Interactive comment on "A scaling approach to Budyko's framework and the complementary relationship of evapotranspiration in humid environments: case study of the Amazon River basin" by A. M. Carmona et al.

Color guide: Referee's comments appear in **black** font, the lines from the manuscript in question appear in **blue** and the answers to the observations appear in **red**.

Answers to Anonymous Referee #2

This paper by A. M. Carmona et al. provides a new perspective on Budyko framework and finds the physical inconsistency of the Budyko curve for humid environments using Ω=E/Ep and Φ=Ep/P. A simple but new scaling approach was proposed to overcome this inconsistency. The results are important and interesting. This manuscript has been well written and I recommend to accept after including comments listed as follows:

1. I found Figure 3 not very helpful as one could not justify how it behaves.

   Figure 3 is included in the manuscript since it presents the three dimensional view of the proposed state space between Φ=Ep/P, Ψ=E/P, and Ω=E/Ep. Data from the United States of America (USA), China, and FAO agro-climatic stations were chosen to prove that there is a surface that can capture the data using a wide range of climates and both in-situ stations and catchments. However, even though this surface comes from a valid mathematical equation, figure 3 shows that it does not necessarily guarantee that all parts of the surface are physically feasible in nature, as explained in the manuscript (section 3.1). Thus, it is this figure that provided the motivation to further explore the bi-dimensional projections of our 3-D space. In conclusion, we are positive that figure 3 is necessary, not only because it shows for the first time the 3-D perspective of our approach, but also because it is through one of the 2-D projections of this figure, the one that captures the physical inconsistency of Budyko-type equations, which provided the main motivation for our study.

2. Line 3-4 in Page 10532, I would not agree. In Yang et al. 2008, it was assumed that the *P* and *Ep* are independent, which is obviously not true in reality. But to my best knowledge, in Fu's derivation, there is no such assumption. That is the reason why Sun (2007) and Yang et al. (2006) used mathematical derivative (your Fig.8) based on Fu's equation instead of using Choudhury 1999 equation to reconcile the complementary relationship and the Budyko curve. Therefore I would suggest to use Fu's equation when expressing Fig.8 for theoretical consistency. (Sun, F.: Study on Watershed

Evapotranspiraiton based on the Budyko Hypothesis, Doctor of Engineering, Tsinghua University, 147 pp., 2007).

"So far, in the analytical deduction of Budyko type equations, P and E_p have been considered completely independent (Fu, 1981; Yang et al., 2008) and thus the terms $\partial P/\partial Ep$ and $\partial Ep/\partial P$ have been neglected".

The reviewer is right. For this reason, we will specify that Yang et al.'s equation is the one that has been derived under the assumption that that $\partial P/\partial Ep = 0$. Accordingly, Line 15 p6, has been reframed to: "Nevertheless, all of these studies have focused on the two-dimensional formulation of the Budyko hypothesis, assuming that E, P and E_p (but mostly P and E_p) are independent on each other. For example, the analytical derivation of the Budyko equation by Yang et al., (2008) (Eq. 4) is carried out under the assumption that $\partial P/\partial Ep$=0. Such an assumption is questionable, given the well-known complementary relationship of evapotranspiration (Bouchet, 1963; Morton, 1983; Hobbins et al., 2001; Xu & Singh, 2005; Szilagyi & Jozsa, 2009; Han et al., 2014 and Lintner et al. 2015), but also having in mind the important role of evapotranspiration in the recycling of precipitation (Shuttleworth, 1988; Elthair & Brass 1994; Dominguez et al., 2006; Zemp et al 2014)".

Also, in line 3-4 in Page 10532 the citation of Fu (1981) will be removed, although it should be pointed out that Yang et al. (2006), who used Fu's equation, did ignore the terms $\partial P/\partial Ep$ and $\partial Ep/\partial P$ when interpreting the complementary relationship in non-humid environments based on the Budyko hypothesis.

In addition, in order to fully address the reviewer's comment, new calculations were made using Fu's equation (Fu, 1981). The main results of the study remained the same, as shown in the figures presented below. On the one hand, Fu's equation (green thick line) also theoretically requires that when $\Omega \rightarrow 1$ (very humid environments), Φ=0 (Fig. 1 below and Fig. 5 in the manuscript). Thus, it also entails the physical inconsistency pointed out in section **2.1.2**. Furthermore, if Fu's equation is used instead of Yang et al.'s equation for the complementary relationship, as suggested by the reviewer (Fig. 2 below and Fig. 8 in the manuscript) the results also remain the same, except for the value of the parameter ($n \neq \omega$). Specifically, fig.2 shows the theoretical relationships between $\partial E/\partial Ep$ and $\partial E/\partial P$ with Ep/P using the differential forms of the equations proposed by Yang et al. (2008), Fu (1981) and our power law relationship, for different values of the parameters n, ω and e. This figure shows that for small values of Ep/P, the value of $\partial E/\partial Ep$ is larger compared to the value of $\partial E/\partial P$ which means that in humid catchments changes in E are mostly governed by changes in Ep rather than in P. Also, it can be seen how traditional Budyko-type equations (Yang et al., 2008 and Fu, 1981) suggest that for very humid environments ($Ep/P \rightarrow 0$) changes in E are equal to changes in E_p ($\partial E/\partial Ep$=1), which is not necessarily true (Granger, 1989; Kahler and Brutsaert, 2006; Szilagyi, 2007, Lintner et al 2015). However, our scaling approach allows E to change more than *Ep*, which is consistent with the asymmetrical nature of the complementary relationship.

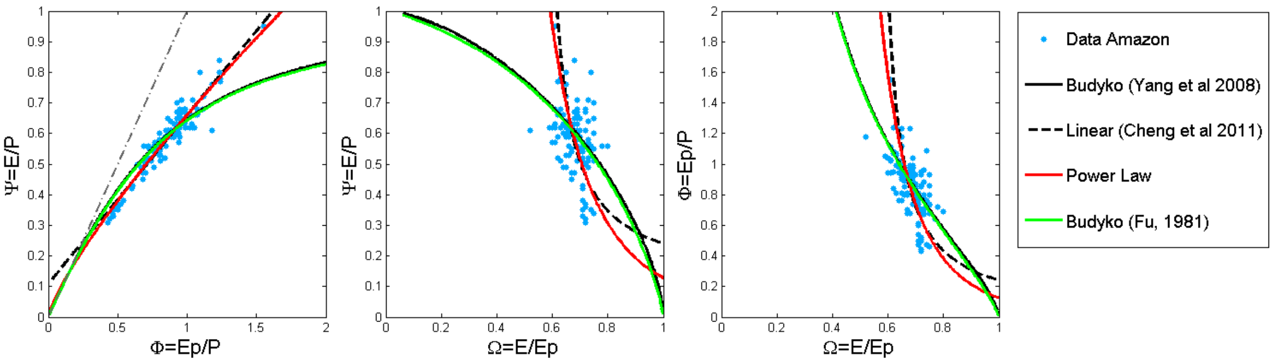

Figure 1. Bi-dimensional projections of the 3-D state space for the Amazon River basin

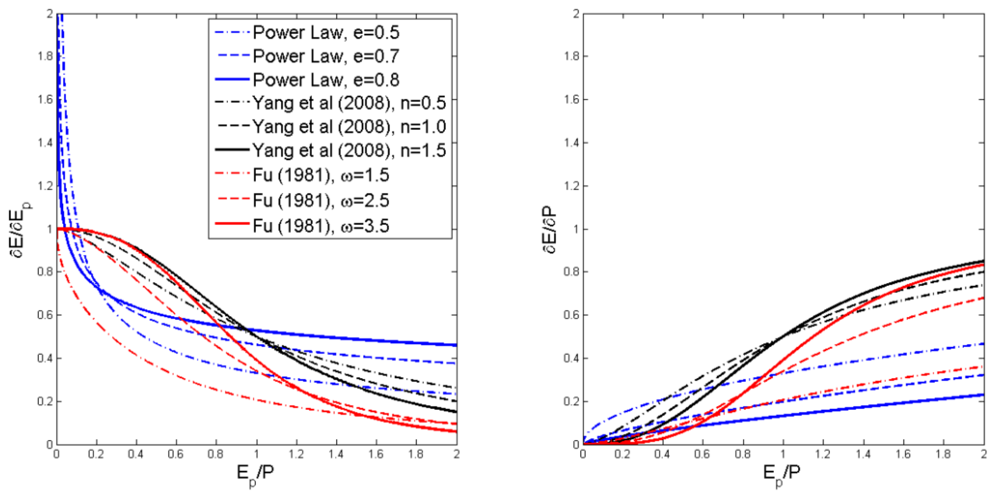

Figure 2. Complementary relationship as presented by Yang et al 2006.

3. Line 18-22 in Page 10519, again I don't agree with that. I don't think anyone could ever demonstrate there is a unique solution. Otherwise how to explain there are many Budyko curves.

   "In particular, Yang et al. (2008) demonstrated analytically that there is a unique solution for the set of partial differential equations representing the coupled water and energy balances in catchments…"

   The reviewer has a valid point: there are many Budyko curves and many equations that represent them. However, Yang et al. (2008) do claim that their equation is a unique solution for the set of partial differential equations, as specified in their paper: "This paper aims to prove the existence of a unique solution to the mean annual water-energy balance equation and to find the analytical solution under general conditions" and later in their conclusions: “Through dimensional analysis and mathematical reasoning, this paper mathematically derived a general solution to the mean annual water-energy balance equation, and proved its uniqueness”.

Nevertheless, and given that we also aim at proposing an alternative equation for the Budyko hypothesis, this paragraph (Line 18-22) will be changed to: “In particular, Yang et al. (2008) mathematically derived a general solution for the set of partial differential equations representing the coupled water and energy balances in catchments…”

4. In terms of the complementary relationship, I think it is more about $\partial Ep/\partial E$ rather than $\partial E/\partial Ep$.

   Both expressions ($\partial E_p/\partial E$ and $\partial E/\partial E_p$) denote changes in one variable given changes in the other. In addition, these changes are neither linear nor straight forward. This means that E_p changes because E changes, but also E changes because E_p changes, that is what the complementary relationship is all about. Nevertheless, previous studies (Granger, 1989 and Yang et al., 2006) have dealt with $\partial E/\partial E_p$ and thus, for comparison purposes (mainly with Yang et al. (2006)) we will continue to analyze this expression instead of the one suggested by the reviewer.

5. Page 10536 Move the description of topography, groundwater levels and vegetation in Section 3.3.4 to Section 2.2 Data sets.

   Thank you for this relevant suggestion. The description of the topography, groundwater levels and vegetation used for section 3.3.4 will be moved to section 2.2.

6. The focus of study area in the manuscript should be humid environment. There is a jump between using the global agro-climatic stations or the data of the arid area in US and China and humid environment.

   We respectfully disagree with the reviewer. The US and China datasets were used as mentioned previously (comment #1) to depict the 3-dimensional state space for a wide range of climates and environments and not just for the humid ones. Besides, these datasets allowed us to identify the physical inconsistency of Budyko-type equations for humid environments, and for this reason we later on focus our study on the Amazon River basin. Nevertheless, at the end of the paper (section 3.3.5) we attempt at generalizing our scaling approach. That is, we show that although the power law equation was derived for humid environments (Amazonia) and suit them better, it could also be used for other catchments such as those in the USA and China.