Spatially shifting temporal points: estimating pooled within-time series variograms for scarce hydrological data

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Abstract

Estimation of pooled within-time series (PTS) variograms is a frequently used technique for geostatistical interpolation of continuous hydrological variables in spatially data-scarce regions. The only available method for estimating PTS variograms averages semivariances, which are computed for individual time steps, over each spatial lag within a pooled time series. However, semivariances computed by a few paired comparisons for individual time steps are erratic and hence they may hamper precision of PTS variogram estimation. Here, we outlined an alternative method for estimating PTS variograms by spatializing temporal data points and shifting them. The data were pooled by ensuring consistency of spatial structure and stationarity within a time series, while pooling sufficient number of data points for reliable variogram estimation. The pooled spatial data point sets from different time steps were assigned to different coordinate clusters on the same space. Then a semivariance was computed for each spatial lag by comparing all point pairs separable by that spatial lag within a pooled time series, and a PTS variogram was estimated by controlling the lower and upper boundary of spatial lags. Our method showed higher precision than the available method for PTS variogram estimation and was developed by using the freely available R open source software environment. The method will reduce uncertainty for spatial variability modeling while preserving spatiotemporal properties of data for geostatistical interpolation of hydrological variables in spatially data-scarce developing countries.

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1 Introduction

Geostatistical interpolation techniques have been extensively applied to mapping spatially continuous hydrological variables, e.g. precipitation (Carrera-Hernández and Gaskin, 2007, Durão et al., 2009, Haberlandt, 2007), stream flow (Skøien et al., 2006, 2014) and runoff (Skøien et al., 2008). Modeling spatial variability, i.e. the spatial variogram, plays a central role in geostatistical interpolation (Webster and Oliver, 2007). The precision of variogram estimation strongly depends on the number of observations, i.e. spatial data points, in a region (Oliver, 2010, Truong et al., 2012). Webster and Oliver (1992, 2007) identified the threshold for satisfactorily precise isotropic and anisotropic variogram estimation as 100 and 250 data points, respectively. Moreover, variograms computed on fewer than 50 data points exhibited little precision, whereas variograms on 400 data points were computed with great precision (Webster and Oliver, 1992, 2007).

In developing countries, hydrological data are scarce because of technological and economical constraints (Bhowmik, 2012, Bhowmik and Costa, 2014). Consequently, spatial variograms are often estimated with less than 50 data points and in turn the resulting variograms are mostly imprecise (Bhowmik and Cabral, 2011, Bhowmik and Costa, 2012, Goovaerts, 2000). Moreover, the smallest separation distance between point pairs for which semivariances are computed, i.e. the smallest spatial-lag, is very high and hence, the uncertainty for short distant spatial variability modeling remains high (Schuurmans et al., 2007).

Estimation of pooled within-time series (PTS) variograms by comparing spatial variability from multiple time steps, e.g. years (similar to pooled within-class (or strata) variograms where spatial variability from multiple attribute classes are compared (Webster and Oliver, 2007)), enables precise variogram estimation in data-scarce regions (Wagner et al., 2012). PTS variograms have been adapted to cases where the available numbers of data points for individual time steps of a hydrological time series were too few to obtain satisfactory precision (Bhowmik, 2012, Rogelis and Werner, 2012, Wagner et al., 2012). The advantages of PTS variograms over individual variograms are: (i) the number of point pairs is considerably increased, reducing the noise in empirical variograms and thus increasing the precision of variogram estimation (Rogelis and Werner, 2012), and (ii) the smallest spatial-lag is considerably decreased by including spatial variability from multiple time steps. For varying lengths of temporal data at different spatial points, some time steps may possess

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smaller spatial-lags than others. Pooling allows to include these small spatial-lags in temporally constant variogram estimation and thus to reduce uncertainties of short distant spatial variability modeling for the time steps that possess only larger spatial-lags. In turn, short distant variability can be modeled for time steps with large spatial-lags using point pairs from time steps with smaller spatial-lags (Schuurmans et al., 2007). Moreover, PTS variograms were shown to be more suitable than spatiotemporal variograms (estimated for interpolation in space-time) and mean variograms (averaging estimated non-singular individual variogram parameters, i.e. nuggets, partial sills and ranges within time series) (Gräler et al., 2011) for cases, where the spatial locations and numbers of available data points vary within a time series and do not meet the threshold for precise individual variogram estimation in any time step (Christakos, 2001, Kerry and Oliver, 2004). This is because temporal variability modeling is uncertain for variable spatial locations of data points and lengths of time series, and, as previously discussed, the estimated spatial variogram parameters for individual time steps are imprecise due to scarce data.

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Averaging empirical variograms (semivariances) (AEV), which are computed by paired comparisons in individual time steps, over each spatial-lag within a pooled time series represents the only method available for PTS variogram estimation (Gräler et al., 2011). Computation of semivariances for individual time steps, where the numbers of data points do not meet the threshold for precise variogram estimation, is erratic because of a few paired comparisons. Hence, averaging erratic semivariances may lead to an erratic semivariance for a spatial-lag within a time series and thus hamper the precision of PTS variogram estimation. Moreover, most studies focused on geostatistical interpolation of hydrological variables in regions with dense spatial data (Haberlandt, 2007, Skøien et al., 2006) whereas there is an increasing need for studies on spatial variability of hydrological variables in spatially data-scarce developing countries (Stocker et al., 2013). Hence, only the AEV method for PTS variogram estimation is insufficient for the anticipated large number of studies on data-scarce countries.

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We outline an alternative method for estimating PTS variograms by spatializing temporal data points and shifting them. We call this method "spatially shifting temporal points (SSTP)". SSTP was developed using the freely available R (R Core Team, 2014) open source software environment. We apply SSTP to estimate PTS variograms for a hydrological series in a spatially data-scarce developing country and compare it with the AEV method.

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2 Materials and Methods

2.1 Data and software

SSTP was applied to the PTS variogram estimation for “annual total precipitation in hydrological wet days (PRCPTOT)” in Bangladesh (Peterson et al., 2001, Figure 1). We used the daily precipitation data from 1948-2007 series that were collected from Bangladesh Meteorological Department (DMICCDMP, 2012). Currently, 32 rain-gauges report daily precipitation in Bangladesh, classifying the country as data scarce (Webster and Oliver, 2007) (Figure 1). Moreover, the numbers of data points exhibit an increasing coverage from 8 in 1948 to 32 in 2007, indicating variably imprecise spatial variograms (all with <50 data points) for individual time steps (Figure S1, Table S1).

The precipitation data were quality controlled and validated using the “RClimdex” routine (Peterson et al., 2001). PRCPTOT was computed for each of the time steps (year) and data points (rain-gauge) where precipitation data were available following the method described in Bhowmik (2012) and Peterson et al. (2001). In general, high values of PRCPTOT were observed at data points with high longitudes and low latitudes (southeastern part of the country) and vice versa (Figure S1). The altitudes of all data points were below 50 m and do not significantly (p=0.8) correlate with PRCPTOT in Bangladesh (Figure 1).

SSTP was developed in R (R Core Team, 2014) using the utilities of the “gstat” (Pebesma, 2004), “intamap” (Pebesma et al. 2011) and “spacetime” (Pebesma, 2012) packages.

2.2 Pooling hydrological time series

Spatial structure and stationarity indicate the strength and pattern of variability of spatial data, respectively (Kravchenko, 2003). Hence, as a PTS variogram represents a constant variability between data points within a pooled time series, spatial structure and stationarity require consistency within that time series (Gräler et al., 2011). Moreover, the number of pooled data points should ensure high enough precision for variogram estimation, i.e. the threshold for reliable variogram estimation (400) should be achieved (Webster and Oliver, 2007). Consequently, we first quantified the spatial structure of PRCPTOT in each year by computing its spatial correlation coefficients along the longitudinal and latitudinal gradients as suggested by Kravchenko (2003). The Pettitt–Mann–Whitney test was then applied to the

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correlation coefficients to identify statistically significant change points between 1948 and 2007 and thus to identify changes in the spatial structure (Kiely et al., 1998, Figure S2). The (sub)time series between the change points were extracted as time series with consistent spatial structure. Next, we checked for the stationarity of PRCPTOT within the previously extracted time series with consistent spatial structure. For the purpose, we conducted an Augmented Dickey-Fuller test for each series (Said and Dickey, 1984). The null hypothesis of the test was that PRCPTOT has a unit root in each series, where rejecting null hypothesis with statistical significance denotes stationarity. In a final step, the time series with consistent spatial structure and stationarity were checked to ensure that the numbers of pooled data points meet the threshold for reliable variogram estimation. The data points of the time series that satisfied the above three criteria were pooled and used for the PTS variogram estimation. For comparison, we also pooled the data points from 1948-2007 series, checked for stationarity and number of pooled data points and used for PTS variogram estimation. The Pettitt–Mann–Whitney test and Augmented Dickey-Fuller test were performed using the R packages “cpm” (Ross, 2013) and “tseries” (Trapletti and Hornik, 2012).

2.3 Estimation of pooled within-time series (PTS) variograms

2.3.1 Spatially shifting temporal points (SSTP)

The data point sets from different years (temporal) within a pooled time series were spatialized, i.e. assigned to different sets of coordinates (clusters) on the same space (Figure 2). Given that s is a data point location vector comprised with coordinate vector touples (x, y), t is a time (year) vector for a pooled time series, $Z(s,t)$ is the vector for computed PRCPTOT value for the data point s in year t and $(s_{i,t}, s_{j,t})$ is the separation distance, i.e. spatial lag of the point pair comprised with points s_i and s_j in year t, we first assigned the data points from the base year (t_1) of a pooled series, e.g. 1948 of the 1948-1975 series, to its original coordinates (x_{t_1}, y_{t_1}). Then coordinates for the data points of the latter years were calculated according to Eq. (1), when $(t_1+1)+4n \le t < (t_1+1)+4(n+1)$; $n \in N$ (N = natural numbers).

$$\begin{aligned}
s_{(t_1+1)+4n} &= (x_{(t_1+1)+4n} + (n+1)d), y_{(t_1+1)+4n} \\
s_{(t_1+1)+4n+1} &= (x_{(t_1+1)+4n+1} - (n+1)d), y_{(t_1+1)+4n+1} \\
s_{(t_1+1)+4n+2} &= x_{(t_1+1)+4n+2}, (y_{(t_1+1)+4n+2} + (n+1)d) \\
s_{(t_1+1)+4n+3} &= x_{(t_1+1)+4n+3}, (y_{(t_1+1)+4n+3} - (n+1)d)
\end{aligned} \quad (1)$$

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For example, for the years t = {1949, 1950, 1951, 1952} within the pooled series of 1948-1975, n=0 because $(1948+1)+4*0 \le t < (1948+1)+4(0+1)$ and hence,

$$\begin{aligned} s_{1949} &= (x_{1949}+d), y_{1949} \\ s_{1950} &= (x_{1950}-d), y_{1950} \\ s_{1951} &= x_{1951}, (y_{1951}+d) \\ s_{1952} &= x_{1952}, (y_{1952}-d) \end{aligned} \quad (2)$$

d in Eqs. (1) and (2) is a shift distance that is bigger than two-fold the largest spatial-lag available within the pooled time series, i.e. $d > 2*\max(s_{i,t}, s_{j,t})$, and shifts the data point sets of different years from each other. This shift distance was chosen because it prevents the influence of data point sets from different years on each other while estimating PTS variograms, i.e. the peripheral data points of the sets from neighboring years are separated by a distance outside of the range of the largest spatial-lag available within the pooled time series (Figure 2). Thus the shift distance represents a spatially rescaled temporal distance (1 year) between data point sets from two consecutive years that preserves the spatiotemporal properties of PRCPTOT. Note that this shift distance is different from the spatially rescaled temporal distance computed for spatiotemporal variogram estimation in Gräler et al. (2011), where temporal variability was examined on a scale analogous to spatial variability. We selected the shift distance as in Eq. (3), but the users can choose any distance that is $> 2*\max(s_{i,t}, s_{j,t})$.

$$d = 2*\max(s_{i,t}, s_{j,t}) + \max(s_{i,t}, s_{j,t})/100. \quad (3)$$

Spatial shifting of the temporal data points was performed using the R package "spacetime" (Pebesma, 2012). This allows for treating all temporal data points within a pooled time series as spatial points on the same space and thus for simultaneously binning and comparing point pairs from all years (spatial clusters) for a temporally constant spatial-lag. For example, for the pooled series of 1948-1975, point pairs with PRCPTOT observations that are separated by 100 km in each of the 25 clusters can be binned and compared simultaneously for a single empirical variogram computation (Figure 2). Consequently, the number of point pairs for comparison can be substantially increased as they are pooled from 25 clusters (years). Moreover, the point pairs in any cluster that are separated by small spatial lag, i.e. < 30 km in 1973 and 1975, are included in the temporally constant empirical variogram computation and

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thus uncertainties of short distance variability modeling for the clusters, where point pairs are only separable by larger spatial lags, are reduced.

2.3.2 Computation of empirical variograms

The empirical variograms (semivariances) were computed by simulatenous comparison of all possible point pairs from the spatially shifted points using the commonly applied Methods of Moments (MoM) (Webster and Oliver, 2007). For the point pair s_i and s_j (both treated as spatial points on the same space), the semivariance $\gamma(s_i,s_j)$ (temporally constant) is a function of the spatial lag s_i,s_j (that is not affected by actual location of data points) and was computed by Eq. (4).

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$$\gamma(s_i,s_j) = \frac{1}{2M(s_i,s_j)}\sum_{i,j}(Z(s_i)-Z(s_j))^2 . \qquad (4)$$

$M(s_i,s_j)$ is the number of point pairs that can be separated by the spatial lag s_i,s_j. Thus, SSTP uses a spatial variogram (empirical) computation method on the spatialized temporal points from a pooled time series and thus computes a temporally constant semivariance for each spatial lag. Departing from the AEV method of computing yearly semivariances for each spatial lag (in our case computing separate semivariance for each coordinate cluster) and averaging them, SSTP computes a single temporally constant semivariance using Eq. (4) by simultaneously comparing point pairs from all years that are separable by a spatial lag (see Figure S3 for details). In turns, SSTP demonstrates two advantages over the AEV: (i) SSTP pools the data points with observations for a series instead of pooling computed semivariances for each year (Figure S3) and (ii) the number of data points that actually participates in semivariance computation using Eq. (4) is substantially higher for SSTP than AEV as it computes one semivariance for a spatial-lag by comparing point pairs from all years rather than computing yearly semivariances and averaging them (Figure S3).

The upper and lower boundaries of s_i,s_j were set to the smallest and largest spatial-lags available within the pooled time series, respectively.

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$$\begin{aligned} s_1,s_2 &= \min(s_{i,t},s_{j,t}) \\ s_{n-1},s_n &= \max(s_{i,t},s_{j,t}) \end{aligned} . \qquad (5)$$

These (Eq. 5) were done to reduce the uncertainty of modeling short distant spatial variability for the time steps with large spatial-lags, i.e. by modeling variability for the smallest spatial-lag within the time series (described above) and to avoid inclusion of temporal variability as pseudo spatial variability in semivariance computation, i.e. points that are temporally apart are not paired for comparison.

In the next step, we checked for anisotropy in the spatial variability of PRCPTOT within the pooled time series. In case that anisotropy was detected, we computed the ratio between the major (A) and minor (B) axes of the anisotropy ellipse and the angle of the anisotropy (ϕ). Computation of semivariances and anisotropy parameters were performed using "gstat" (Pebesma, 2004) and "intamap" (Pebesma et al. 2011) packages of R.

2.3.3 Estimation of pooled within-time series (PTS) variograms

We estimated PTS variograms, i.e. fitted variogram models to the PTS empirical variograms (semivariances) for each pooled time series. The available variogram models were fitted to the computed semivariances by a weighted least square approach providing $M(s_i,s_j)/(s_i,s_j)^2$ as weights (see Pebesma (2004) for details). However, variogram models can also be fitted by the maximum likelihood approach as described in Marchant and Lark (2007) or by providing different weights than ours if using weighted least square approach (Pebesma, 2004). The parameters of the fitted models, i.e. nugget and sill variances, and range (a) were extracted. In case that anisotropy was detected, a was replaced by the anisotropy parameter where geometric anisotropy was made isotropic according to Eq. (5) through a linear transformation of coordinates with reference to the anisotropy ellipse described above (Oliver, 2010).

$$a = \sqrt{(A^2 \cos^2 \phi + B^2 \sin^2 \phi)}. \qquad (6)$$

2.4 Precision of variograms

Precision of the estimated PTS variograms was evaluated by (i) variogram model-fit to the empirical variograms and (ii) cross-validation of an ordinary kriging (OK) interpolation of PRCPTOT using the best-fit models (Webster and Oliver, 1992, 2007). We computed the "weighted mean of squared error (MSE)" as a model-fit statistic (Pebesma, 2004). The MSEs of the previously fitted variogram models were compared and the best-fit model with the lowest MSE was identified for each pooled series. Then the best-fit model form was used in a

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leave-one-out cross-validation of the OK interpolation of PRCPTOT in each year of the each pooled series. The OK interpolation method was chosen because it gives unbiased evaluation of how well the variogram model fits the data (Oliver, 2010). Finally, the root means squared error (RMSE) was computed for each model by comparing the observed and OK interpolated PRCPTOT values through the cross-validation (Pebesma, 2004). Note that we avoided the recalibration of the model form based on the RMSE computed through the cross-validation because RMSEs in the cross-validation can be related to many factors other than the variogram model, such as the implementation of parameters related to the search neighborhood and the specific interpolation algorithm used (Goovaerts, 2000).

For comparison, we also estimated PTS variograms for the above pooled series by applying the averaging empirical variogram (AEV) method of pooled estimation following the steps described in Gräler et al. (2011) (method c) and Pebesma and Gräler (2014). The MSEs and RMSEs were also computed for the AEV variograms following the method described above and compared with the MSEs and RMSEs of the SSTP variograms.

We provide a commented R-script as a supplementary material (SM) detailing the SSTP method for PTS variogram estimation (SM2). The sample data for reproducibility is also provided as a supplementary material (SM3).

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3 Results

Statistically significant change points were detected in 1976 and 1993, and in 1976 for the spatial correlation coefficients of PRCOTOT along the longitudinal and latitudinal gradients, respectively, within the 1948-2007 series (Figure S2). These change points indicated changes in spatial structure from 1976 and 1993. Consequently, spatial structure within the entire 1948-2007 series was inconsistent whereas the (sub)time series 1948-1975, 1976-1992 and 1993-2007 showed consistent spatial structure. The Dickey-Fuller statistics obtained for the 1948-1972, 1976-1992, 1993-2007 and 1948-2007 series were -4.5, -3.4, -5.0 and -4.0 respectively, and they were statistically significant at $p < 0.01$. Therefore, for each of these series null hypothesis was rejected and thus PRCPTOT showed stationarity. Moreover, the number of total data points within the 1948-1975, 1976-1992, 1993-2007 and 1948-2007 series met the threshold for reliable variogram estimation (Table 1). PRCPTOT values did not vary much between the pooled series though the spatial variation of PRCPTOT within the pooled time series were high (CV≥41%) (Table 1, S1).

The distance d used for spatial shifting in each of the pooled series was 1111 km (~10^0) because the largest spatial-lag available within these series was approximately 550 km (~5^0) (Figure 2, Table 1, S1). Thus the shifted peripheral data points of sets from neighboring years showed a distance >550 km, i.e. ≥(1111-550) km (Figure 2), and thus the spatiotemporal properties of PRCPTOT were preserved, i.e. the data points from a year did not influence data points from other years and temporal autocorrelation was coherent with the spatial autocorrelation of the spatialized point clusters. The smallest spatial-lags available within the three pooled series were similar and allowed for modeling spatial variability of PRCPTOT at ≤29 km (Table 1, S1).

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Anisotropy was detected in the spatial variability of PRCPTOT for all pooled series in the northwest-southeast direction ($90^0 > \phi > 0^0$ from normal north to anticlockwise) indicating a strong variability of PRCPTOT in that direction (Figure 3, S1). Moreover, 1948-1975 series depicted weak anisotropy ($A:B$ = 0.8), i.e. relatively weak variability whereas 1976-1992 and 1993-2007 series depicted strong anisotropy ($A:B$ = 0.4), i.e. relatively strong variability (Figrue 3).

The SSTP computed semivariances were much less noisy than the semivariances computed by AEV, especially for large spatial lags (Figure 3). Consequently, the PTS variograms estimated by SSTP showed better model-fit (lower MSE) and in turn entailed better performance of OK interpolation in cross-validation, showing higher precision than the PTS variograms estimated by AEV (Table 2). The "Power" (Pow) model showed the best fit for both methods in all pooled series except for the SSTP in 1948-2007 series, where the "Hole" (Hol) model showed the best fit (Figure 3). The PTS variograms estimated for the time series with inconsistent spatial structure, i.e. 1948-2007, by both methods showed lower precision than the variograms estimated for the time series with consistent spatial structure (Table 2). For the time series with consistent spatial structure, precision of PTS variogram estimation increased with the increasing number of pooled data points (Table 2).

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4 Discussion

In this paper, we developed and implemented spatially shifting temporal points (SSTP), an alternative method for estimating pooled within-time series (PTS) variograms in spatially data-scare regions. Contrasting with the available method of averaging empirical variograms (AEV) computed for individual time steps, SSTP computed empirical variograms (semivariances) by simultaneously comparing all point pairs separable by a spatial-lag within a pooled time series. Consequently, when compared to the PTS variograms estimated by AEV, SSTP variograms showed higher precision (Table 2). The numbers of available data points did not meet the threshold for satisfactorily precise variogram estimation in any of the individual time steps (year) within 1948-2007 series and hence the available numbers of point pairs for comparisons were not sufficient for reliable semivariance computation (Table S1). As a result, computed semivariances for those years were likely erratic that induced noisy and erratic semivariances when averaged by AEV method (Figure 3). Thus model fitting to AEV semivariances showed a lower goodness-of-fit and ordinary kriging (OK) interpolation of PRCPTOT using the AEV variograms showed worse performance than the SSTP variograms (Figure 3, Table 2). By contrast, SSTP computed semivariances were reliable because of subtantially higher number of comparisons than by AEV (Figure S3) and thus entailed higher precision in PTS variogram estimation. These results are in line with Webster and Oliver (1992, 2007).

Semivariances computed for small spatial-lags by SSTP and AEV methods were similar whereas semivariances for large spatial-lags were largely different (Figure 3). Moreover, semivariances computed by AEV showed much more noise at large spatial-lags than small spatial-lags. The number of erratic semivariances averaged by AEV for large spatial-lags were higher than for small spatial lags because point-pairs from more years were separable by large spatial-lags than by small spatial-lags due to data availability (Table S1). For example, point pairs from only two years (1973 and 1975) were separable by the smallest spatial-lag for 1948-1975 series whereas point pairs from 20 years were separable by the largest spatial-lag (Table S1). In addition, the numbers and spatial locations of available data points are highly variable within the pooled series and spatial variability of PRCPTOT was high (Table 1, S1, Figure S1). Hence, we argue that the average semivariances computed by AEV was representative of the small number of semivariances at small spatial-lags but unrepresentative of the large number of semivariances at large spatial-lags because of the variable number and spatial location of data points and high spatial variability of PRCPTOT. As a result,

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semivariances for large spatial lags computed by SSTP and AEV could be similar if the numbers and spatial locations of data points were the same for all time steps and spatial variability of PRCPTOT was low (Gräler et al., 2011). Moreover, for variable number and spatial locations of data points, the noise in the semivariances computed by AEV can be partly reduced if the average of the semivariances per spatial lag is weighted by the corresponding number of data points available per time step (see Figure S4 for details on weighted AEV).

The PTS variograms estimated for the 1948-2007 series (inconsistent spatial structure) showed lower precision than the variograms estimated for the series with consistent spatial structure, although PRCPCTOT was stationary within 1948-2007 series and the number of data points (higher than for the series with consistent spatial structure) met the threshold for reliable variogram estimation (Webster and Oliver, 1992, 2007) (Table 1, 2). Moreover, higher precision was obtained for PTS variogram estimation with higher number of pooled data points among the series with consistent spatial structure (Table 1, 2). However, this may also be related to the inherent spatial structure within the time series, i.e. spatial variability of PRCPTOT may be estimated with higher precision for the data points with the spatial structure observed for 1993-2007 than for 1948-1975. Furthermore, the Hole model showed the best fit for the series with inconsistent spatial structure that did not represent the variability for individual time steps (Power variability was representative as depicted by the models for consistent spatial structure). These results suggest that the consistency of spatial structure, i.e. the strength of spatial variabilty within pooled time series is crucial for PTS variogram estimation (Kravchenko, 2003) and increasing the number of pooled data points may increase the precision of PTS variogarm estimation if the spatial structure is persistent. Many studies pooled data points only by assuming the consistency of spatial structure within time series (Bhowmik, 2012, Gräler et al., 2011, Rogelis and Werner, 2012, Wagner et al., 2012). We recommend that time series should be checked for consistency of spatial structure before pooling. Notwithstanding, if the required number of data points for reliable variogram estimation is unavailable users should comply with the threshold for precise isotropic (100) and anisotropic (250) variogram estimation (Webster and Oliver, 2007).

A weaker anisotropy, i.e. variability was detected in the northwest-southeast direction for the 1948-1975 series than for 1976-1992 and 1993-2007 series (Figure 3). This is presumably because of the lower number of spatial points per year in the 1948-1975 series than in 1976-1992 and 1993-2007 series, and thus a loss of anisotropy information (Table S1). However, higher PRCPTOT values were observed in the southeast than the northeast of Bangladesh and

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a high spatial variation (average CV = 42%) was observed for 1948-1975 series (Figure S1, Table S1). Hence, it can be claimed that the anisotropy, i.e. variability of PRCPTOT was equally strong for 1948-1975 series although not captured due to lower number of spatial points per year.

The PTS variograms allowed for modeling spatial variability at ≤29 km distance for all time steps (constant) within the pooled series although the smallest spatial-lags available for many years, e.g. 1948-1950 were much higher (>95 km) (Table 1, S1). Thus, the PTS variograms estimated by SSTP reduce uncertainties for short distant spatial variability modeling for the time steps with large spatial lags. This is done by including point pairs separable by smaller spatial-lags available in any time step in empirical variogram computation. However, the smallest spatial-lag for which spatial variability can be modeled for a pooled series is inherently dependent on the availability of spatial-lags in individual time steps, i.e. at least one point pair should be separated by a small spatial-lag in a time step. For example, if the smallest spatial-lags between point pairs in all years within the 1948-1975 series were ≥100 km, spatial variability could not be modeled at ≤29 km and could only be modeled at ≥100 km. Moreover, although SSTP reduces uncertainties for short distant spatial variability modeling, it does not reduce uncertainties for spatial prediction of hydrological variables at short distant, i.e. spatial prediction is uncertain if the variable is not gauged at short distance. Thus, modeling short distant spatial variability by PTS variograms can be further improved if smaller spatial-lags are available or more point pairs are available for comparison, i.e. more point pairs in individual time steps are separable by the smallest spatial-lags (Rogelis and Werner, 2012; Schuurmans et al., 2007).

Modeling spatial variability across time should consider temporal dependence or autocorrelation (Christakos, 2001, Said and Dickey, 1984). PTS variograms estimated by AEV do not account for temporal autocorrelation as the spatial variability from time steps are averaged. Although SSTP preserves temporal autocorrelation by spatialization, i.e. spatial clusters from neighboring years are closer on space than the clusters from distant years, it also excludes temporal autocorrelation for PTS variogram estimation (spatial variability is assumed to be temporally constant). Hence, future studies should include temporal autocorrelation in PTS variogram estimation by SSTP as performed by spatiotemporal variograms (Gräler et al., 2011). Inclusion of temporal autocorrelation could be achieved by weighting spatial distances using rescaled temporal distances. This will allow for using PTS

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variograms in modeling time series across space, e.g. estimating time series structure for an ungauged location.

SSTP was developed in the freely available open source R software environment (R Core Team, 2014), and thus ensures reproducibility and wide spread application to geostatistical interpolation for resource constraint developing countries (Pebesma et al., 2012). The method is also applicable to PTS variograms estimation for geostatistical interpolation of non-hydrological spatially continuous variables in data-scarce regions. Spatiotemporal variogram estimation techniques by modeling time as a separate dimension (Gräler et al., 2011) were criticized for time series with variable spatial locations and numbers of data points (Christakos, 2001, Kerry and Oliver, 2004). This can be empirically examined if future studies compare the precision of the spatiotemporal variograms with the SSTP variograms for time series with variable lengths.

SSTP increases precision for spatial variability modeling at both short and long distances by including variability of the smallest spatial-lag within a time series and comparing many point pairs for large distances. Inclusion of external variables that correlate with the variable for interpolation, e.g. altitude with precipitation (although did not correlate in our case), will also increase the precision of PTS variogram estimation by SSTP (Diodato, 2005, Pebesma, 2006). To conclude, SSTP method can be further improved by integrating with the expert elicitation technique (Truong et al., 2013).

Author contribution

A.K.B. conceived the study. A.K.B. developed the method under supervision of P.C. A.K.B. drafted the manuscript. A.K.B. and P.C. revised the manuscript.

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Table 1. Number of data points, smallest and largest spatial-lags, and summary statistics, i.e. minimum (Min.), mean, maximum (Max.) and coefficient of variation (CV) of annual total precipitation in hydrological wet days (PRCPTOT) within the pooled time series.

Pooled time series	Number of pooled data points	Spatial lag		PRCPTOT			
		Smallest (km)	Largest (km)	Min. (mm)	Mean (mm)	Max. (mm)	CV (%)
1948-1975	441	29.16	550	17	1659	4036	42
1776-1992	465	26.61	550	84	1759	4499	42
1993-2007	475	27.51	550	29	1789	4516	41
1948-2007*	1381	26.61	550	17	1738	4516	41

* Pooled time series with inconsistent spatial structure

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Table 2. Precision statistics of the pooled within-time series (PTS) variograms estimated by spatially shifting temporal points (SSTP) and averaging empirical variograms (AEV) methods. The weighted mean of squared errors (MSE) as the variogram model-fit statistic and root means squared error (RMSE) as the ordinary kriging interpolation performance statistic are presented.

Pooled time series	MSE		RMSE	
	SSTP	AEV	SSTP	AEV
1948-1975	2.55 X 10^{7}	6.63 X 10^{8}	524.82	634.15
1776-1992	2.47 X 10^{7}	4.49 X 10^{8}	511.29	624.40
1993-2007	2.43 X 10^{7}	3.34 X 10^{8}	501.17	612.97
1948-2007*	1.07 X 10^{8}	1.56 X 10^{9}	572.06	683.32

* Pooled time series with inconsistent spatial structure

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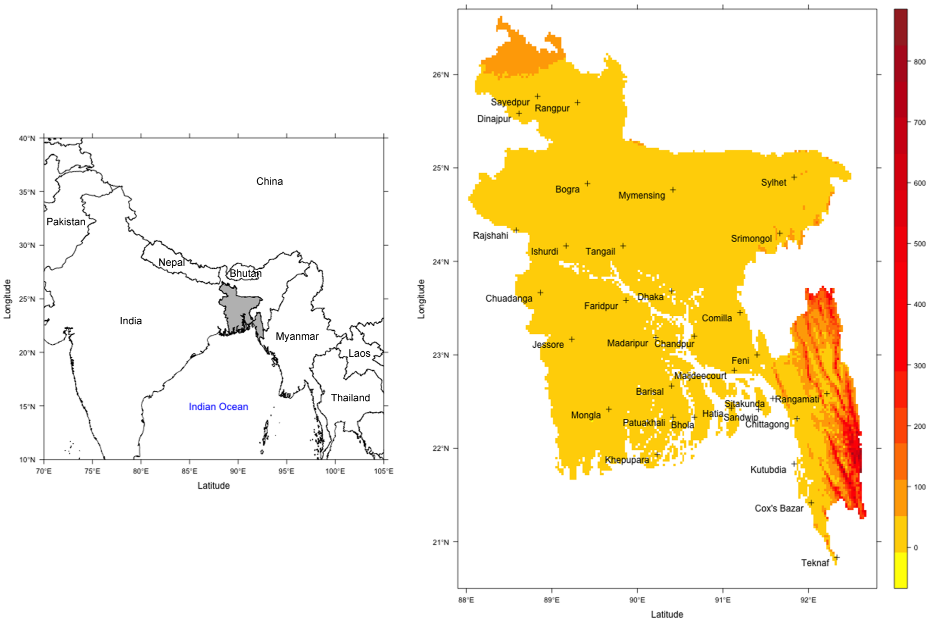

Figure 1. Geographic location of Bangladesh (left) in Southeast Asia within the coastal belt of Indian Ocean and the spatial distribution of currently active 32 rain-gauges (right) with altitudes (m above mean sea level) in the background. The coordinate reference system is WGS 1984.

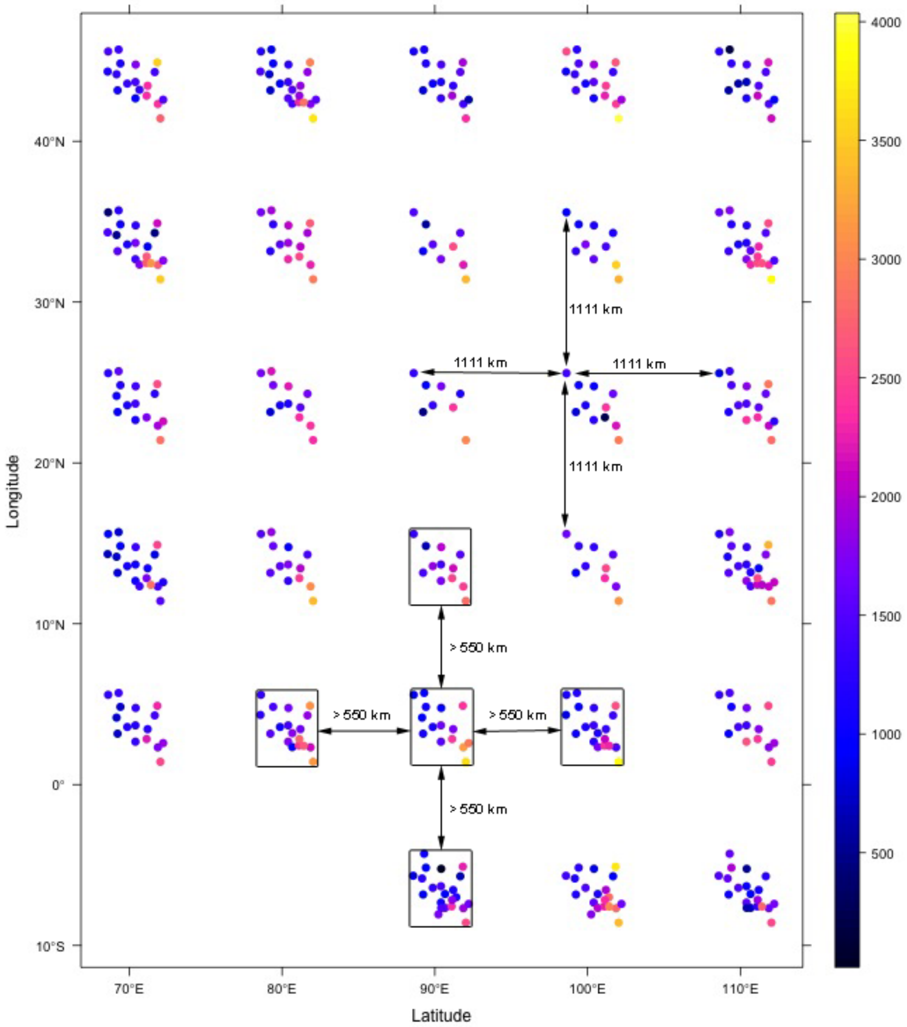

Figure 2. Spatially shifted (according to Eq. (1)) temporal data points for the pooled 1948-1975 series. Shift distance (d = 1111 km) is calculated based on the largest-spatial-lag (550 km) available within the series (Eq. 3). The data point sets from neighboring years are shifted by 1111 km (~10^0), which ensures that the peripheral points of the sets are shifted by >550 km (~5^0). The rectangles and legend indicate peripheries (convex hull) of data points in a year and PRCPTOT in mm, respectively.

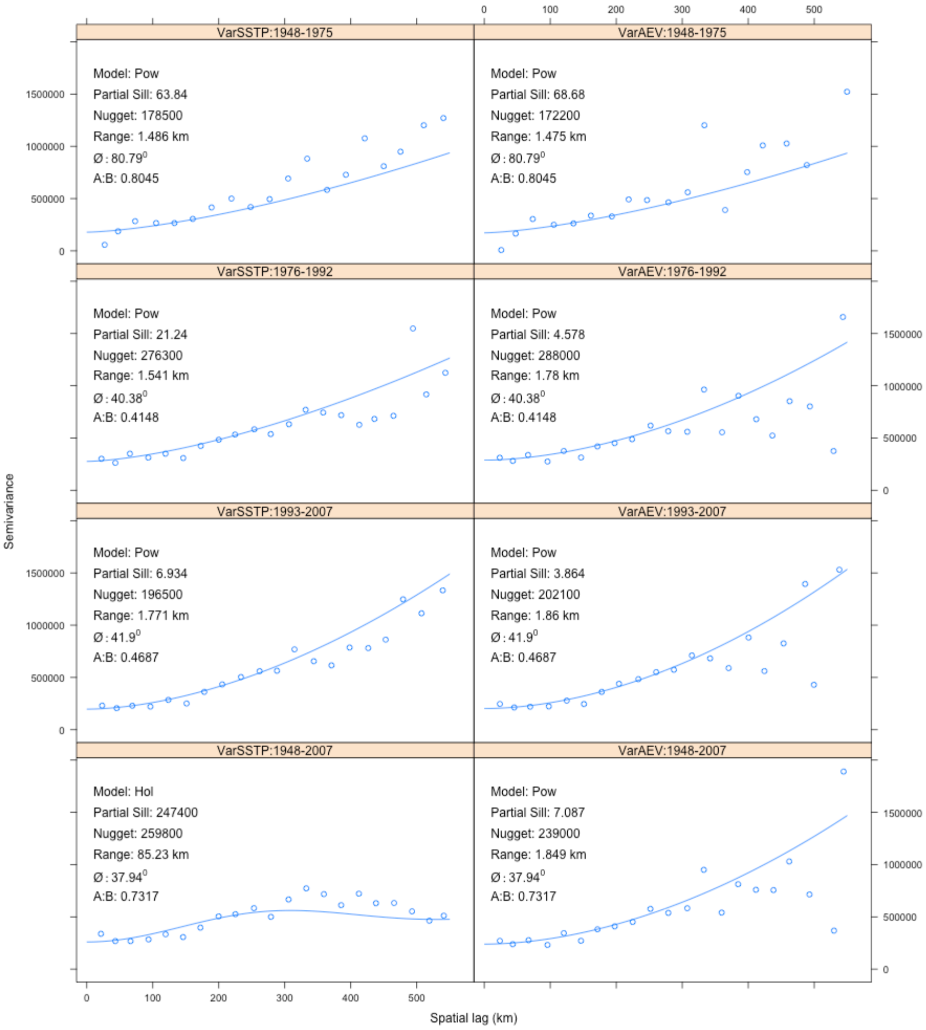

Figure 3. Estimated pooled within-time series (PTS) variograms (fitted best models to empirical variograms) by spatially shifting temporal points (SSTP) and averaging empirical variograms (AEV) methods. Fitted variogram models (“Power” (Pow) and “Hole” (Hol)), partial sill and nugget variance, range, anisotropy angle (ϕ) and the ratio between major and minor axes of the anisotropy ellipse ($A:B$) are presented. Figure captions depict variogram(Var) estimation method: pooled series.

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Additional supplementary figures

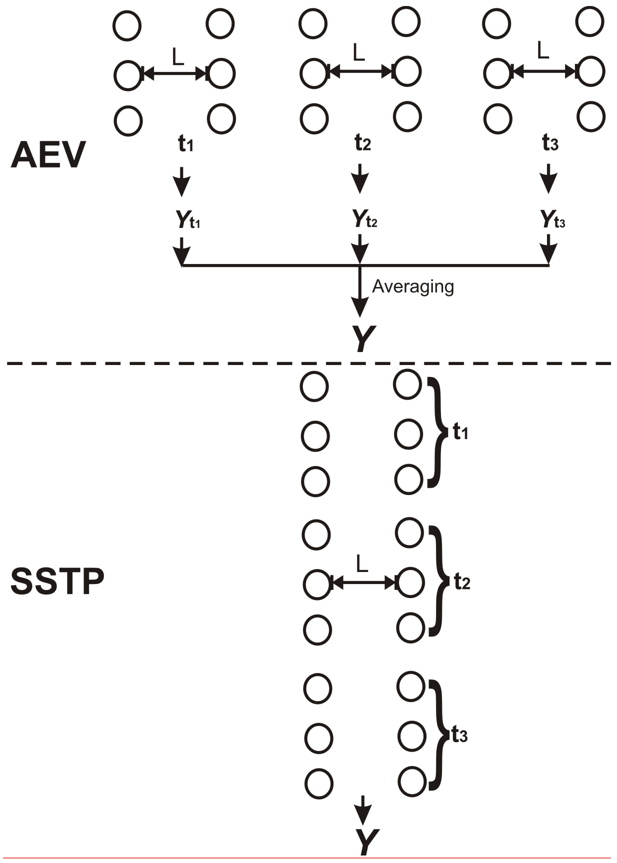

Figure S3. Methodological difference between the empirical variogram (semivariance) computation methods of averaging empirical variograms (AEV) and spatially shifting temporal points (SSTP). In each of the time steps t_1, t_2 and t_3, three point pairs are separated by the spatial lag L. AEV computes separate semivariances Y_{t1}, Y_{t2} and Y_{t3} according to Eq. (4) by comparing three point pairs available in each time step, and then averages Y_{t1}, Y_{t2} and Y_{t3} to yield the pooled semivariance Y. Whereas, SSTP compares the spatialized nine point pairs (on the same space) from three time steps simultaneously in Eq. (4) to yield Y.

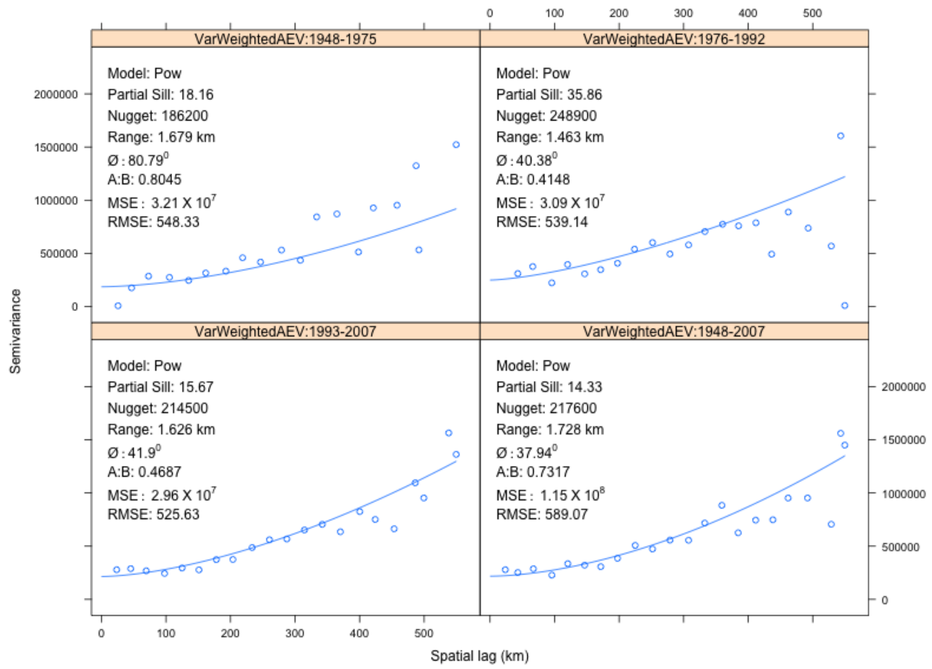

Figure S4. Estimated pooled within-time series (PTS) variograms (fitted best models to empirical variograms) by weighted averaging empirical variograms. Semivariance computation followed the same procedure as in Gräler et al. (2011) (method c) and Pebesma and Gräler (2014) but the average semivariances were weighted by the number of data points in individual time steps. Fitted variogram models ("Power" (Pow)), partial sill and nugget variance, range, anisotropy angle (ϕ), the ratio between major and minor axes of the anisotropy ellipse ($A : B$), weighted mean squared error (MSE) as a variogram model fit statistic and root means squared error (RMSE) as the ordinary kriging interpolation performance statistic are presented. Figure captions depict variogram(Var) estimation method: pooled series.