Responses to Dr Sinclair's comments on "Singularity-sensitive gauge-based radar rainfall adjustment methods for urban hydrological applications" by L.-P. Wang et al.

1. General comments:

(0) The authors present a technique for merging weather radar and rain gauge data which aims to preserve the spatially variable character of the radar rainfall while simultaneously reducing the bias between radar and gauges. They assess the technique by evaluating the estimated rainfall against alternative merging/adjustment methods. In addition they evaluate modelled flows from a small urban catchment. My overall impression is that the paper is well written and presents an interesting and useful method of rainfall data merging which is new to me (although it builds on ideas of separating scales of variability in radar rainfall that have been explored previously). I think that the paper can be published essentially as is, subject to some clarifications in the description of the singularity extraction and recovery procedure in sections 2.2 and 2.3.

Answer: We would like to thank the reviewer for taking time reviewing this manuscript and for the positive impression of the proposed data merging method.

(1) It's unclear to me how the initial unknown constant value $c(\mathrm{x})$ and singularity index $\alpha(\mathrm{x})$ are computed from eqn (3) before applying the iterative procedure described. The authors discuss a spatial-scale range on page 11 (line 6 and following), so I assume that the two unknowns are fitted based on data from "a small number of data samples" from radar grid cells in the neighbourhood of each target grid cell? I think this requires clarification.

Answer: The schematic of estimating the initial constant value $c(\mathrm{x})$ and singularity index $\alpha(\mathrm{x})$ is illustrated in Figure **1**. For a given radar pixel location (i.e. the centre of the top plot in Figure **1**), we can calculate the corresponding 'mean' rainfall intensities of this location at different spatial scales (e.g. the rainfall intensities ρ_1, $\rho_2$and ρ_3, respectively at scales ϵ_1, ϵ_2 and ϵ_3). Then, the logarithms of these mean rainfall intensities and the associated spatial scales are compared (the bottom plot in Figure **1**). The constant value $c(\mathrm{x})$ and singularity index $\alpha(\mathrm{x})$ of the dataset can be therefore derived by applying a simple linear regression analysis, where the terms $(\alpha(\mathbf{x}) - E)$ and $\log c(\mathbf{x})$ are respectively the 'slope' and the 'y-intercept' of the regression line. However, it is worth to mention that the estimation of $c(\mathrm{x})$ and $\alpha(\mathrm{x})$ can be trusted only if a well linear relation is observed (i.e. the scaling behaviour is well followed).
In this paper, the scales of interest ranged from 1 to 9 km, so in total five rainfall intensity samples (at scales 1, 3, 5, 7 and 9 km) were used to fit the regression line at each radar pixel location. Working with a small number of samples inevitably increases the uncertainty of the estimation; however, it preserves better local features of the rainfall fields. Based upon our analyses, a well linear behaviour was generally observed within this scale range. In addition, valuable small-scale structures were indeed preserved in the resulting rainfall product. So

we believe the spatial-scale range we selected is a good balance between estimation uncertainty and local feature preservation.

A detailed explanation of computing $c(\mathbf{x})$ and $\alpha(\mathbf{x})$ from eqn (3) can be also found in previous studies (Agterberg, 2012; Chen et al., 2007; Cheng et al., 1994).

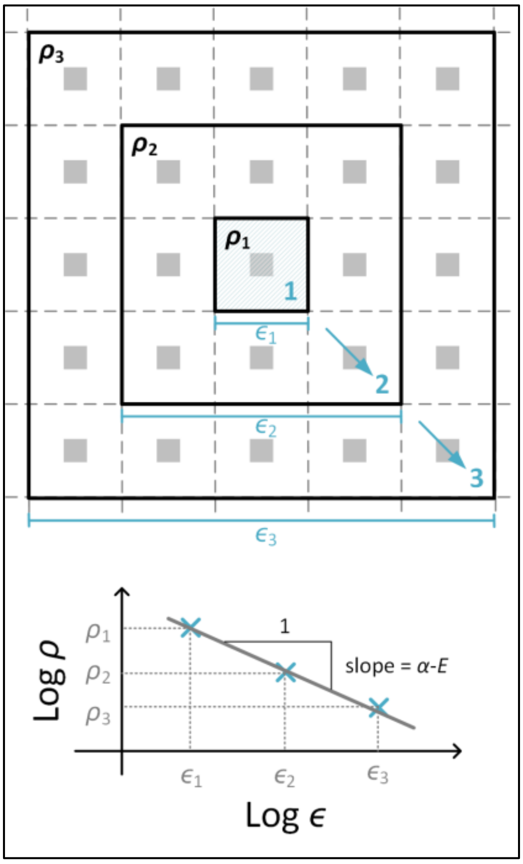

Figure 1: Schematic of the local singularity analysis (Wang et al., 2014).

(2) Is the singularity map simply the difference between the original radar field and the computed c*(x) field? I think that it should be more explicit if this is the case and would appreciate more detail on how it is proportionally applied back to the NS-BAY field.

Answer: The singularity map is composed of the iteratively-derived singularity exponents $\alpha^*(\mathrm{x})$ (see eqn (6) for the definition). Based upon it, a ratio at each radar pixel location can be derived:

$$r(\mathrm{x}) = \epsilon^{\alpha^*(\mathrm{x})-E},$$

which is equal to the difference between original radar field and the computed NSRD (non-singular radar field): $r(\mathrm{x}) = \rho(\mathrm{x}, \epsilon)/c^*(\mathrm{x})$. After the merging between NSRD and BK rain gauge fields is conducted, the ratio at each pixel location is multiplied back to the resulting NS-BAY field to retain the local singularity structures.

The description of this part will be improved in the revised manuscript, and more details will be given.

2. Detailed and editorial comments:

(1) Page 8, line 24 - References error in my version of the PDF.

Answer: This will be corrected in the revised manuscript.

(2) Page 9, line 5 - Agterberg, 2017?

Answer: It should be Agterberg, 2007. This will be corrected in the revised manuscript.

(3) Figure 2 - Circled flow gauges FM * don't match the caption text, missing section number at end of caption.

Answer: These should be gauges FM 3, 10, 14 and 19. This will be corrected in the revised manuscript.

(4) Page 14, line 19 - Only one of the 4 storms chosen in the analysis were not used for calibration of the flow model. Discussion in section 3.3.2 should proceed with this in mind.

Answer: As suggested by the reviewer, we will highlight this fact in the section 3.3.2 that three of the selected storm events in this paper were previously used for model calibration. This indeed caused some differences in the hydraulic performance between the events used and not used for model calibration. As can be seen in Fig 8, the performance measures for the hydraulic outputs of the Storm 4 were in general lower than other three events.

Ideally, the result of this paper could be more convincing if the selected events were not overlapped with those used for the model calibration. However, as specified in the section 3.1.4, the measurements used in this paper were from a flow survey which lasted only three months, so the period that was possible to access this amount of measurements was actually short. Moreover, during this period, only a very limited number of events fulfilled the UK standards for calibration and verification of the urban drainage models, so we had to include the events that were used for model calibration in the analysis.

(5) Page 16, line 20 - How is the areal rain gauge estimate calculated? Is it different from the block-kriged gauge estimate?

Answer: In this paper, we used the average of the 'point' rain gauge records to represent the areal rain gauge estimates. Due to the fact that the rain gauge network used in this study is of high density, we believe that this point average is a good approximation to the areal rain gauge estimate.

In theory, this point average should be equivalent to the areal averages obtained from the block-kriged rain gauge estimates because (block-) kriging is a unbiased estimator. However, according to the statistics given in Table 2, we can see some minor difference between these two estimates. This could be due to the following two reasons:

1. In order to obtain 'block' estimate using point rain gauge records, it is required to conduct 'integration' of semi-variogram model over the block area (i.e. radar pixel square). In this paper, the Gaussian variogram model was employed and its integration

(i.e. the error function) was numerically calculated, so some numerical round-off errors appeared during the block-kriging process.

2. In order not to obtain negative kriged estimate, an addition constraint (i.e. the weights of linear combination cannot be negative values) was imposed into the kriging system. This converted the original kriging into a quadratic programming (optimisation) problem (Barnes and Johnson, 1984). Similarly, this optimisation process was done numerically, which also brought out some numerical round-off errors.

(6) Page 17, lines 8-10 - Is the regression forced to pass through zero?

Answer: The regression was not forced to pass through zero; however, the y-axis intercepts we obtained were usually close to zero.

(7) Fig 7 - It's tough to read the hydrographs, too small, too busy

Answer: In the revised manuscript, the figures of hydrographs will be improved.

(8) Page 30, line 18 - In addition to Wavelets consider data driven techniques like Empirical Mode Decomposition, PCA etc.

Answer: Thank you for the suggestion. These data-driven techniques will be added in the Section 4 (Conclusions and Future Work).

(9) What are the BAY merging artifacts in fig 3 (c1) (e1) from the supplement? Very sharp discontinuities in regular blocks.

Answer: The data used to produce the BAY and NSBAY images in Fig 3 (c1) and (e1) from the supplementary files were generated using RainMusic software tool. Some thresholds (or rules) might be given in the tool for the numerically stable purpose, and consequently caused some artefacts in the images. Because it is a commercial software, we had no access to the source code to find more evidences of this.

Nonetheless, throughout the period of this research, a C/C++ tool has been implemented to replicate the function of the RainMusic but was customised for urban applications. As can be seen from the images produced by the new tool (see Fig 6 in the manuscript), the problem of generating artefacts has been resolved.

References

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