Hydrol. Earth Sys. Sci. Discuss., 2, 1835–1864, 2005
www.copernicus.org/EGU/hess/hessd/2/1835/
SRef-ID: 1812-2116/hessd/2005-2-1835
European Geosciences Union

Hydrology and Earth System Sciences Discussions

Papers published in *Hydrology and Earth System Sciences Discussions* are under open-access review for the journal *Hydrology and Earth System Sciences*

On the asymptotic behavior of flood peak distributions – theoretical results

E. Gaume

Centre d'Enseignement et de Recherche Eau Ville Environnement, Paris, France

Received: 2 August 2005 – Accepted: 20 August 2005 – Published: 1 September 2005

Correspondence to: E. Gaume (gaume@cereve.enpc.fr)

Abstract

This paper presents some analytical results and numerical illustrations on the asymptotic properties of flood peak distributions obtained through derived flood frequency approaches. It confirms and extends the results of previous works: i.e. the shape of the flood peak distributions are asymptotically controlled by the rainfall statistical properties, given limited and reasonable assumptions concerning the rainfall-runoff process. This previous result is partial so far: only two types of rainfall intensity distributions have been considered (extreme value distributions of types I and II), and the impact of the rainfall spatial heterogeneity has not been studied. From a practical point of view, it provides a general framework for analysis of the outcomes of previous works based on derived flood frequency approaches and leads to some proposals for the estimation of very large return-period flood quantiles. This paper, focussed on asymptotic distribution properties, does not propose any new approach for the extrapolation of flood frequency distribution to estimate intermediate return period flood quantiles. Nevertheless, the large distance between frequent flood peak values and the asymptotic values as well as the simulations conducted in this paper help quantifying the ill condition of the problem of flood frequency distribution extrapolation: it illustrates how large the range of possibilities for the shapes of flood peak distributions is.

1. Introduction

Eagleson (1972) was the first to combine a rainfall stochastic model and a rainfall runoff model to generate synthetic “derived flood frequency distributions” (DFFD). The objective of this approach was twofold. Firstly, it aimed at understanding the relationship between the flood peak distributions (FPD) and the climatic and hydrologic characteristics of a watershed: to identify the main control parameters of the FPD shape, to compare the FPD of various watersheds or to anticipate the effect of changes, for instance land use or climatic evolutions on FPD. Secondly, it appeared intellectually more satisfactory, for statistical interpolation and extrapolation purposes, to derive the shape of flood peak distributions from the selection of a rainfall statistical model and of a rainfall-runoff model adapted to the considered case study, rather than to directly select a theoretical distribution chosen on the basis of the extreme value theory or for mathematical convenience.

As statistical extrapolation tools, DFFD, if properly used, probably have performances comparable to the conventional procedure based on theoretical distribution fitting. But there is, for the moment, no reason to think that they perform better. The unavoidable simplifications of the runoff generating processes and of the rainfall statistical structure in the DFFD tools, the limited extrapolation capacities of the available simplified rainfall-runoff models reduce their potential advantage over conventional statistical extrapolation methods as illustrated by some works (Raines and Valdes, 1993; Moughamian et al., 1987). This probably explains why despite the numerous works conducted on DFFD since the first paper of Eagleson (De Michele and Salvadori, 2002; Loukas, 2002; Arnaud and Lavabre, 1999; Blaskova and Beven, 2002; Gupta et al., 1996; Goel et al., 2000; Cameron et al., 2000; Iacobellis and Fiorentino, 2000; Raines and Valdes, 1993; Smith, 1992; Sivapalan et al., 1990), such procedures are, to our knowledge, seldom used in an operational context (Lamb and Kay, 2004; Blaskova and Beven, 2004; Arnaud and Lavabre, 2000).

Nevertheless, DFFD are also interesting tools to study the functional relationship between the FPD shape and the climatic and hydrologic characteristics of the corresponding watershed. But what general conclusions about the shape of the FPD can be drawn on the basis of DFFD approaches? This question is still wide open. A large variety of rainfall stochastic models and rainfall-runoff models have been tested in the previous works on DFFD. Some of these works are purely numerical approaches based on Monte-Carlo simulations (Loukas, 2002; Arnaud and Lavabre, 1999; Hashemi et al., 2000). With an ad hoc choice of rainfall and rainfall-runoff models it is sometimes possible to derive an approximate (Diaz-Granados et al., 1984; Eagleson, 1972) or a completely analytical form of the resulting FPD (De Michele and Salvadori, 2002; Goel et al., 2000). The impact of the parameters of the rainfall and the rainfall-runoff models used on the FPD shape are generally analyzed but it is not possible through these various works to evaluate the influence of the models themselves.

The shape of a FPD, as will be shown hereafter, highly depends of course on the dynamics of the rainfall-runoff processes and the range of possibilities is quite large. But, general conclusions can be drawn concerning the asymptotic behavior of the FPD (i.e. the shape of the FPD as the return period or the peak discharge tends to infinity) for a large variety of rainfall-runoff dynamics and DFFD tools.

This paper explores the link between the rainfall intensity statistical characteristics and the asymptotic behavior of FPDs. The results presented are a generalization of results already obtained on specific DFFD tools (De Michele and Salvadori, 2002; Eagleson, 1972). Limited and reasonable assumptions are made concerning the rainfall-runoff process. The demonstration is first conducted with a simple rainfall stochastic model used in many previous DFFD works: the rainfall events are supposed to be rectangular pulses with a given duration and a constant intensity. Two rainfall intensity distribution types are considered: exponential distribution and extreme value distribution of type II.

The results are then generalized to any rainfall temporal structure. Numerical results obtained with a DFFD tool combining a 5-min point rainfall stochastic model (Mouhous et al., 2001) and a rainfall-runoff model presented in Appendix A1 are shown as an illustration.

The last part of the paper is devoted to the discussion. The "Gradex" statistical extrapolation method based on assumptions concerning the asymptotic behavior of flood peak distributions as well as the limits of the common practice consisting of fitting theoretical statistical distributions to small peak discharge series are questioned in light of the results presented herein.

This work remains partial: the influence of the spatial heterogeneity of rainfall is for instance not studied.

2. Basic concept

2.1. The key idea: the simplification of the rainfall-runoff process in a DFFD framework

Let us begin with a very simple representation of a rainfall event: a rectangular pulse (i.e. a constant intensity event over a given duration). This is the representation selected in many papers dealing with DFFD, including the paper of Eagleson (1972).

In this very simple case, any rainfall-runoff model can be summarized in the following form, as far as the peak discharge is concerned:

$$Y = CX \tag{1}$$

The peak discharge Y is a given proportion of the rainfall intensity X. If the baseflow is neglected, this proportion C, a kind of runoff rate, is included in the interval $[0, 1]$ and may depend on the duration of the rainfall event, the rainfall-runoff dynamics and the state of the watershed (antecedent soil moisture for instance). Note that the condition imposed on C implies reasonable conditions concerning the rainfall-runoff process: the peak discharge can not be a negative value and can not exceed the intensity of the rainfall event. If $p(x)$ is the probability density function of the rainfall intensity X, then the survival function of Y has the following form:

$$P(Y \geq y) = F(y) = \int_0^1 \int_{y/c}^{\infty} p(c|x)p(x)dxdc \tag{2}$$

where $p(c|x)$ is the conditional density of C given X. We will now try to find an approximation of $F(y)$ as y tends to infinity for various types of density functions $p(x)$ and with as few assumptions as possible concerning the function $p(c|x)$. An exponential density function will be considered for X in the following text. Results concerning the extreme value type II distribution are presented in the Appendix C1.

2.2. C is independent of X and X is exponentially distributed

Let us begin with the very simple case where C is uniformly distributed over the interval $[0, 1]$. The survival function of the flood peak Y then has the following form:

$$F(y) = \int_0^1 \int_{y/c}^{\infty} \lambda e^{-\lambda x} dx dc \tag{3}$$

or

$$F(y) = \int_0^1 e^{-\lambda y/c} dc. \tag{4}$$

We are looking for an approximation of this integral when y tends to infinity. One method consists of finding an upper and a lower boundary for this integral have the same limit when y tends to infinity. Let us first note that $e^{-\lambda y/c}$ is an increasing function of c. Therefore, obviously:

$$F(y) \le e^{-\lambda y} \int_0^1 dc \tag{5}$$

or

$$\log F(y) \le -\lambda y. \tag{6}$$

Moreover, the function $e^{-\lambda y/c}$ takes positive values over the interval $[0, 1]$. Then

$$F(y) \ge \int_{\epsilon}^1 e^{-\lambda y/c} dc \tag{7}$$

for any ϵ in $[0, 1]$. Recalling that $e^{-\lambda y/c}$ is an increasing function of c:

$$F(y) \ge e^{-\lambda y/\epsilon} \int_{\epsilon}^1 dc \tag{8}$$

or

$$F(y) \geq e^{-\lambda y/\epsilon}(1-\epsilon) \tag{9}$$

or

$$\log F(y) \geq -\lambda y \left(\frac{1}{\epsilon} - \frac{\log(1-\epsilon)}{\lambda y} \right) \tag{10}$$

for any $\epsilon<1$, as close to 1 as wished, the right hand term of Eq. (10) tends to $-\lambda y/\epsilon$ when y tends to infinity. We can then write:

$$-\lambda y \geq \log F(y) \geq \frac{-\lambda y}{\epsilon} [1+o(1)] \tag{11}$$

where o(1) stands for a function of y that tends to 0 when y tends to infinity. As y tends to infinity the lower boundary of the inequality (11) can be taken as close to the upper one as desired. In other words, $\log F(y)$ converges to $-\lambda y$ as y tends to infinity (i.e. the flood peak distribution will asymptotically appear as a straight line with slope λ on a conventional semi-logarithmic plot). According to the inequality (11), $\log F(y)$ has the following asymptotic shape.

$$\log F(y) = -\lambda y[1+o(1)]. \tag{12}$$

Equation (12) is equivalent to

$$F(y) \approx h(y)e^{-\lambda y} \tag{13}$$

or

$$\log F(y) \approx -\lambda y + \log h(y) \approx -\lambda y \left(1 - \frac{\log h(y)}{\lambda y}\right) \tag{14}$$

with $h(y)$ any function verifying $\lim_{y\to\infty} \log h(y)/(\lambda y)=0$. In the particular case where C is uniformly distributed over the interval $[0, 1]$ and independent of Y, it can be shown that $h(y)=1/(\lambda y)$ (see Appendix B1).

The same demonstration leads to a similar result if C has any distribution independent of X and Y with a strictly positive density over the interval $[0, 1]$:

$$F(y) = \int_0^1 p(c) \int_{y/c}^{\infty} \lambda e^{-\lambda x} dx dc \tag{15}$$

$$F(y) = \int_0^1 p(c) e^{-\lambda y/c} dc \tag{16}$$

$$F(y) = \int_0^1 e^{-y(\lambda/c + \log[p(c)]/y)} dc \tag{17}$$

In this case, whatever the function $p(c)$, the term $\log[p(c)]/y$ tends to 0 when y tends to infinity for any value of c. Integrals (4) and (17) have the same asymptotic behavior as y tends to infinity. Figures 1 and 2 show the distributions of $Y = CX$ obtained through Monte Carlo simulations with X exponentially distributed (mean equal to 1) and C being a Beta random variable taking values in the interval $[0, 1]$. Different values for the mean and the variance of this Beta variable have been tested. In each case, the slope of the distribution of Y appears to converge, even though very slowly, towards 1: the slope of the distribution of X (continuous lines in Figs. 1A and 2A). It may appear more clearly on Figs. 1B and 2B that the slope of the distribution of $-\log F(y) - y$ tends towards 0 as $-\log F(y)$ or y tend to infinity. The asymptote when C is uniformly distributed is $-\log F(y) - y = \log(y)$ (see Appendix B1).

Let us finally note, to be more general, that the function $p(c)$ can take non-zero values over a reduced interval $[c_1, c_2]$ with $0 \le c_1 \le c_2 \le 1$. In this case the preceding developments will obviously lead to the following asymptotic relation:

$$\log T(y) = \frac{\lambda y}{c_2}[1 + o(1)] \tag{18}$$

3. Generalization of the previous results

3.1. Rainfall events still have a constant intensity but the density function of C may depend on X

The statistical independence between C and X, that is the independence between the rainfall intensity and the "runoff rate" is an unrealistic assumption if the model is supposed to simulate a runoff process. In the case of an "infiltration excess" also called "Horton" runoff process, C and X are clearly linked. If a "saturation excess" process is simulated, C is related to the rainfall amount of each event. The evolution of the density $p(c|x)$ with x will, in this last case, depend on the relation between the intensity and the rainfall amount of an event. Since rectangular pulse events are considered, it will depend on the relation between the intensity and the duration of the rainfall events. The conditional expectancy of the rainfall volume of an event may decrease as its intensity increases in the very tricky situation where the rainfall event duration and intensity are highly negatively correlated (case tested by Goel et al., 2000). But usually, even if the rainfall event intensity and duration are slightly negatively correlated, the expectancy of the rainfall volume and therefore, the expectancy of the "runoff rate" will increase as the rainfall intensity of an event increases. Moreover, the event intensity distribution seems to control the asymptotic shape of the volume distribution (De Michele and Salvadori, 2003). It can therefore be foreseen, for "realistic" rainfall stochastic model and rainfall-runoff model combinations, that the runoff rate expectancy will have a general tendency to grow as the magnitude of the intensity of the rainfall event grows.

The function $p(c|x)$ can have two types of behaviors as x tends to infinity. It can either converge towards a limit density function $p^*(c)$ defined over an interval $[c_1, c_2]$ with $c_1 < c_2$ or it can concentrate around one value c_2. In this last case, Y is asymptotically equal to $c_2 X$: i.e. the asymptotic distribution of the flood peak discharges is the distribution of the rainfall intensities multiplied by c_2. Let us add that if $c=1$ belongs to the domain of the possible, which is the case for most hydrological models if the rainfall spatial heterogeneity is not considered, than $c_2=1$. In other words, the asymptotic statistical distribution of the peak discharges of a watershed obtained through a DFFD framework will either have the same shape parameter than the distribution of the rainfall event intensities or be the distribution of the rainfall intensities.

This result confirms and extends the conclusions drawn from the detailed analysis of some specific DFFD frameworks: i.e. for DFFD frameworks in which rainfall events are considered as rectangular pulses, asymptotically "the shape parameter of the flood distribution is the same as that of the rainfall (intensity) distribution" (De Michele and Salvadori, 2002). Figure 3 illustrates this for two previous DFFD tools. Note that in the study of Eagleson, the runoff is supposed to be produced on a part of the total catchment area: the direct runoff producing area. The specific peak discharges have been computed considering this direct runoff producing area.

3.2. Generalization to any type of rainfall representation

In the more general case, where the temporal variations of rainfall intensities during an event are considered, the peak discharge expressed in mm/h can be higher than the mean intensity of the event. The coefficient C in Eq. (1) is not limited to 1 any more. But a formula, comparable to Eq. (1) can be proposed to summarize the rainfall-peak discharge relation.

$$Y = CY_m \tag{19}$$

where Y_m represents the peak discharge of the watershed obtained for a runoff coefficient equal to 1: i.e. if the watershed is supposed to be impervious. Again, in Eq. (19), the coefficient C is included in the interval $[0, 1]$ if the baseflow is not considered. Let us note that there is a link between the distribution of Y_m and the statistical characteristics of the rainfall and particularly the so-called intensity-duration-frequency curves. This relation depends on the transfer function of the flood flows on the watershed.

If this transfer function is linear, the peak discharge is highly correlated with the maximum mean rainfall intensity over a duration generally lower than the time of concentration of the watershed (see Fig. 4a). Hence, the statistical distribution of Y_m is the distribution of the maximum mean rainfall intensities over this duration (Fig. 5a). Just recall that the high correlation between peak discharge values and mean rainfall intensity over a given duration is at the basis of the development of the well known rational method.

If the transfer function is not linear, the link between the distribution of Y_m and the Intensity-duration-frequency curves is less direct (Fig. 5b). When the "kinematic wave" model is used, the time of concentration of the watershed decreases as the discharge and the water mean velocity increases. For the chosen theoretical watershed and parameters, this time of concentration appears to be about 1 h for a discharge equal to 20 mm/h and 30 min for a discharge equal to 120 mm/h. Looking at Fig. 5b, it appears that the maximum mean intensity of the rainfall event over a duration close to the time of concentration of the watershed still controls the shape of the distribution of Y_m in the non-linear transfer case. But then, the time of concentration depends on the discharge or the return period.

Recalling the theoretical results of the first part of this paper, the flood peak discharge distribution obtained with any rainfall-runoff model should asymptotically either (1) have the same shape parameters than the distribution of $c_2 Y_m$ if the density function of the coefficient C tends to a dense function on the interval $[c_1, c_2]$, or (2) be the distribution of $c_2 Y_m$ if the density function of C concentrates around c_2 as Y_m tends to infinity.

Looking in detail at the rainfall-runoff simulations (Figs. 4b, 6, see Appendix A1 for a description of the model), it appears that the expectancy of the parameter C increases and seems to tend towards 1 as Y_m increases for the chosen DFFD framework. The asymptotic distribution of the peak discharge is therefore the distribution of Y_m. This convergence is very slow when the standard "soil conservation service" (SCS) model is used and does not appear clearly on the Fig. 6a. Does the convergence of C towards 1 depend on the rainfall-runoff used? Hydrological models simulating infiltration or saturation excess runoff generating processes will all show an increase of the runoff coefficient expectancy with the mean rainfall intensity over a given duration. But, the hydrological model may have a major influence on the shape of the flood peak distribution and on the convergence speed of c towards its asymptote. To illustrate this, a second series of computations where conducted with the same rainfall-runoff model including a modified version of the SCS model (see Appendix A1). The conventional SCS model simulates a very progressive convergence of the runoff coefficient towards 1 as the rainfall amount increases, behavior which seems not to be in accordance with some recent observations (Gaume et al., 2004). The proposed modified version of the SCS model simulates a rapid evolution of the runoff coefficient over a given rainfall amount threshold. This behavior induces a change of the convergence speed of the flood peak distribution towards its asymptote (Fig. 6). The flood peak distribution resulting from the DFFD framework has a strange "S" shape with two extremes dominated by the pre and post-threshold behaviors of the rainfall-runoff model and a large transition phase. This is of course a purely theoretical example, but it shows that flood peak distributions may have a large variety of shapes depending on the dynamics of the rainfall-runoff process. Considering the distance between the actual and the asymptotic distributions for low return periods, the range of possibilities is certainly quite large.

4. Discussion

4.1. On the "Gradex" method

The result concerning the asymptotic behavior of flood peak distributions has some similarities with the "Gradex" theory (Naghettini et al., 1996; Guillot and Duband, 1967), popular in France and in some other countries. This theory states (1) that the distributions of the daily rainfall amount is exponential, (2) that over a given return period value, the mean daily discharge distribution will have the same slope on a semi-log plot than the daily rainfall amount, if both are expressed in the same unit, and (3) that the ratio between mean daily and peak discharges is independent of the return period. But there are some differences.

Firstly, the slope of the peak discharge Y appears rather linked to the slope of the distribution of Y_m which is more related to the so called intensity-duration-frequency curves than to the slope of the mean daily rainfall amount distribution. Both would be asymptotically equivalent if the ratio between the quantiles of Y_m and of the mean daily rainfall amounts were constant, which is generally not the case.

Secondly, we have presented here an asymptotic result. The shape of the flood peak distribution and the convergence speed towards its asymptote are highly dependent on the dynamics of the rainfall-runoff process summarized in the density function $p(c|x)$. A large variety of FPD shapes can be produced especially if there are thresholds in the rainfall-runoff relation as illustrated herein or previously by Sivapalan et al. (1990). The Gradex is not necessarily the maximum possible slope of the flood peak distribution.

4.2. About statistical extrapolations

The debate on the estimation of extreme values in hydrology is still lively (Koutsoyiannis, 2004; Klemes, 2000). The results presented herein provide some elements of discussion on this issue. Concerning the quantile estimations of very large return period floods, the distribution of the maximum mean rainfall intensity over a duration of the order of the time of concentration of a watershed should be considered as the possible flood peak asymptotic distribution. Concerning the estimation of medium return period flood quantiles, typically 50 to 500 years, the present paper does not lead to any new proposal. But the awareness of the distance between the asymptotic distribution and the low return period flood quantiles gives an idea of the range of the possibilities for the shape of the flood peak distributions (i.e. for the way the actual distribution will converge towards its asymptote). Let us illustrate this last idea with the real example of the flood peak distribution of a small river (watershed area of 40 km^2) located in the south of France (see Fig. 7). This example will also show that the simulated "S" shaped distribution (Fig. 6) is realistic. 24 years of measured discharges are available on the Clamoux river. On 13 November 1999 occurred an extreme flood event whose estimated peak specific discharge (16 mm/h) lies far over the measured ones. The extrapolation based on the measured series leads to a return period for this discharge of a few thousand years if the EV II distribution is used and of some hundred million years if the EV I distribution is used. But some clues indicate that the return period can not be so high: comparable floods seem to have been observed on the same river during the last century. The 100-year maximum mean rainfall intensity over two hours, estimated time of concentration of the Clamoux watershed during the 1999 flood, is about 70 mm/h in this area. According to the preceding conclusions, the return period of the 1999 peak specific discharge can therefore be of the order of 100 to a few hundred years. This relatively low return period seems to be confirmed by the analysis of the data existing on the major historical flood since 1850 as illustrated on Fig. 7 (Payrastre et al., 2005).

This example is typical for small watersheds in the French Mediterranean area (Payrastre et al., 2005; Gaume et al., 2004). The conclusion is that the "range of the possibilities" for the shape of flood peak distributions is large and, of course, not limited to the theoretical distributions generally used for extrapolation purposes, as for instance the extreme value distributions of type I or II. Thus, how meaningful is it to extrapolate tendencies identified on short series of data for the estimation of larger return-period flood quantiles? As mentioned in the introduction, DFFD tools due to their inherent simplifications, can hardly be considered as an efficient alternative to the conventional statistical extrapolation methods. As a conclusion, the only way to reduce the large uncertainties on estimated flood quantiles is to enlarge the studied series of data using the available information on historical floods as illustrated here and/or combining various data sets in a regional approach.

5. Conclusions

In summary, it has been shown herein that:

(a) The asymptotic statistical distribution of flood peaks Y obtained through a DFFD approach is of the same type and has the same shape parameter than the distribution of the rainfall mean intensity X for simple DFFD frameworks, i.e. of the maximum possible peak discharge Y_m for DFFD accounting for the temporal variations of the intensity during the rainfall event. Of course, Y, X and Y_m must have the same units for this result to be valid. In other words, the distribution of Y will appear linear and with the same slope as the distribution of X or Y_m on a semi-log plot for exponential type distributions or on a log-log plot for "extreme value" of type II distributions. If the maximum possible value of C is c_2 rather than 1, the same conclusions can be drawn for Y/c_2.

(b) From a practical point of view the asymptotic properties presented herein shed a new light on previous DFFD results as illustrated in Fig. 3. They also reveal that the distribution of Y_m or more simply of the mean maximum rainfall intensity over a duration of the order of the time of concentration of a watershed should be considered and used as a guideline for any extrapolation of a flood peak distribution, especially for large return periods. It should be nevertheless taken into account that the impact of the rainfall spatial heterogeneity has not been considered herein.

Appendix A1: Presentation of the rainfall-runoff model used in the DFFD numerical simulations

The rainfall-runoff simulation results presented here have been obtained with a simplified rectangular shaped watershed composed of two rectangular slopes and a central river reach having a rectangular cross-section (see Fig. A1). The main characteristics of the rainfall-runoff model used are as follows: 1) the flood flows are assumed to be essentially composed of surface runoff water, and other sources are set aside, 2) The SCS ("soil conservation service") model is used to calculate the evolution of the mean runoff coefficient on each sub-watershed during the storm event (see Eq. A1), and 3) the flood flows can be either routed through the watershed using a linear transfer function or the "kinematic wave" model.

$$C_t = 1 - \left(\frac{S}{(P_t + 0.8S)}\right)^2. \tag{A1}$$

The standard SCS model simulates a progressive and asymptotic growth of the runoff coefficient C_t towards 1 as the rainfall amount increases (see Fig. A2). This SCS function shape has been selected for mathematical reasons: an asymptotic convergence is the only possible solution for functions including one parameter only. But this asymptotic dynamics does not correspond to some recent hydrological observations (Gaume et al., 2004, 2003). Therefore, an other model including a threshold has been tested: the SCS model is used to compute the runoff rate C_t until it reaches 30% and the runoff rate is set equal to 1 over this threshold (Fig. A2).

Appendix B1: Shape of the function $h(x)$ when C is uniformly distributed over the interval $[0, 1]$

The mathematical developments of this appendix are due to Alain Mailhot of the Institut National de la Recherche Scientifique (Québec, Canada) who suggested them during a discussion about the content of the present paper.

We are looking for an approximation of the following integral as y tends to infinity:

$$F(y) = \int_0^1 e^{-\lambda y/c} dc. \tag{B1}$$

Changing the variable in this integral $u = \lambda y/c$ and $du = -\lambda y/c^2 dc$ leads to:

$$F(y) = -\lambda y \int_{+\infty}^{\lambda y} \frac{e^{-u}}{u^2} du. \tag{B2}$$

This integral can be evaluated using integration by parts:

$$F(y) = -\lambda y \left[\frac{e^{-u}}{u} \right]_{+\infty}^{\lambda y} + \lambda y \int_{+\infty}^{\lambda y} \frac{e^{-u}}{u} du \quad \text{(B3)}$$

or

$$F(y) = e^{-\lambda y} - \lambda y \int_{\lambda y}^{+\infty} \frac{e^{-u}}{u} du. \quad \text{(B4)}$$

The second term of Eq. (B4) has a well known Taylor expansion:

$$F(y) = e^{-\lambda y} - \lambda y \frac{e^{-\lambda y}}{\lambda y} \left(1 - \frac{1!}{\lambda y} + \frac{2!}{(\lambda y)^2} - \ldots \right) \quad \text{(B5)}$$

so

$$F(y) = \frac{e^{-\lambda y}}{\lambda y} \left(1 - \frac{2!}{\lambda y} + \frac{3!}{(\lambda y)^2} - \ldots \right). \quad \text{(B6)}$$

We can then conclude that when y tends to infinity:

$$F(y) \approx \frac{e^{-\lambda y}}{\lambda y}. \quad \text{(B7)}$$

Appendix C1: C is independent of X and the statistical distribution of X is an extreme value distribution of type II (Fréchet)

The density function of the rainfall intensities has the following form:

$$p(x) = \frac{\alpha}{b} \left(\frac{x-a}{b} \right)^{-(\alpha+1)} e^{-\left(\frac{x-a}{b}\right)^{-\alpha}} \quad \text{(C1)}$$

HESSD

2, 1835–1864, 2005

On the asymptotic behavior of flood peak distributions

E. Gaume

with α>0. If $p(c)$ is the density function of the runoff rate C, the survival function of the peak discharges is:

$$P(Y > y) = F(y) \int_0^1 p(c) \left(1 - e^{-\left(\frac{y/c-a}{b}\right)^{-\alpha}}\right) dc. \tag{C2}$$

The term $e^{-\left(\frac{y/c-a}{b}\right)^{-\alpha}}$ tends to $1 - \left(\frac{y/c-a}{b}\right)^{-\alpha}$ when y tends to infinity. Then

$$F(y) \approx \int_0^1 p(c) \left(\frac{y/c-a}{b}\right)^{-\alpha} dc. \tag{C3}$$

This integral can be furthermore simplified when y tends to infinity:

$$F(y) \approx \int_0^1 p(c) \left(\frac{y}{cb}\right)^{-\alpha} dc \tag{C4}$$

or

$$F(y) \approx \int_0^1 p(c) c^{\alpha} \left(\frac{y}{b}\right)^{-\alpha} dc \tag{C5}$$

which is equivalent to

$$F(y) \approx \left(\frac{y}{b}\right)^{-\alpha} \int_0^1 p(c) c^{\alpha} dc. \tag{C6}$$

α is greater than 0 by definition of the EV II distribution and $p(c)$ is the density function of a variable defined over the interval $[0, 1]$:

$$\int_0^1 p(c) dc = 1. \tag{C7}$$

Necessarily

$$0 \le \int_0^1 p(c) c^{\alpha} dc \le 1. \tag{C8}$$

Let A be the limit of this integral when y tends to infinity, $A \in [0, 1]$:

$$A = \lim_{y \to \infty} \left(\int_0^1 p(c) c^{\alpha} dc \right). \tag{C9}$$

The result is much more simple than the one obtained in the exponential case. Unless A is equal to zero, which is non-realistic for a rainfall-runoff model since it implicates that c tends to zero when y tends to infinity, the asymptote of the survival function of Y is a EV II function with the same shape parameters α as the rainfall intensity survival function.

$$F(y) \approx A \left(\frac{y}{b} \right)^{-\alpha} \tag{C10}$$

References

Arnaud, P. and Lavabre, J.: Using a stochastic model for generating hourly hyetographs to study extreme rainfalls, Hydrol. Sci. Journal, 44, 433–446, 1999. 1837

Arnaud, P. and Lavabre, J.: La modélisation stochastique des pluies horaires et leur transformation en débits pour la prédétermination des crues, Revue et Sciences de l'Eau, 13, 441–462, 2000. 1837

Blaskova, S. and Beven, K.: Flood ferquency estimation by continuous simulation of a catchment treated as ungauged (with uncertainty), Water Resour. Res., 38, 1–14, 2002. 1837

Blaskova, S. and Beven, K.: Flood ferquency estimation by continuous simulation of subcatchment rainfalls and discharges with the aim of improving dam safety assessment in a large basin in the Czech Republic, J. Hydrol., 292, 153–172, 2004. 1837

Cameron, D., Beven, K., and Naden, P.: Flood frequency estimation by continuous simulation under climate change (with uncertainty), Hydrol. Earth Syst. Sci., 4, 393–405, 2000, SRef-ID: 1607-7938/hess/2000-4-393. 1837

De Michele, C. and Salvadori, G.: On the derived flood frequency distribution : analytical formulation and the influence of the antecendent soil moisture condition, J. Hydrol., 262, 245–258, 2002. 1837, 1838, 1844

De Michele, C. and Salvadori, G.: A generalized Pareto intensity-duration model of storm rainfall exploiting 2-Copulas, J. Geophys. Res., 108, 4067, doi:10.1029/2002JD002534, 2003. 1843

Diaz-Granados, M., Valdes, J., and Bras, R.: A physical based flood frequency distribution, Water Resour. Res., 20, 995–1002, 1984. 1837

Eagleson, P.: Dynamics of flood frequency, Water Resour. Res., 14, 878–898, 1972. 1836, 1837, 1838

Gaume, E., Livet, M., and Desbordes, M.: Study of the hydrological processes during the Avene river extraordinary flood (south of France): 6–7 October 1997, Phys. Chem. Earth, 28, 263–267, 2003. 1850

Gaume, E., Livet, M., Desbordes, M., and Villeneuve, J.-P.: Hydrological analysis of the river Aude, France, flash flood on 12 and 13 November 1999, J. Hydrol., 286, 135–154, 2004. 1846, 1848, 1850

Goel, N., Kurothe, R., Mathur, B., and Vogel, R.: A derived flood frequency distribution for correlated rainfall intensity and duration, J. Hydrol., 228, 56–67, 2000. 1837

Guillot, P. and Duband, D.: La méthode du GRADEX pour le calcul de probabilités des crues partir des pluies, IAHS, publication 84, 1967. 1846

Gupta, V., Castro, S., and Over, T.: On scaling exponents of spatial peak flows from rainfall and river network geometry, J. Hydrol., 187, 81–104, 1996. 1837

Hashemi, A., Franchini, M., and O'Connell, P.: Climatic and basin factors affecting the flood frequency curve: Part I – A simple sensitivity analysis based on the continuous simulation approach, Hydrol. Earth Syst. Sci., 4, 463–482, 2000, SRef-ID: 1607-7938/hess/2000-4-463. 1837

Iacobellis, V. and Fiorentino, M.: Derived distribution of floods based on the concept of partial area coverage with a climatic appeal, Water Resour. Res., 36, 469–482, 2000. 1837

Klemes, V.: Tall tales about tails of hydrological distributions, I and II, J. Hydrol. Eng., 5, 225–239, 2000. 1847

Koutsoyiannis, D.: Statistics of extremes and estimation of extreme rainfall: I.Theoretical investigation, Hydrol. Sci. J., 49, 575–590, 2004. 1847

Lamb, R. and Kay, A.: Confidence intervals for a spatially generalised continuous simulation flood frequency model for Great Britain, Water Resour. Res., W07501, 40, doi:10.1029/2003WR002776, 2004. 1837

Loukas, A.: Flood frequency estimation by a derived distribution procedure, J. Hydrol., 255,

69–89, 2002. 1837
Moughamian, M., McLaughlin, D., and Bras, R.: Estimation of flood frequency : an evaluation of two derived distribution procedures, Water Resour. Res., 23, 1309–1319, 1987. 1837
Mouhous, N., Gaume, E., and Andrieu, H.: Influence of the highest values on the choice of log-poisson random cascade model parameters, Phys. Chem. Earth, 26, 701–704, 2001. 1838
Naghettini, M., Potter, K., and Illangasekare, T.: Estimating the upper tail of flood-peak frequency distributions using hydrometeorological information, Water Resour. Res., 32, 1729–1740, 1996. 1846
Payrastre, O., Gaume, E., and Andrieu, H.: Use of historical data to assess the occurrence of floods in small watersheds in the French mediterranean area, Adv. Geosci., in press, 2005. 1848
Raines, T. and Valdes, J.: Estimation of flood frequency for ungaged catchments, J. Hydraul. Eng., 119, 1138–1154, 1993. 1837
Sivapalan, M., Wood, E., and Beven, K.: On hydrologic similarity 3. A dimensionless flood frequency model using a generalized geomorphologic unit hydrograph and partial area runoff generation, Water Resour. Res., 26, 43–58, 1990. 1837
Smith, J.: Representation of bassin scale in flood peak distributions, Water Resour. Res., 28, 2993–2999, 1992. 1837

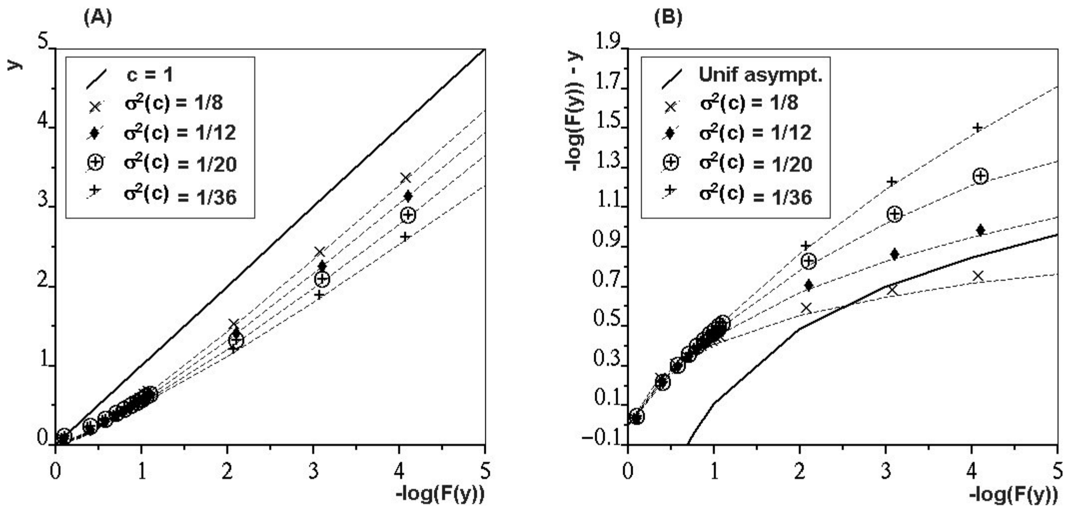


Fig. 1. Distributions of $Y = CX$ obtained with various values of the variance of C. The mean of C is equal to 0.5. The black diamonds correspond to a uniform distribution for C over $[0, 1]$.

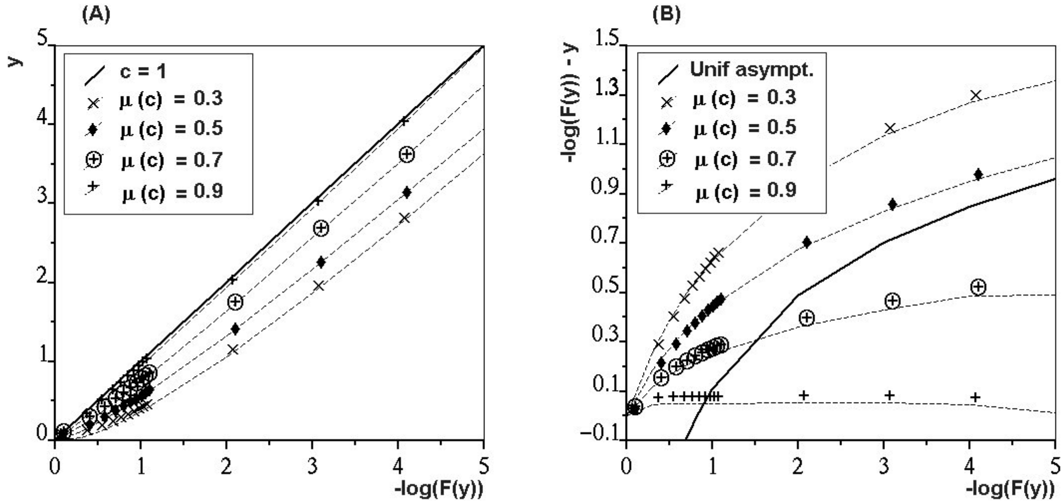


Fig. 2. Distributions of $Y = CX$ obtained with various values of the mean of C. The variance of C is equal to $1/12$. The black diamonds correspond to a uniform distribution for C over $[0, 1]$.

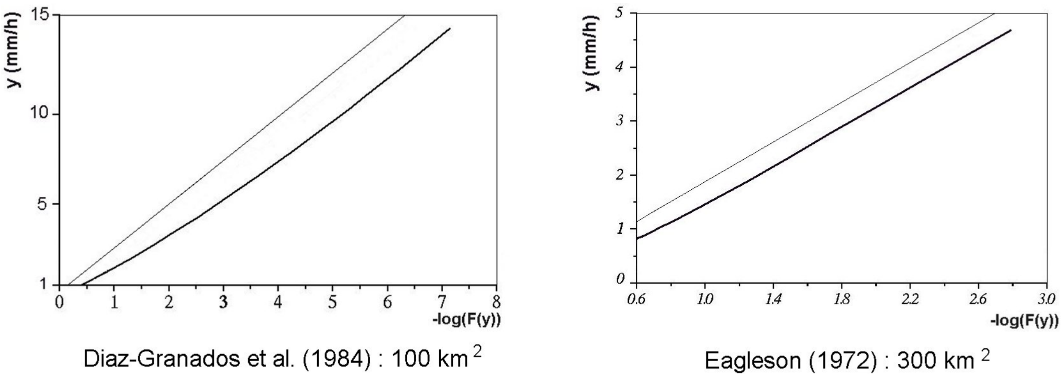


Fig. 3. Example of peak specific discharge frequency distributions obtained with two DFFD models (thick lines) and comparison with the distributions of the rainfall event intensities (thin lines).

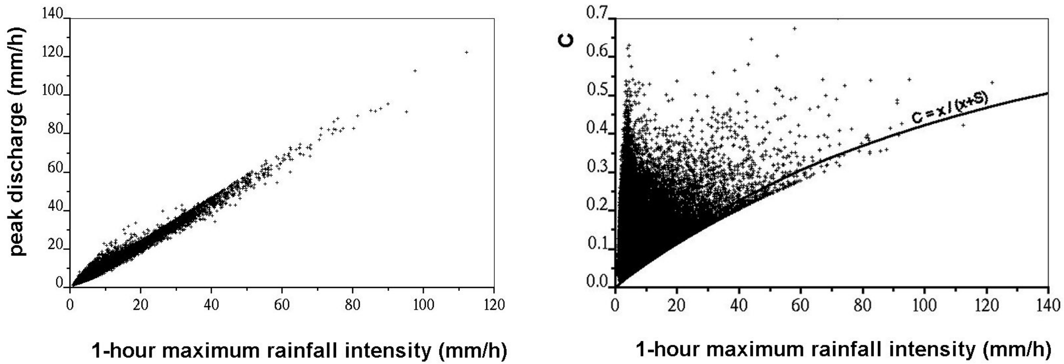


Fig. 4. Relations between the simulated peak discharge (CN=100), the simulated runoff coefficient C and the rainfall event 1-h maximum rainfall intensity.

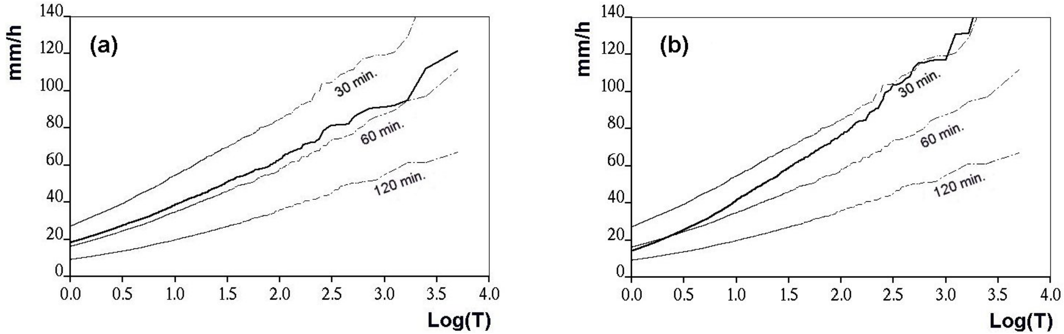
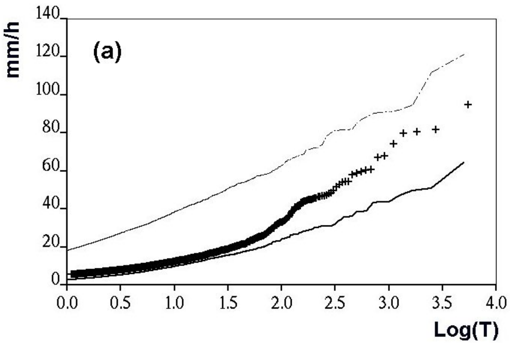


Fig. 5. Simulated flood peak (mm/h) distributions with CN=100 (bold line) compared to the rainfall event maximum mean intensities over various durations (other lines): **(a)** linear tranfer and **(b)** kinematic wave.



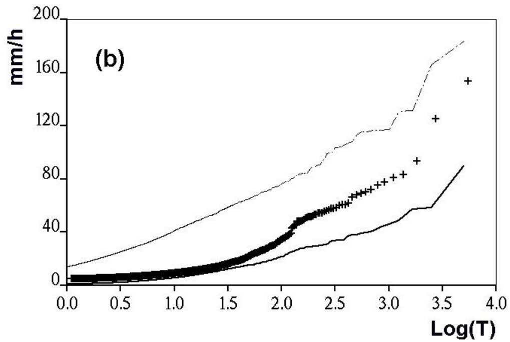


Fig. 6. Simulated flood peak (mm/h) distributions for CN=100 (thin line), CN=65 (bold line), CN=65 and modified SCS (dots): **(a)** linear tranfer and **(b)** kinematic wave.

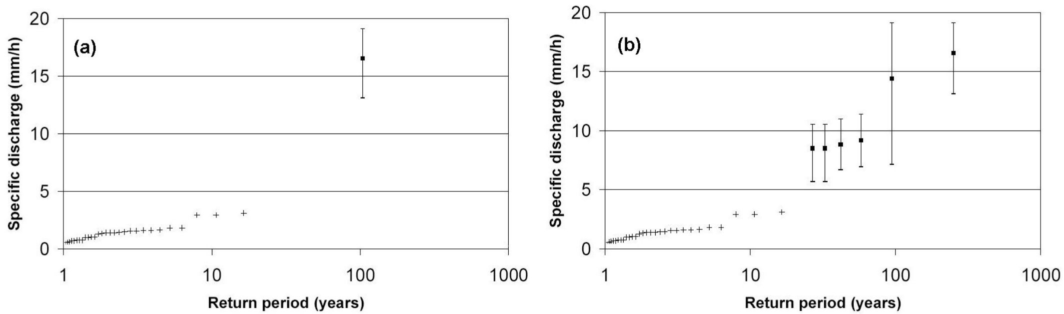


Fig. 7. Empirical flood peak specific discharge distributions of the Clamoux river (French Mediterranean area): **(a)** based on a series of 24 years of measured data, **(b)** including the estimated peak discharges of the major floods since 1850 with their estimated ranges of uncertainty.

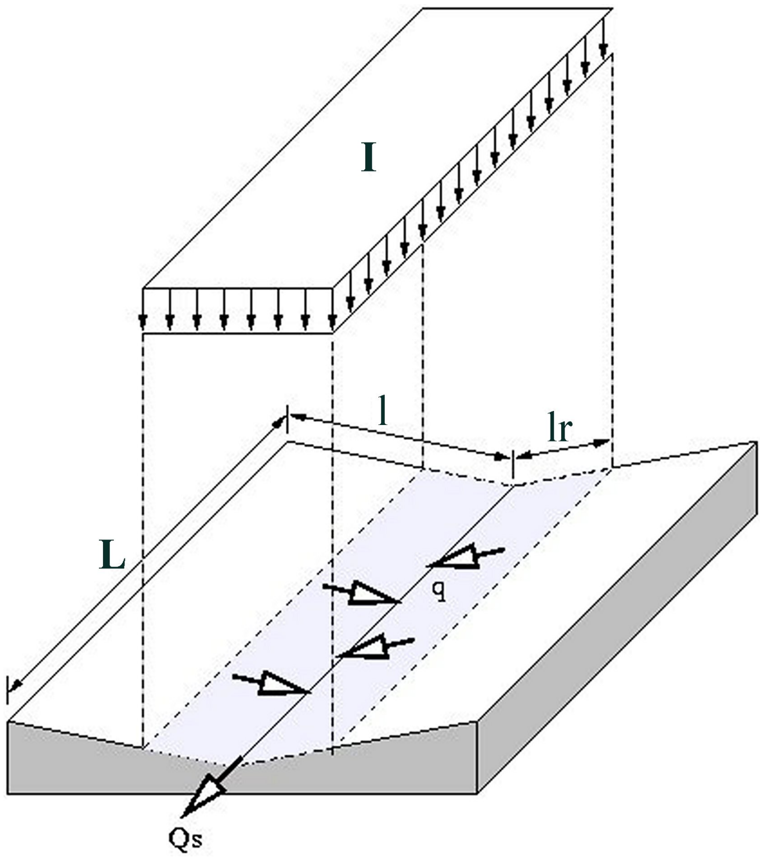


Fig. A1. Representation of a watershed in the rainfall-runoff model.

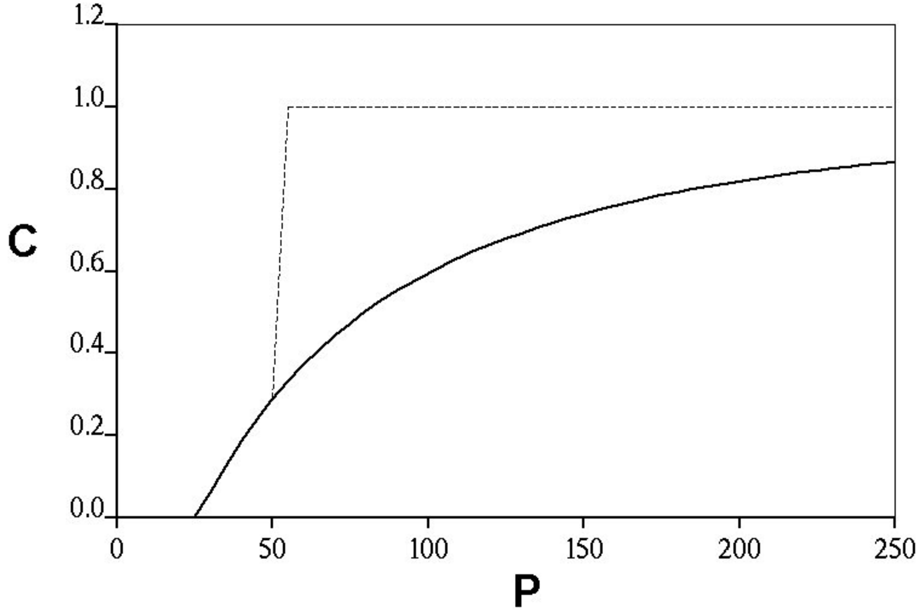


Fig. A2. Relation between the runoff coefficient C and the total rainfall amount P: standard SCS model (bold line) and modified SCS model (dotted line).

HESSD

2, 1835–1864, 2005

On the asymptotic behavior of flood peak distributions

E. Gaume

Title Page
Abstract
Introduction
Conclusions
References
Tables
Figures
Back
Close
Full Screen / Esc
Print Version
Interactive Discussion

EGU