^{N-1}(1 - v^N)i^{1-1/N}(0) \quad (8)$$

$$t = \frac{F(v,N)}{ci^{1-1/N}(0)} \quad (9)$$

HESSD
2, 2111–2151, 2005

where

$$F(v,N)=\int_{v=0}^{v}\frac{dv}{1-v^N} \tag{10}$$

$F(v,N)$ is the well-known Bakhmeteff (1932) varied-flow function. Conceptually, v is not a dummy variable, but a normalized flow rate, $[q(t)/i(0)]^{!/N}$.

Note in Eqs. (8) and (9), not only does the IUH ordinate vary directly, but also the elapsed time inversely, with the rainfall excess intensity raised to a power of $(1-1/N)$ so that the area under the IUH remains unity. The effect of parameter N on the IUH shape is complicated by the fact that it amplifies the impact of the rainfall excess intensity as well as having its own. The effect of parameter c is straightforward, as it affects the IUH ordinate directly and elapsed time inversely.

Substituting $u(t)$ in Eq. (8) into Eq. (7), the convolution integral becomes:

$$q(t)=Nc\int_{\tau=0}^{t}v^{N-1}(1-v^N)i^{2-1/N}(t-\tau)d\tau \tag{11}$$

Equations (11) and (9) constitute the 2-parameter, variable IUH model.

6.2. Bakhmeteff varied-flow function

To calculate the value of the varied-flow function, Bakhmeteff (1932) expands the integrand in Eq. (10) by the Taylor series and sums the successive higher-order terms:

$$F(v,N)=\sum_{p=1}^{\infty}\frac{v^{(p-1)N+1}}{(p-1)N+1} \tag{12}$$

$$R_p\le\frac{v^{pN+1}}{pN+1}\bullet\frac{1}{1-v^N} \tag{13}$$

where R_p is the residue of the series after p number of terms. He sets the residual error at less than or equal to 0.0005.

HESSD

2, 2111–2151, 2005

Title Page
Abstract | Introduction
Conclusions | References
Tables | Figures
◀◀ | ▶▶
◀ | ▶
Back | Close
Full Screen / Esc
Print Version
Interactive Discussion

EGU

As an example of calculation, for $N=1.67$ by Manning friction and $v=0.473$, the latter yields the IUH peak as shown in Section 6.8 below:

$$F(0.473, 1.67) = 0.473 + \frac{(0.473)^{2.67}}{2.67} + \frac{(0.473)^{4.34}}{4.34} + \frac{(0.473)^{6.01}}{6.01} + .\frac{(0.473)^{7.68}}{7.68}. + ...$$
$$= 0.473 + 0.051 + 0.009 + 0.002 + 0.000 = 0.535$$

6.3. Variable IUH peak characteristics

In Eq. (8), the peak ordinate of the IUH corresponds to the maximum value of the dummy-variable factor, $v^N(1-v^{N-1})$. Maximizing the factor yields:

$$v(t_p) = \left(\frac{N-1}{2N-1}\right)^{1/N} \quad (14)$$

where t_p is time to the peak.

Substituting $v(t_p)$ in Eq. (14) into Eqs. (8) and (9), the peak characteristics are expressed as follows:

$$u(t_p) = Eci^{1-1/N}(0) \quad (15)$$

$$t_p = t_L = \frac{F}{ci^{1-1/N}(0)} \quad (16)$$

where:

$$E = \frac{N^2(N-1)^{1-1/N}}{(2N-1)^{2-1/N}} \quad (17)$$

$$F = F[v(t_p), N] \quad (18)$$

Note these peak functions depend on the value of N only. In Eq. (16), t_p is the time to IUH peak measured from the start of the rainfall-excess storm, and t_L is the time to the peak from the mid-point of rainfall excess, the latter known as the basin lag or the lag time. For the IUH in which Δt approaches zero, t_p and t_L are identical. More on this will be discussed in Sect. 6.4 below.

The product of $u(t_p)$ and t_p defines the shape of an IUH and is known as a shape factor. The product of Eqs. (15) and (16) yields:

$$u(t_p) \bullet t_p = u(t_p) \bullet t_L = E \bullet F \tag{19}$$

Note the IUH shape factor also is a function of N only.

6.4. Discretization of the variable instantaneous unit hydrograph model

The variable IUH model and its peak characteristics summarized above are mathematically derived treating the rainfall-runoff transformation as a continuous process. For application, the process will have to be sampled or discretized along the time axis.

Equations (11) and (9) in the integral form are approximated by a summation form as follows:

$$q(j) = Nc \sum_{k=1}^{j} i^{2-1/N}(j-k+1) v^{N-1}(1-v^N)\Delta t \tag{20}$$

$$k = \frac{F(v, N)}{c i^{1-1/N}(j-k+1)\Delta t} \tag{21}$$

where indices j and k are non-negative integers (note: in accordance with Fortran programming language convention, the index of a subscripted variable starts from 1, and not 0).

Note the IUH as represented by Eqs. (7) to (19) thus becomes a Δt-unit hydrograph (or Δt UH for short). Rainfall excess values are accumulated over Δt, and runoff rates measured at the end of each Δt. Since the midpoint of the rainfall excess, rather the starting point, is more representative of the input variable, t_L will be used as a characteristic time. In a discrete form, the relation between the time to peak and the lag time is:

$$t_p = \frac{\Delta t}{2} + t_L \tag{22}$$

The IUH shape factor in Eq. (19) is approximated by its ΔtUH shape factor, which will be used to determine the degree of nonlinearity for both the Edwardsville and Naugatuck watersheds in Sects. 7 and 8 below.

6.5. Standardization of the scale parameter

The watershed discharge coefficient c is known as a scale parameter. Calibrated c values are derived usually for storms of different duration. For comparison of calibrated results, parameter c need be standardized to one, say parameter C_h, having a common time unit of 1 h. Let the units of parameter C_h be $(\text{mm/h})^{1/N}$/mm, and note that 1 h=$(1/|\Delta t|)\Delta t$ in which $(1/|\Delta t|)$ is a conversion factor and $|\Delta t|$ has the value of the variable Δt. If the nonlinear storage-discharge relation is expressed in terms of C_h, and in order to keep the same units of outflow rate (q) in mm/Δt, it becomes:

$$q = C_h^N s^N |\Delta t| = (C_h |\Delta t|^{1/N})^N s^N \tag{23}$$

Equating the coefficients of storage in Eqs. (2) and (23) gives the following relation:

$$C_h = \frac{c}{|\Delta t|^{1/N}} \tag{24}$$

Since $|\Delta t|$ and Δt have the same value, the value sign for $|\Delta t|$ will be dropped unless confusion is expected.

6.6. Conversion of the outflow rate

In applications of the variable IUH model, it has been found more intuitive to express both the variables and parameters in terms of the depth of water over the watershed.

As a final step in hydrograph synthesis, the outflow rate q in mm/Δt is converted to a new variable Q having the familiar volumetric units of $m^3\ s^{-1}$. Let A be the watershed area in km^2, the relation between the two is:

$$Q = qA/3.6 \tag{25}$$

6.7. Variable IUH equations for unit pulse input

For direct runoff hydrograph generated by a single block of rainfall excess, i.e. $i(j-k+1)=i(1)$ when indices $j=k$, and $i(j-k+1)=0$ otherwise, and making use of Eq. (24), Eqs. (11) and (9) become:

$$q(j) = NC_h i^{2-1/N}(1) v^{N-1}(1 - v^N)(\Delta t)^{1+1/N} \tag{26}$$

$$j = \frac{F(v, N)}{C_h i^{1-1/N}(1)(\Delta t)^{1+1/N}} \tag{27}$$

At the time to peak, making use of Eqs. (14), (17), (18) and (22), the above reduce to the following:

$$q(j_p) = EC_h i^{2-1/N}(1)(\Delta t)^{1+1/N} \tag{28}$$

$$j_p = 0.5 + \frac{F}{C_h i^{1-1/N}(1)(\Delta t)^{1+1/N}} \tag{29}$$

where j_p is a multiple of Δt denoting the peak time.

6.8. Variable IUH by the Manning friction law

For N=1.67 by Manning friction, the variable IUH shape factor is calculated in several steps: by Eq. (14), $v(t_p)$=0.473; Eq. (17), E=0.722; Eq. (18), F=0.535; and finally by Eq. (19), $u(t_p)t_L$=0.386. Table 1 lists some other values of the IUH shape factor, which are extracted from a VUH Model manual (Ontario Ministry of Natural Resources, 1983). It can also be constructed following the steps outlined above. This table will be used in Sects. 7 and 8 below.

Let $i(0)=R_E/\Delta t$ where R_E is the rainfall excess amount. Substituting the values of peak flow functions above into Eqs. (28) and (29) yields the following:

$$q(j_p) = 0.722C_h(R_E/\Delta t)^{1.4}(\Delta t)^{1.6} \tag{30}$$

$$j_p = 0.5 + \frac{0.535}{C_h(R_E/\Delta t)^{0.4}(\Delta t)^{0.6}} \tag{31}$$

Equation (30) illustrates the relative effects on the peak discharge, of the storm duration, rainfall excess intensity and the watershed discharge coefficient in that order, if the Manning friction law applies on a watershed. Other things being equal, given the same intensity, a longer duration storm would produce a higher peak discharge than a shorter one. More on this will be discussed in Sect. 7.4 below.

As a final step, the peak flow rate $q(j_p)$ in mmh^{-1} is converted by Eq. (25) to the peak discharge $Q(j_p)$ in m^3 s^{-1} as follows:

$$Q(j_p) = 0.201C_h(\Delta t)^{0.2}R_E^{1.4}A \tag{32}$$

6.9. Model calibration methodology

In the context of the variable IUH, the storage exponent N in Eq. (2) defines the degree of watershed nonlinearity. Ding (1998) conducted a survey of the variable IUH model applications mainly in Ontario, Canada and elsewhere in China (Collins and Moon Ltd., 1981; Tsao, 1981; Wisner et al., 1984; Chen and Singh, 1986) and reported that the calibrated N values on watersheds ranging in size from one to 1900 km^2 vary from 1.2 to 3.4.

As a form of simplification, Collins and Moon Ltd. (1981), in a calibration study in Ontario, Canada, fixed the N value at 1.5 according to Chezy friction, thus leaving only

HESSD

2, 2111–2151, 2005

the scale parameter C_h to be determined. For the normal range of storm events used in calibration, they found that the 1-parameter model does not suffer significant loss in its flexibility to fit observed hydrographs. For some 10 watersheds in southwestern Ontario, they found that the scale parameter is inversely proportional to watershed area to a power of 0.31, i.e. the larger the watershed, the smaller the discharge coefficient

Given a pair of rainfall excess hyetograph and direct runoff hydrograph, the variable IUH model parameters can be simultaneously calibrated or optimized by the process of reversing the convolution integral (Eqs. 20 and 21), i.e. de-convolution. A parameter optimization procedure based on the method of differential corrections is given by Ding (1974) (note: in Eq. 43 of the paper, the factor: $v^{n0}/(1-v^{n0})$ should read $v^{n0}\ln v/(1-v^{n0})$). However, this approach will not be followed because only the unit hydrograph peak characteristics will be used for calibration, as explained in Sect. 7.1 below.

Instead, an alternate approach called the variable IUH shape factor method will be used to determine or calibrate the shape parameter N, which in turn determines the scale parameter C_h. To verify the accuracy of calibrated parameters, hydrographs including the peak characteristics will be regenerated by applying the convolution integral for comparison with observed one.

7. Analysis of the Minshall unit hydrograph data for the Edwardsville catchment

7.1. Shape parameter

The Minshall (1960) family of five unit hydrographs for the 11-hectare Edwardsville catchment is among the oft-cited examples of watershed nonlinearity. These storm events have a much wider range of rainfall values and provide an excellent data set for another closer look at the watershed nonlinearity.

Since Minshall (1960) provided data in the finished form of unit hydrographs, especially the peak rates and the time to peak, these lend themselves to the use of the IUH shape factor for model calibration.

Table 2a shows the unit hydrograph data for the Edwardsville catchment. Columns (1) to (8) are reproduced from one of Minshall's more extensive tables, with the data converted from the imperial units to the metric. The "unit" hydrograph as used in this paper refers to that produced by a unit storm having 1 mm in rainfall excess instead of 1 inch (25.4 mm) in Minshall's paper. The headings are slightly modified to reflect the present-day usage. The data are arranged in the descending order of the rainfall intensity in Column (4). Note the time to peak in Column (8), when expressed in the multiple of the storm duration Δt in Column (2), is an integer of 1 to 2, i.e. the response time is very short.

Table 2b shows the calculations of the variable ΔtUH model parameters. In Column (9), the range of rainfall *excess* intensity is found much narrower than that of rainfall intensity in Column (4) and, in terms of the former, the lowest event is out of the order. In unit hydrograph analysis, data for the rainfall excess intensity, and not the rainfall intensity, are required, hence reference will be made to the former.

The lag time in Column (10) is computed from t_p in Column (8) and Δt in Column (2). The IUH shape factor is approximated by the ΔtUH shape factor in Column (11). According to Minshall (1960), periods of high rainfall intensity all occurred late in the storm for all five events. These imply that computed values of the lag time may be too long, which may in turn cause an over-estimation of parameter N values because, as can be seen from Table 1, N value increases as does the IUH shape factor. Because of absence of the observed data, their effects on N values will not be pursued. The degree of nonlinearity in Column (12) is interpolated using Table 1 for a given value of the ΔtUH shape factor.

For the five unit hydrographs, the N value varies from 1.47 to 1.84, with an average of 1.72, as also shown in Fig. 1. All events, except the largest one, have an average N value of 1.79, which is 7% higher than the theoretical value of 1.67 by Manning friction law. The highest event alone has a lower N value of 1.47. This is close to the theoretical value of 1.5 by Chezy friction, which, as mentioned in Sect. 2 above, is the value chosen by Singh (1975) for his laboratory watershed. An examination of Tables 2a and 2b shows that in comparison with other events, this has an atypical unit hydrograph in that it peaked before the storm ended, and is an outlier because its rainfall excess intensity is three and half times higher than the rest.

As mentioned in Sect. 2 above, in a review of the Amorocho and Orlob (1961) laboratory experimental data, Dooge (2005) concludes that the characteristic time is inversely proportional to the characteristic discharge to a power of 0.4. Note that the Dooge relation is of the same form as the Manning friction-based IUH peak time equation expressed by Eq. (31). It follows that for Amorocho and Orlob's overland flow plane, the N value is 1.67. This is in contrast to an N value of 1.5 for the Singh (1975) laboratory watershed having a converging surface.

7.2. Scale parameter

When parameter N has been determined, parameter C_h can be determined from the IUH peak characteristics either by Eq. (15) or (16), plus Eq. (24), and the results are shown in Table 2b and Fig. 1. The peak ordinate function in Column (13) is computed by Eq. (17), and parameter c in Column (14) is computed by Eq. (15). Values of parameter C_h in Column (15) are computed by Eq. (24). The C_h values vary from 0.9 to 3.5, with an average of 1.91. The calibrated C_h values have a much wider scatter than do the N values, with the highest C_h value, as well as the lowest N, associated with the largest event The lowest C_h value is associated with the 20 July 1948 storm which has the longest duration of 17 min, compared to that of 10 to 14 min for the rest.

7.3. Regeneration of unit hydrograph peak characteristics

The accuracy of parameters calibrated by the shape factor method in Sects. 7.1 and 7.2 above can be verified by applying the convolution integral to regenerate hydrograph for comparison with the observed one.

Based on the calibrated N and C_h values shown in Table 2b, hydrographs for each of the five events are regenerated by convolution. Computations are done on a spreadsheet using a discrete form of the convolution integral with a variable IUH (Eqs. 20 and 21). The simulation results are shown in Table 2c (a template showing calculations for 27 May 1938 event, in which the storm duration of 14 min is divided into two 7-min periods as discussed in Sect. 7.4 below, is included as Appendix A).

For the largest, 27 May 1938 event, when compared with the peak characteristics generated by convolution, the calibrated model under-estimates the peak rate by about 42%. The inability to capture the peak rate may be due to its being an atypical unit hydrograph, as explained in Sect. 7.1 above.

For the four other moderate events, the calibrated models under-estimate by convolution the hydrograph peak rates by an average of 0.7% and the peak time by $1\Delta t$. Therefore, it may be concluded that for the Edwardsville catchment, parameter values calibrated by the shape factor method for typical hydrographs are correct.

7.4. Size of the time step

To test the sensitivity of the peak characteristics to change in the size of time step, the unit hydrographs for the two largest events are regenerated using different sizes of time step varying from 1 min to the full duration of the storms, i.e. 14 and 12 min, respectively. Although periods of high rainfall intensity all occurred late in the storm for all five events according to Minshall (1960), the rainfall excess is assumed, for sensitivity test, uniformly distributed within the storm duration, i.e. having the same intensity throughout the whole period. The full storm duration Δt, called the "calibrated" time step, is divided into n periods of $(\Delta t/n)$-steps. By convolution (Eqs. 20 and 21), successive incremental hydrographs for each of the n periods are generated and the ordinates are added at the same time steps to yield a composite hydrograph. Appendix A shows an example of calculations for the 27 May 1938 storm using two 7-min steps.

Table 3 shows the peak characteristics of the regenerated hydrographs for these two events. The results show that as the size of time step becomes smaller, the hydrograph peak becomes more attenuated and the peak time much delayed, and that a single time step of the full storm duration, i.e. the calibrated time step, is the best available to reproduce or approximate the peak magnitude. Decreasing the size of time step does not improve, contrary to expectation, simulation accuracy. This is explained by examining, for example, the peak flow equation by Manning friction (Eq. 30) that for a unit pulse input, the peak discharge varies with the size of time step Δt to a power of 1.6, and that the sum of two $(\Delta t/2)$-hydrograph peak discharges is less than that of the Δt-hydrograph, let alone one $(\Delta t/2)$-hydrograph peak lagged by $(\Delta t/2)$ after the other.

These have profound implications for calibration and application of nonlinear models such as the 2-parameter, variable IUH model. It is obvious from these results that the principle of superposition does not strictly apply to a nonlinear system for even a rainfall excess input uniformly distributed in time. The model parameters are applicable to the size of time step for which they are calibrated.

Since the variations of regenerated peak discharges with the sizes of time step as shown in Table 3 appear systematic, these point to the need for an adjustment factor or additional parameter to account for the difference between various time step sizes used in simulation.

To illustrate the development of such an adjustment factor, let's us consider any one case from Table 3. Let $q^{'}(j_p, \Delta t/m)$ be the peak discharge of the composite hydrograph comprising m successive incremental hydrographs each of $(\Delta t/m)$ duration, where the second argument in $q^{'}$ denotes the time step size. By definition, $q^{'}(j_p, \Delta t/m) = q(j_p, \Delta t/m) + q(j_p - 1, \Delta t/m) + q(j_p + 1, \Delta t/m) + \ldots + q(j_p - m/2, \Delta t/m)$, the right-hand side includes the first m incremental hydrograph ordinates at the peak time of the composite hydrograph. To simplify analysis, let's ignore the time lag for each successive $(\Delta t/m)$-incremental hydrograph so that $q^{'}(j_p, \Delta t/m) = mq(j_p, \Delta t/m)$. Let a_m be an adjustment factor, in which the subscript m denotes the number of $(\Delta t/m)$-periods in the calibrated time step Δt. The purpose of a_m is to bring the peak discharge of the composite hydrograph upward to that of the whole hydrograph, i.e.

$q(j_p, \Delta t) = a_m m q(j_p, \Delta t/m)$. Note the value of a_m thus derived will be at a minimum.

Let's also assume the peak time of the composite hydrograph and that of the whole hydrograph resulting from storm of Δt duration are the same. By making use of the peak flow equation in Eq. (28) in which parameters N and C_h and the intensity $i(1)$ are the same for both Δt- and $(\Delta t/m)$-hydrographs, the adjustment factor becomes:

$$a_m = m^{1/N} \quad (33)$$

This adjustment factor is applied to the regenerated hydrograph peak discharges for the two large events as shown in Table 3. These results show that the adjusted regenerated hydrograph peak discharges are closer to, but still below, the observed ones. As mentioned in Sect. 7.1 above and also shown in Table 2a, the heaviest storm on 27 May 1938 is both atypical in the timing of the unit hydrograph peak, and an outlier in terms of the magnitude of peak discharge. The use of a smaller time step than the full storm duration, i.e. the calibrated time step, coupled with the adjustment factor greatly improves the accuracy of regenerated peak discharges for this unique event. For the more typical 02 September 1941 event, the calibrated time step still gives the best estimate of the peak discharge.

Note that to capture the peak ordinate of a hydrograph due to a single block of rainfall excess and pinpoint its time of occurrence, one can make direct use of the peak equations given by Eqs. (28) and (29).

7.5. Layers of the rainfall excess depth

The adjustment approach in the time domain as described in Sect. 7.4 above may be applied to the rainfall excess in the space domain as well. Imagine the depth of rainfall excess R_E is sliced into p layers of (R_E/p) each but of the same duration Δt. Let $q'(j_p, \Delta t, R_E/p)$ be the peak discharge of the composite hydrograph comprising p incremental hydrographs all resulting from the rainfall excess of (R_E/p) in Δt, where the third argument in q' denotes the rainfall excess amount. The peak ordinate of the composite hydrograph is simply p times the peak ordinate of an incremental hydrograph, i.e. $q^{'}(j_p, \Delta t, R_E/p) = pq(j_p, \Delta t, R_E/p)$.

Let b_p be a second adjustment factor, the subscript p denotes the number of (R_E/p) layers in the rainfall excess depth. The purpose of b_p is again to bring the peak discharge of the composite hydrograph upward to that of the whole hydrograph, i.e. $q(j_p, \Delta t, R_E) = b_p pq(j_p, \Delta t, R_E/p)$. Let's also assume the peak time of the composite hydrograph is equal to that of the whole hydrograph, an assumption which is clearly approximate because the difference between two rainfall excess intensities $(R_E/\Delta t)$ and $(R_E/p)/\Delta t$ and that the peak time varies inversely with the intensity to a power of $(1-1/N)$ as indicated by Eq. (29). Regardless of the validity of this assumption, by making use of the peak flow equation Eq. (28) in which parameter N, C_h, and Δt are the same for both hydrographs of R_E and (R_E/p), the adjustment factor becomes:

$$b_p = p/p^{1/N} \tag{34}$$

This adjustment factor is applied to the regenerated hydrograph peak discharges for the two large events as shown in Table 4. These results show that the adjusted regenerated hydrograph peak discharges are capable of reproducing the peak discharges using 10-plus layers of rainfall excess depth for the heaviest storm and atypical hydrograph on 27 May 1938, and using 3 to 5 layers for the second largest event.

It is of interest to compare the magnitudes of these two adjustment factors. Let $r = a_m/b_p$ and $m = p$, i.e. the number of division is the same in the time step and the rainfall excess depth. From Eqs. (33) and (34):

$$r = m^{2/N}/m \tag{35}$$

This shows that for $N=2$, $r=1$ and therefore $a_m = b_p$, and that for $N<2$, $a_m > b_p$, and for $N<2$, $a_m < b_p$, where m and p are equal in value.

8. Analysis of the Childs unit hydrograph data for the Naugatuck River

8.1. Shape parameter

As mentioned in Sect. 5 above, the Childs (1958) family of unit hydrographs for the Naugatuck River is an earlier but rarely cited example of watershed nonlinearity. Since he associated the variation of the unit hydrographs with the observed (and thus effected) peak discharges, not the causative rainfall excess intensities, thus one key piece of data was missing as required for analysis by the variable IUH model

The IUH shape factor is a function of parameter N only. Given the duration of rainfall excess, N can be calculated from the unit hydrograph peak characteristics alone. The calculation of C_h, however, requires the amount of rainfall excess as well.

Table 5a shows the 3h unit hydrograph peak characteristics for four events on the Naugatuck River as provided by Childs (1958) and converted to metric units from the imperial ones. As is the case for the Edwardsville catchment, the "unit" hydrograph refers to that produced by a unit storm having 1 mm in rainfall excess. Data are arranged in the descending order of the observed peak discharge in Column (2). Column (3) shows the traditional "unit" hydrograph peak rates, i.e. for 1 inch (25.4 mm) of rainfall excess, which are read off the Childs graph or chart. The unit hydrograph peak ordinate in Column (4) is computed from the peak rate in Column (3) divided by the drainage area of 186.2 km^2. Values for the time to peak in Column (5) are also read off his graph. In terms of the storm duration of 3 h, the time to peak is an integer of 2 to 3 in comparison with that of only 1 to 2 for the Edwardsville catchment. The ΔtUH shape factor and degree of nonlinearity for each of the events are computed in the same manner as described in Sect. 7.1 above for the Edwardsville.

For the four 3 h-unit hydrographs, the N value varies from 1.92 to 2.68, with an average of 2.28. The smallest N value of 1.92 and the largest of 2.68 are associated with the smallest and largest flood events, respectively. They all lie between the theoretical value of 1.67 by Manning friction for turbulent overland flow, and that of 3.0 for laminar overland flow (Ding, 1967a).

When compared to the average nonlinearity of 1.72 for the 11-hectare Edwardsville catchment, the larger Naugatuck River with a drainage area of 186.2 km^2 has a much higher nonlinearity of 2.28. According to Eq. (2), between these two watersheds, the large river is more efficient in converting the flood storage into flood flow than the small catchment.

8.2. Scale parameter

The calculation of scale parameter C_h requires data for the causative rainfall excess intensity, which were not given in the Childs paper.

For the August 1955 Hurricane Diane, Childs (1958) reported that the computed peak discharge of 41 600 c.f.s. was equivalent to a rate of runoff of 0.9 inches per hour from the entire drainage area of 72 sq. mi., and that the rate of rainfall probably did not greatly exceed a basin-wide average of 1 inch per hour, thus the Naugatuck River becoming a proverbial *"tin-roof"* (in Childs' word) under extreme flood conditions.

Based on his estimated rainfall excess intensity of 0.9 inches per hour, parameter C_h is calculated by the same shape factor method, which gives a C_h value of 0.017 as shown in Table 5b. This is very much smaller than the average C_h value of 1.91 for the Edwardsville catchment, i.e. the larger the watershed size, the smaller the discharge coefficient.

8.3. Regeneration of unit hydrograph peak characteristics

Based on calibrated N and C_h values shown in Tables 5a and 5b, hydrograph for the August 1955 flood event is regenerated and shown in Table 5c. For the Naugatuck River with a computational time step of 3 h, the calibrated model by the shape factor method under-estimates the peak discharge generated by convolution by about 16%. Increasing the C_h value from 0.017 to 0.019 as shown also in Table 5c would over-estimate the peak discharge by about 3%.

9. Summary and conclusions

The author has described connections between nonlinear overland flow, channel routing and catchment runoff processes mainly through the use of an input-dependent kernel or variable IUH. A 2-parameter variable IUH model has been applied to two watersheds of vastly different sizes. The calibration for the Edwardsville and Naugatuck watersheds both is carried out using their shape factor, because of the availability of the unit hydrograph data in a finished form. Otherwise, parameters are usually estimated from pairs of rainfall excess hyetograph and direct runoff hydrograph by de-convolution, i.e. optimization, or by convolution, i.e. sensitivity test. The sensitivity of peak characteristics to changes either in the computational time step or the rainfall excess depth has been carried out for the Edwardsville catchment. These tests show instability problems associated with analysis of even some simple nonlinear systems such as the 2-parameter variable IUH model. Based on analysis of these well-documented storm events, but only on two watersheds, a number of conclusions regarding the model are summarized below.

Shape parameter

a. The storage exponent N in the nonlinear watershed storage-discharge, $q = c^N s^N$, has been proposed as an index or measure of the watershed nonlinearity (Ding, 1974). It measures the efficiency of a watershed in converting the flood storage to flow.

b. The Amorocho and Orlob (1961) laboratory experimental data, as analyzed by Dooge (2005), substantiate a nonlinearity of 1.67 by the Manning friction law for flow in wide rectangular channels, while the Singh (1975) experimental data for a laboratory watershed having a converging surface substantiate an N value of 1.5 by Chezy friction. The difference between the two N values is less than 12%.

c. The Minshall (1960) unit hydrograph data for the 11-hectare Edwardsville catchment show mixed results. For moderate storms, the degree of nonlinearity averages 1.79, or 7% higher than the theoretical value of 1.67 by Manning friction. For the largest event, which has an atypical unit hydrograph in that it peaked prior to the end of the storm, and is an outlier in terms of the peak discharge, it has an N value of 1.47, close to the theoretical value of 1.5 by Chezy friction.

d. The Childs (1958) unit hydrograph data for the Naugatuck River having a drainage area of 186.2 km^2 indicate a highly nonlinear river basin with N values ranging from 1.92 to 2.68 with an average of 2.28. These lie between the theoretical value of 1.67 for turbulent overland flow by Manning friction, and that of 3.0 for laminar overland flow.

Scale parameter

e. The watershed discharge coefficient c defines the time scale of an IUH. For comparison of calibrated c values from storms of different duration and among different watersheds, the scale parameter c is standardized by Eq. (24) to C_h having a common time unit of 1 h.

f. The larger Naugatuck River has a C_h value of 0.017 calibrated from a hurricane-induced flood, and the smaller Edwardsville catchment has an average value of 1.91. Given similar N values, the larger the watershed size, the smaller the discharge coefficient.

Computational time step

g. The peak discharge in the variable IUH model is very sensitive to change in the storm duration or computational time step. The use of a single time step of the full storm duration is the best available to reproduce or approximate the peak magnitude. Decreasing the size of time steps without the use of an adjustment factor does not improve simulation accuracy.

h. To account for decrease in regenerated peak discharges caused by the storm duration or calibrated time step Δt being divided into m times ($\Delta t/m$) steps, an adjustment factor a_m, which is the Nth root of m periods (Eq. 33), can be applied to the regenerated peak discharges to improve their accuracy.

Interaction of parameters and the time step

i. Parameters N and C_h are calibrated by the shape factor method, and verified by convolution. For the Edwardsville catchment having storm durations in the order of 10 min, both methods give similar peak rates for moderate events. For the Naugatuck River having a storm duration of 3 h for the hurricane-induced August 1955 flood, the calibrated parameters would under-estimate the peak discharge by about 16%.

j. The model parameters are applicable to the size of time step for which they are calibrated.

k. To calculate hydrograph peak characteristics produced by a block of uniform rainfall excess, the IUH peak equations (Eqs. 28 and 29) are available for such a purpose.

Application to ungauged basins

l. For small ungauged watersheds, by defaulting the degree of nonlinearity N to the theoretical value of either 1.67 by Manning friction or 1.5 by Chezy, the variable IUH model reduces to a single parameter one, leaving only the scale parameter C_h to be determined. Parameter C_h has a very appealing property in that the IUH peak ordinate varies directly and the peak time inversely with C_h. The standard scale parameter C_h, when calibrated for more watersheds under a wide range of storm sizes, may be regionalized to provide guidance for prediction purposes in ungauged basins.

Appendix A. The variable IUH template for generating the direct runoff hydrograph

Part 1. Watershed characteristics

Watershed and storm date (1)	**A** km^2 (2)	**N** (3)	**C_h** (4)	**Δt** h (5)	c (6)	$A/3.6$ (7)
Edwardsville, Illinois – 27 May 1938	**0.11**	**1.470**	**3.500**	**0.117**	0.813	0.031

(4) in $(\text{mm}\bullet\text{h}^{-1})^{1/N}/\text{mm}$

(6) in $(\text{mm}\bullet(\Delta t)^{-1})^{1/N}/\text{mm}$. $c{=}C_h(\Delta t)^{1/N}$

(7) Runoff conversion factor in $(\text{m}^3\text{s}^{-1})/\text{mm}$. $Q{=}q(A/3.6)$

Part 2. Incremental hydrograph generator

k – time step index (inner loop)	(a)	Total	1	2	3	4
$i(k)$ – rainfall excess in mmh^{-1}	(b)		**71.62**			
$ci^{1-1/N}(k)\Delta t$	(c)		0.373			
$Nci^{2-1/N}(k)\Delta t$	(d)		39.251			
$F(v,N)$	(e)		0.373	0.746	1.118	1.491
v	(f)		0.340	0.589	0.751	0.852
$v^{N-1}(1-v^N)$	(g)		0.479	0.422	0.300	0.194
$q(k)$ – direct runoff in mmh^{-1}	(h)	66.19	**18.801**	16.559	11.776	7.619

(e) = (c) * (a)

(f) – interpolated from table of Bakhmeteff function (not shown)

(h) = (g) * (d)

Part 3. Composite hydrograph composer

j – time step index (outer loop)	(i)	Total	1	2	3	4
$i(j)$ – rainfall excess in mm	(j)	16.76	**8.38**	**8.38**		
Incre. $q(j-k+1)$ due to $i(k=1)$ – mmh^{-1}	(k1)	66.19	**18.801**	16.559	11.776	7.619
Incre. $q(j-k+1)$ due to $i(k=2)$ – mmh^{-1}	(k2)	66.19		**18.801**	16.559	11.776
Composite $q(j)$ – mmh^{-1}	(l)	132.39	18.801	**35.361**	28.336	19.395
Composite $Q(j)$ – m^{3}s^{-1}	(m)	4.05	0.57	**1.08**	0.87	0.59

(k1) = (h) starting at time step 1
(k2) = (h) starting at time step 2
(l) = (k1)+(k2)
(m) = (1)*(7)

References

Amorocho, J.: The nonlinear prediction problem in the study of runoff cycle, Water Resour. Res., 3, 861–880, 1967.

Amorocho, J. and Orlob, G.T: Nonlinear analysis of hydrologic systems, Water Resour. Center, Univ. California (Berkeley), Contrib. 40, 1961.

Ashfag, A. and Webster, P.: The timing of runoff response in design flood analysis, Hydrol. Processes, 14, 1217–1233, 2000.

Bakhmeteff, B. A.: Hydraulics of open channels, McGraw-Hill, New York, USA, 1932.

Chang, Cheng-Nan, Singer Da Motta, E., and Koussis, A. D.: On the mathematics of storage routing. J. Hydrol., 61, 357–370, 1983.
Chen, S.J. and Singh, V.P.: Derivation of a new variable instantaneous unit hydrograph, J. Hydrol., 88, 25-42, 1986.
Childs, E. F.: Northeastern floods of 1955: flood control hydrology, J. Hydraul. Div., Am. Soc. Civ. Eng., 84(HY3), 1663.1–1963.24, 1958.
Collins and Moon Ltd.: Watershed model calibration methodology study, Report prepared for Conservation Authorities and Water Management Branch, Ontario Ministry of Natural Resources, Toronto, Canada, 1981.
Cranmer, A. J., Kouwen, N., and Mousavi, S. F.: Proving WATFLOOD: modelling the nonlinearities of hydrologic response to storm intensities, Can. J. Civ. Eng. 28, 837–855, 2001.
Ding, J. Y.: Flow routing by direct integration method, Proc. Int. Hydrol. Symp., Fort Collins, Colo., 1, 113–120, 1967a.
Ding, J. Y.: Comments on mathematical models for hydrologic systems, Proc. Int. Hydrol. Symp., Fort Collins, Colo., 2, 421–422, 1967b.
Ding, J. Y.: Variable unit hydrograph, J. Hydrol., 22, 53–69, 1974.
Ding, J. Y.: Variable unit hydrograph model revisited, Eos Trans. AGU, 79 (24), Western Pacific Geophysics Meet. Suppl., W27, 1998.
Dooge, J. C. I.: Bringing it all together, Hydrol. Earth Syst. Sci., 9, 3–14, 2005, SRef-ID: 1607-7938/hess/2005-9-3.
Gill G. A.: Flood routing by the Muskingum method, J. Hydrol., 36, 353–363, 1978.
Horton, R. E.: The interpretation and application of runoff plot experiments with reference to soil erosion problems. Proc. Soil Sci. Soc. Am., 3, 340–349, 1938.
Izzard, C. F.: Hydraulics of runoff from developed surfaces, Proc. 26th Ann. Meet., Highway Res. Board, 26, 129–146, 1946.
Kokkonen, T., Koivusalo, H., Karvonen, T., Croke, B., and Jakeman, A.: Exploring streamflow response to effective rainfall across event magnitude scale, Hydrol. Processes, 18, 1467–1486, 2004.
Kundzewicz, Z.: On the mathematics of storage routing – a comment, J. Hydrol., 69, 359–364, 1984.
Lee, K. T. and Yen, B. C.: Unit hydrograph theory: a 60-year unfulfilled promise, Section 37, Chapter 37 in: Building Partnerships: Proceedings of 2000 Joint Conference on Water Resource Engineering and Water Resources Planning & Management, edited by: Hotchkiss,

R. H. and Glade, M., Am. Soc. Civ. Eng., 2000.
Minshall, N. E.: Predicting storm runoff on small experimental watersheds, J. Hydraul. Div., Am. Soc. Civ. Eng., 86, 17–38, 1960.
Ontario Ministry of Natural Resources: Flood Hydrology: VUH Model User's Manual, Toronto, Canada, 1983.
Overton, D. E. and Meadows, M. E.: Stormwater modeling, Academic Press, New York, USA, 1976.
Paik, K. and Kumar, P.: Hydraulic geometry and the nonlinearity of the network instantaneous response, Water Resour. Res., 40, W03602, 2004.
Robinson, J. S., Sivapalan, M., and Snell, J. D.: On the relative roles of hillslope processes, channel routing, and network geomorphology in the hydrologic response of natural catchments, Water Resour. Res., 31, 3089–3101, 1995.
Singh, V. P.: A laboratory investigation of surface runoff, J. Hydrology, 25, 187–200, 1975.
Singh, V. P.: Hydrologic systems: Rainfall-runoff modeling. Prentice-Hall, volume 1, 1988.
Singh, V. P. and Scarlatos, P. D.: Analysis of nonlinear Muskingum flood routing, J. Hydraul. Engineering, Am. Soc. Civ. Eng., 113, 61–79, 1987.
Sivapalan, M., Jothityangkoon, C., and Menabde, M.: Linearity and nonlinearity of basin response as a function of scale: Discussion of alternative definitions, Water Resour. Res., 38, 1012, doi:10.1029/2001WR000482, 2002.
Tsao, W.-W.: Application of the variable unit hydrograph in the nonlinear discharge computations, Chinese J. Hydrol., 4, 7–12, (in Chinese), 1981.
Tung, Y.-K.: River flood routing by nonlinear Muskingum method, J. Hydraul. Engineering, Am. Soc. Civ. Eng., 111, 1447–1460, 1985.
Wisner, P. E., Fraser, H., Png, C. E., and Consuegra-Zammit, D.: An investigation of the runoff component in the HYMO, OTTHYMO and VUH models for selected Ontario watersheds, Department of Civil Engineering, University of Ottawa, Canada, 1984.

Table 1. Variable instantaneous unit hydrograph (IUH) shape factor.

Degree of nonlinearity N (1)	Normalized unit peak $v(t_p)$ (2)	Peak ordinate function E (3)	Peak time function F (4)	IUH shape factor $u(t_p)t_L$ (5)
1.4	.342	.709	.378	.268
1.5	.397	.709	.444	.315
1.6	.444	.715	.500	.358
1.67	***.473***	***.722***	***.535***	***.386***
1.7	.484	.725	.549	.398
1.8	.520	.738	.590	.435
1.9	.550	.753	.627	.472
2.0	.577	.770	.658	.507
2.1	.601	.788	.686	.541
2.2	.623	.807	.711	.574
2.3	.642	.826	.733	.605
2.4	.660	.847	.752	.637
2.5	.675	.867	.770	.668
2.6	.690	.889	.785	.698
2.7	.703	.910	.799	.727

Source: Ontario Ministry of Natural Resources (1983)
Column (5): $u(t_p) \bullet t_L = E \bullet F$

Table 2a. Unit hydrograph data for the Edwardsville catchment. Relation between rainfall intensity, unit hydrograph peak rate and time to peak.

Date	Rainfall producing UH			Runoff used in computing UH		UH peak	Time to
	Duration	Amount	Intensity	Peak rate	Amount	ordinate	peak
	Δt			$q(t_p)$	R_E	$u(t_p)$	t_p
	min	mm	mm•h^{-1}	mm•h^{-1}	mm	h^{-1}	min
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
27 May 1938	14	28.19	120.81	60.45	16.76	3.61	12
2 Sept. 1941	12	13.46	67.30	9.65	4.32	2.23	18
17 April 1941	13	10.67	49.25	6.35	3.56	1.78	20
22 Oct. 1941	10	5.59	33.54	3.56	2.54	1.40	24
20 July 1948	17	6.86	24.21	6.35	5.33	1.19	30

Source: adapted from Minshall (1960) and converted to metric units
Catchment area 11 hectare

Table 2b. Unit hydrograph data for the Edwardsville catchment. Variable instantaneous unit hydrograph (IUH) model parameters (based on Δt step).

Date	Rainfall excess intensity $i(0)$ mm•h^{-1}	Lag time t_L h	Δt-UH shape factor $u(t_p) \bullet t_L$	Degree of nonlinearity N	Peak ordinate function E	Scale parameter	
						Internal c	Standard C_h
(1)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
27 May 1938	71.83	0.08	0.30	1.47	0.708	1.30	3.50
2 Sept. 1941	21.60	0.20	0.45	1.84	0.744	0.74	1.77
17 April 1941	16.43	0.23	0.40	1.71	0.726	0.77	1.88
22 Oct. 1941	15.24	0.32	0.44	1.81	0.739	0.56	1.51
20 July 1948	18.81	0.36	0.43	1.79	0.737	0.44	0.90
Average				1.72			1.91

Column (14): c in $(\text{mm} \bullet (\Delta t)^{-1})^{1/N}/\text{mm}$

Column (15): C_h in $(\text{mm}\bullet\text{h}^{-1})^{1/N}/\text{mm}$

Table 2c. Unit hydrograph data for the Edwardsville catchment. Regeneration of unit peak characteristics by convolution.

Date	Hydrograph peak		Hydrograph peak time	
	Peak rate $q(t_p)$ $mm \bullet h^{-1}$	Estimation error %	Time to peak t_p Δt	Estimation error Δt
(1)	(16)	(17)	(18)	(19)
27 May 1938	34.94	−42.2	1	0
2 Sept. 1941	9.64	−0.1	1	−1
17 April 1941	6.35	0	1	−1
22 Oct. 1941	3.56	0	2	0
20 July 1948	6.17	−2.8	1	−1

Table 3. Sensitivity of unit hydrograph peak characteristics to the number and size of time steps for the Edwardsville catchment.

Number and size of time step	27 May 1938 (N=1.47)				2 Sept. 1941 (N=1.84)			
	Hydrograph peak		Time to peak		Hydrograph peak		Time to peak	
	Regen'd	Adjusted			Regen'd	Adjusted		
$m\times(\Delta t/m)$	$q(t_p)$	$a_m q(t_p)$	t_p		$q(t_p)$	$a_m \bullet q(t_p)$	t_p	
min	mm•h^{-1}		Δt	min	mm•h^{-1}		Δt	min
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Observed								
1×14	60.45			12				
1×12					9.65			18
Regenerated								
1×14	34.94		1	14				
1×12					9.64		1	12
2×7	35.36	56.66	2	14				
2×6					6.38	9.30	4	24
3×5	25.91	54.71	4	20				
3×4					5.21	9.47	7	28
5×3	16.29	48.69	7	21				
4×3					4.49	9.54	10	30
7×2	15.82	59.44	13	26				
6×2					3.61	9.56	19	38
14×1	8.64	52.02	35	35				
12×1					2.27 *	8.76	36 *	36

Columns (3) and (7): Adjustment factor $a_m = m^{1/N}$
* The dimension of the template is 36 only

Table 4. Sensitivity of unit hydrograph peak characteristics to the number and size of layers in the rainfall excess depth.

Number and size of layers in rainfall excess $p\times(R_E/p)$	27 May 1938 (N=1.47) Hydrograph peak Regen'd $q(t_p)$	Adjusted $b_p pq(t_p)$	Time to peak t_p		2 Sept. 1941 (N=1.84) Hydrograph peak Regen'd $q(t_p)$	Adjusted $b_p pq(t_p)$	Time to peak t_p	
mm	$mm\bullet h^{-1}$		Δt	min	$mm\bullet h^{-1}$		Δt	min
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Observed								
1×16.76	60.45			12				
1×4.32					9.65			18
Regenerated								
$1\times R_E$	34.94		1	14	9.64		1	18
$2\times(R_E/2)$	17.87	44.61	1	14	3.30	9.06	1	18
$3\times(R_E/3)$	11.59	49.40	1	14	1.90	9.41	2	36
$5\times(R_E/5)$	6.49	54.29	1	14	0.96	10.01	2	36
$10\times(R_E/10)$	2.81	58.67	1	14	0.34	9.73	3	54

Columns (3) and (7): Adjustment factor $b_p = p/p^{1/N}$

HESSD

2, 2111–2151, 2005

EGU

Table 5a. 3-hour unit hydrograph data for the Naugatuck River. Peak characteristics and degree of nonlinearity.

Date	Observed peak discharge $Q(t_p)$ $m^3 s^{-1}$	25.4 mm times UH peak rate 25.4 mm•$u(t_p)$ $m^3 s^{-1}$	UH peak ordinate $u(t_p)$ h^{-1}	Time to peak t_p h	Lag time t_L h	Δt-UH shape factor $u(t_p)\bullet t_L$	Degree of nonlinearity N
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Aug. 1955	1178.1	211.3	0.16	6.0	4.5	0.72	2.68
Dec. 1948	288.9	141.6	0.11	6,5	5.0	0.54	2.10
Sept. 1938	282.4	117.5	0.09	8.7	7.2	0.64	2.42
June 1952	90.6	85.0	0.06	9.0	7.5	0.48	1.92
Average							2.28

Source: adapted from Childs (1958) and converted to metric units
Drainage area A=186.2 km^2
Storm duration Δt=3 h

Title Page
Abstract | Introduction
Conclusions | References
Tables | Figures
Back | Close
Full Screen / Esc
Print Version
Interactive Discussion
EGU

Table 5b. 3-hour unit hydrograph data for the Naugatuck River. Scale parameter of the variable instantaneous unit hydrograph (IUH) model.

Date	Peak ordinate function E	Rainfall excess intensity $i(0)$ mm•h^{-1}	Scale parameter: Internal c $(\text{mm}\bullet(\Delta t)^{-1})^{1/N}/\text{mm}$	Scale parameter: Standard C_h $(\text{mm}\bullet\text{h}^{-1})^{1/N}/\text{mm}$
(1)	(9)	(10)	(11)	(12)
Aug. 1955	0.906	22.86	0.025	0.017

Table 5c. Unit hydrograph data for Naugatuck River. Regeneration of peak characteristics for August 1955 flood by convolution.

Scale parameter	68.58 mm times	Hydrograph peak		Hydrograph peak time	
	UH peak rate	Peak rate	Estimation error	Time to peak	Estimation error
C_h	68.58 mm $u(t_p)$	$q(t_p)$		t_p	
$(\text{mm}\bullet\text{h}^{-1})^{1/N}/\text{mm}$	m^3s^{-1}	m^3s^{-1}	%	Δt	Δt
(9)	(10)	(11)	(12)	(13)	(14)
0.017	570.51	476.74	−16.4	1	−1
0.018	570.51	531.89	−6.8	1	−1
0.019	570.51	586.80	2.9	1	−1

Parameter N=2.68

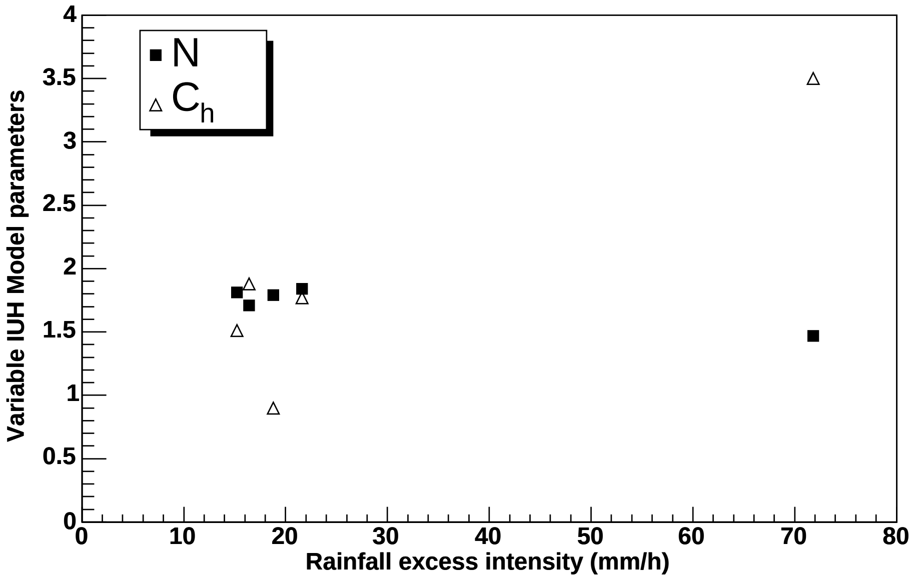

Fig. 1. Variations of the variable IUH model parameters with the rainfall excess intensity for the Edwardsville catchment.