Final author comments

Title:	Dissolved oxygen prediction using a possibility-theory based fuzzy neural network
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Journal*:*	*Hydrology and Earth System Sciences*
Manuscript No.:	hess-2015-446

Response to Anonymous Referee #1

> **RC 1:** "The authors present the application of the Fuzzy Neural Network (originally proposed by Alvisi and Franchini, 2011) for the prediction of the dissolved oxygen concentration in a river. The topic is of interest and within the scope of the journal. The manuscript is well written and technically sound, even though some sections could be shortened. As properly pointed out by the authors in the conclusions, *"the proposed model refines the exiting model by (i) using possibility theory based intervals to calibrate the neural network (rather than arbitrarily selecting confidence intervals), and (ii) using fuzzy number inputs rather than crisp inputs."* Indeed, the first aspect represents a valuable, but rather limited, step forward with respect to the existing model. As far as the second aspect concerns, I really appreciate both the idea of considering the inputs of the FNN as fuzzy numbers and the approach used to define these fuzzy inputs."

Thank you for these positive comments. We will endeavour to shorten the length of the final revised manuscript where possible.

> **RC 2.** "Unfortunately, the manuscript misses to point out the benefits of using the fuzzy inputs. A comparison of the performances of the prediction model featuring fuzzy inputs with respect to the prediction model using non-fuzzy inputs is completely missing. Does the application of fuzzy inputs allows for a more accurate prediction of the DO and, most important, for a reduction of the output uncertainty? Indeed, the discussion of the result is mainly focused on the benefits of using a FNN with respect to a traditional NN in which uncertainty is disregarded, but this should not be the main task of the manuscript, given that benefits of FNN have already been pointed out in other studies, whereas the attention should be focused on the application of Fuzzy inputs."

We apologise for not including more details of the specific advantages of the fuzzy number inputs in the FNN. We did include the following statement in the original manuscript:

"The method is adapted to be able to handle fuzzy number inputs to produce fuzzy weights and biases, and fuzzy outputs. The advantage is that the uncertainties in the input observations are also captured within the model structure." (Section 1.2, page 12318, line 3).

We will make an effort to highlight this in the revised manuscript, along with the fact that, if possible (i.e. data is available as is the case with this research), the uncertainty in the input data should not be ignored and should be included in the model, and our method proposes a method to do this. Existing methods do not allow for this and thus, this is the major advantage of our research.

We will also highlight the fact that the proposed method to create the fuzzy inputs have limited assumptions of the underlying distribution of the data and relies only on objective information (i.e. no subjective information is used to construct the fuzzy numbers as is the case in many studies). This addresses a major concern when using fuzzy numbers for many numerical applications.

We are happy to include the suggested comparison (between the existing FNN method with crisp inputs and the proposed FNN with fuzzy inputs) in the in final revised manuscript. However, for clarification we would like to highlight a few issues related to this comparison:

1. Conceptually, an FNN model with crisp inputs and fuzzy inputs are completely different, making a direct comparison difficult (or at least not straightforward). It is not just a case of comparing error metrics, or percent of data captured within intervals (e.g. as shown in Tables 3 and 4 in the initial manuscript). This is because these two approaches are essentially modelling the system completely differently. In the crisp input case, the input uncertainty is completely ignored even though this data is available. This is essentially making a complex problem less complex by limiting the amount of data that is used in the model. Also, there is no definitive answer to what the crisp input should be: is it the mean daily value? The median value? Or the corresponding value from the fuzzy number at $\mu = 1$? (n.b. this final option is what we have selected for our comparison to allow for the closest approximation between the two approaches). On the other hand, the fuzzy number based input use all the available information (i.e. hourly observations), and condenses it into one fuzzy number. Arguably, in this approach the complexity of the system is not being ignored (by reducing the highly variable/uncertain inputs into crisp, single-values inputs). Thus, any analysis of the performance of these two methods should highlight that the proposed method accomplishes something that the existing method cannot.

2. Currently, they are no suitable performance metrics to compare fuzzy number based models with each other (or for that matter even with other crisp models). While the Nash-Sutcliffe Efficiency (or other similar metrics) can be calculated on an α-cut interval basis, these values do not represent the overall model performance nor does calculating the amount of extreme values within the fuzzy interval. A suitable alternative may be to use the training method of the FNN: to see if the percentage of data captured within each interval is similar (see Table 4 in the manuscript). However, given that this is an optimisation problem, and both methods will have the same tolerances, the result is expected to be the same for both (at least for the training dataset), although the computation time will differ. Thus, the computation time may be used as an effective metric, but even this does not account for the fact that in the proposed method the uncertainty in the inputs is being included in the model, and hence, the extra computational cost is acceptable if the input uncertainty is high (as in the case in this research where flowrate and water temperature are used as inputs).

3. With respect to “more accurate predictions”: please refer to Figure 1 attached to this response that includes a comparison of trend plots for 2004 for crisp and fuzzy inputs, as an example of a comparison between both methods. Using fuzzy inputs means that the output is not necessarily symmetrical about the modal value (as is the case with this crisp inputs). This means that the outputs are more skewed, resulting in more of the “very low DO” values being captured within the predicted intervals. It also shows that the upper limit for fuzzy inputs is much closer to the observations. Thus, from this point of view, the benefits of using fuzzy number inputs are clear, though the accuracy (in this case this

may be the amount of data captured within intervals, which by definition is the same for both) or precision (e.g. width of various α-cut intervals) may not be different. Again, it is worthwhile to point out that in the proposed system more data (due to the construction of fuzzy numbers) was used to train the model, and represents the real uncertainty in the system.

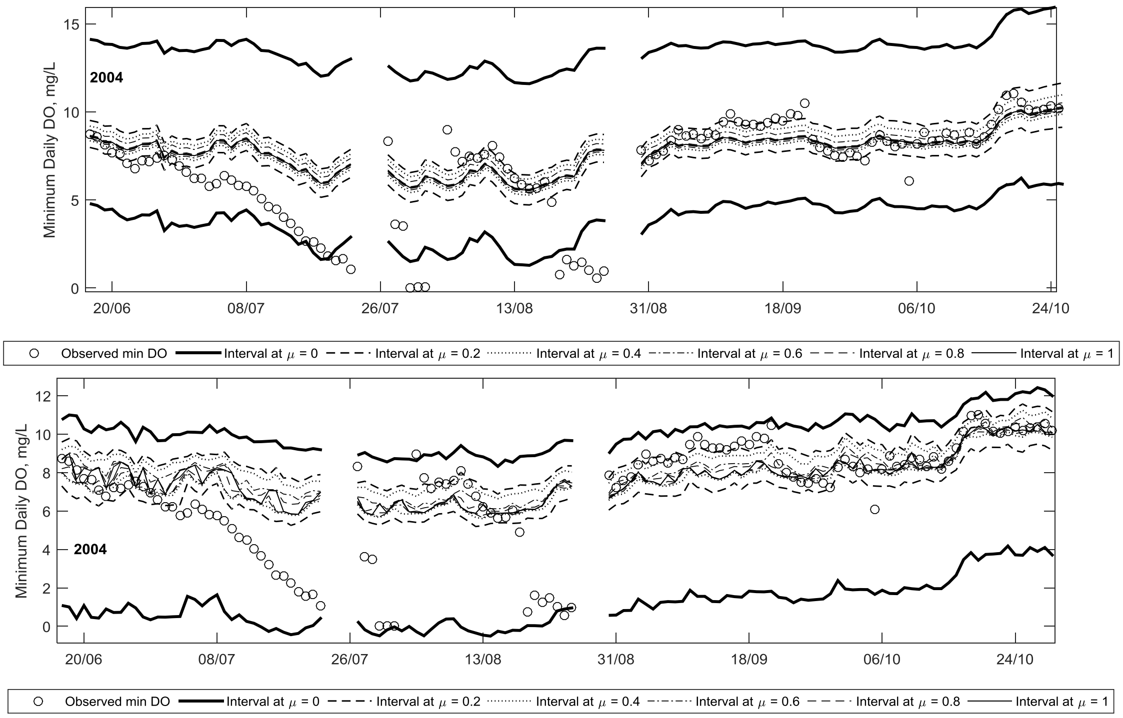

Figure 1: A comparison of trend plots for 2004 using (top) crisp inputs, and (bottom) fuzzy inputs in the FNN

4. With respect to a "reduction in output uncertainty": the uncertainty is reduced because by using fuzzy inputs all the observed data is used to calibrate the model, and this gives a full spectrum of *possible* outcomes. In other words, the uncertainty is lower because all possibilities of the output value have been mapped out using all available input data. In the crisp input case, the uncertainty is by definition higher, since the variability or uncertainty in the input data has not been accounted for (only one crisp value is used). Using this definition of uncertainty means that there is a reduction in output uncertainty (since more information is used), while not necessarily meaning that the predicted intervals are smaller.

5. Based on our experience using fuzzy number based data-driven methods for hydrological and environmental applications, there is still a need and demand to compare new methods (like the one proposed in this research) with existing non-fuzzy methods to provide a baseline reference with other literature (see a brief discussion on page 12331, line 4). Thus, we think it is important to include this comparison in this manuscript so that readers who may be unfamiliar with fuzzy number and possibility theory based methods may be able to directly compare results from this research to other NN based results. Secondly, while we agree that the benefits of the FNN method has been highlighted in previous studies, we would like to highlight that the FNN used in this research uses a different training criteria (i.e. the selection of P_{CI} shown in Table 2 in the manuscript) compared to previous work, so there is a benefit in showing the results for this comparison.

> **RC 3.** "Furthermore, I have some concerns also on the benefits of using the FNN with respect to a deterministic NN. Indeed, the authors state that (page 12351) "the FNN method predicts a probability of low DO (even if it is relatively small) on days when the crisp ANN does not predict a low DO event. This value can be used as a threshold by water resource managers for estimating the risk of low DO. For example, if forecasted water temperature and flow rate are used to predict minimum fuzzy DO using the calibrated model, if the risk of low DO reaches 14 %, the event can be flagged." Capability of the FNN of identifying very low DO values is certainly appreciable, but on the other hand, by looking at figure 6, it seems that most of the predicted fuzzy DO numbers features a support which in some way intersects very low (i.e. <5 mg/l) DO values. In other words, according to the criteria proposed by the authors how many events would be flagged? And, how many of these flagged events were low (i.e. <5 mg/l) observed DO events and how many would have been false alarms?"

First, we would like to highlight that the data has been filtered to include only data from the April to October period for each year (to remove the ice-free period in the river). This means that the entire analysis has been conducted on the time period that is most susceptible to low DO (due to high water temperature). In other words, we are focusing on the most critical time period already. Thus, it is expected that the majority of the days will have some *possibility* of low DO. This phenomenon is correctly reproduced in Figure 6 that shows that indeed there is a possibility (though typically at low membership levels) to predict "very low DO" (< 5mg/L) values.

Second, it is worth noting that using possibility theory means that "something should be possible before it is probable", i.e. Zadeh's consistency principle. Thus, the fact the FNN model predicts a possibility of very low DO does not necessarily mean that there will be a significant or high probability of this event to occur. In fact, this can be seen in the trend plots (Figs. 7 and 8 in the manuscript) that show that the produced fuzzy number membership functions are highly skewed (see page 12348 Line 10 and more examples in Figure 10), i.e. the predictions at the lower limit of the α-cut at $\mu = 0$ are much lower than the rest of the membership function. In the possibility-probability framework adopted in this research (see Sections 2.2 and 2.4 in the manuscript), this means that the highly skewed membership functions translate into very low probability events (based on Equation 20 in the manuscript).

Third, we have identified (page 12348, Line 20) that the 2004 data has contributed to the wide intervals in the predictions. The rapid decrease in DO in the 2004 data (which are likely due to instrument error), along with the optimisation constraints that requires 99.5% of the data to be included in the predicted interval at $\mu = 0$, means that the produced output will include these outliers at the expense of creating wider intervals. However, we have noted that as more data is available and include in the model, the 0.5% of data points that will no longer be captured within the $\mu = 0$ interval are likely to be these outliers.

Finally, we are happy to include in the final revised manuscript a summary of the flagged days using our criteria for low DO as well as those incorrectly identified at the given threshold. We hope that this will satisfy all of the Referee's comments.

Response to Anonymous Referee #2

RC 1. What is the novelty of this study? What do the authors expect international readers (who are not interested in the study region) to learn from reading this paper.

There are three novel contributions presented in this study, and in addition to this an approach for hydrological prediction, uncertainty and risk analysis that can be extended to many other applications. In more specific detail:

- a new method to construct fuzzy numbers from observed environmental and hydrological data is presented. Many fuzzy number based applications suffer from the fact that there is no widely accepted, consistent and objective method to construct fuzzy numbers from observations. We have attempted to address this issue by introducing a new two-step procedure where we first estimate the underlying, unknown probability mass function using a bin-size optimisation procedure, and then use a probability-to-possibility transformation to convert this to fuzzy number membership function. A number of different examples are used to demonstrate the advantage and suitability of this method.
- An existing fuzzy neural network (FNN) method is improved in this paper by proposing the use of possibility theory-based intervals for training the neural network. This replaces a somewhat arbitrary training criteria with a more objective criterion. Specifically, the original FNN uses pre-selected confidence intervals to define the amount of data captured within each fuzzy interval (i.e. α-cut), for example 100% at $\mu = 0$, 99% at $\mu = 0.25$. We use a relationship proposed by Serrurier & Prade (2013) to define the amount of data captured within α-cut. In doing so, the full spectrum of possible values are included in these calculations. This is so that modellers and end-users who are interested in events not included in the original, pre-determined criteria can use an objective (i.e. based on possibility theory) method to design their FNN.
- The existing FNN is further refined by allowing the use of fuzzy inputs, along with the fuzzy weights, biases and outputs. Current methods only allowed crisp (i.e. non-fuzzy inputs) in the FNN. This has significant advantages over current methods, namely that the uncertainty in the input data is also accounted for in predicting DO concentration. In other words, the model output has accounted for the total uncertainty, in the weights and biases, as well as the inputs.
- The approach used in this study (data-driven modelling with fuzzy numbers when the underlying physical system is complex and poorly understood) can be extended to many other applications dealing with water quality in rivers, in flood risk predictions, or hydrological and environmental applications that suffer from similar issues, namely a complex system with many source of uncertainty. International readers will benefit from potentially applying this technique in their own watersheds to improve water quality prediction, and the associated risk analysis presented in this research. As mentioned above, this paper also presents a new method to construct fuzzy numbers that relies on minimal assumptions of the underlying data. This directly addresses a major need in the hydrological community. Lastly, readers will benefit from seeing the refinements to an existing FNN model; these refinements create a more transparent model structure (i.e. objective criteria for training) and include the use of fuzzy inputs (which is necessary in many hydrological cases where input uncertainty is present).

RC 2. The authors didn' t define statistical parameters of input and output variables. The study will make more sense in interpretation of statistical parameters.

We apologize for these omissions. The following will be included in the final revised manuscript (Section 2.1):

"The mean annual water temperature ranged between 9.23 and 13.2◦C, the annual mean flow rate was between 75 and 146 m^3s^{-1}*, and the mean annual minimum daily DO was between 6.89 and 9.54* mgL^{-1}*, for the selected period."*

RC 3. How many datas are used in this study? The authors didn't define to use training datas and test datas this study.

We apologize for this omission as well. A total of 9 years of data was used for this research (from 2004 to 2012); the data were filtered to include data only from the ice-free period (April to October of each year). The total amount of daily data was 1639 days (a yearly breakdown is shown in Table 1 and this will be included in the revised manuscript).

Table 1: Summary of amount of data used from each year

Year	Number of days
2012	206
2011	204
2010	207
2009	96
2008	163
2007	211
2006	209
2005	208
2004	135
Total	**1639**

The amount of data used for training, validation and testing followed a 50–25–25% Split (randomly divided into each section). This is outlined in Section 2.3.3 of the manuscript.

RC 4. The authors didn' t write key board. What are key board for the manuscript?

The key words associated with this manuscript are listed below, and we will include these in the final version**:**

dissolved oxygen; water quality; artificial neural networks; fuzzy numbers; risk analysis; uncertainty

RC 5. Why is not continuous in Figüre 7, 8, and 9.

There are a number of missing data throughout the dataset due to numerous reasons, ranging from sampler error or no data recorded (as received from the data providers Environment Canada or the City of Calgary), or due to the data filters used for reasons highlighted in Section 2.1 (only ice-free period was considered). We ignored all missing data from our analysis. The data that we used was thus for days where error-free data existed for each of the three parameters (flowrate, temperature and DO). Thus, Figures 7, 8 and 9 show some gaps in the trends for days when no data was collected, and hence no subsequent prediction was made.

RC 6. Fuzzy neural networks method is too large, it should be less the part.
RC 7. Results and discussion is too large, The authors should reduce the part.

Thank you for these suggestions, we will endeavour to reduce the length of the final manuscript.