Interactive comment on "A scaling approach to Budyko's framework and the complementary relationship of evapotranspiration in humid environments: case study of the Amazon River basin" by A. M. Carmona et al.

Color guide: Referee's comments appear in **black** font, the lines from the manuscript in question appear in **blue** and the answers to the observations appear in **red**.

Answers to Anonymous Referee #1

The manuscript of Carmona et al presents an interesting perspective on the Budyko curve. I liked the idea of looking at covariations across the three variables and the discussion on the complementary relationship within this framework. The paper is of potential interest to HESS but needs to be made clearer and easier to read. I think a lot of the discussion cold be reduced to avoid some circular discussion. Nonetheless I want to reiterate my interest so that the authors do not feel discouraged. Also the authors should discuss the results of Lintner et al. 2015 which discusses the role of the complementary relationship and Budyko curve, published in HESS and on the Amazon!

We would like to thank you for taking the time to go over our manuscript and for the constructive comments. An effort will be made towards reducing the discussion so it becomes clearer and less repetitive.

Line 1 3D is unclear, I think you should maybe give another name because we think in terms of physical x,y, z space.

"This paper studies a 3-D generalization of Budyko's framework designed to capture the mutual interdependence among long-term mean actual evapotranspiration (*E*), potential evapotranspiration (*Ep*) and precipitation (*P*)"

Thank you for your observation. Accordingly, this sentence can be reframed as: "This paper studies a three-dimensional state space representation of Budyko's framework designed to capture the mutual interdependence along the three dimensions of long-term mean actual evapotranspiration (*E*), potential evapotranspiration (*Ep*) and precipitation (*P*)"

In the abstract and introduction you should refer to Lintner et al who found things along the same lines as what you found and the fact that the complementary relationship and Budyko curve in very humid catchments are modified.

Thank you for bringing this manuscript to our attention. As far as we know, references are not allowed in the abstract, but it will definitely be included in the introduction. Specifically in Line 18, page 6.

Line 15 p6 the statement on independence is incorrect: what you mean is that *ET* and *Ep* have been assumed independent again it is clear that the authors should refer to Lintner et al line 20 as it is very close to the discussion of the paper and uses the complementary relationship as well, as the problem on the wet end was mentioned in that paper.

"Nevertheless, all of these studies have focused on the bi-dimensional approach of the Budyko hypothesis, assuming that *P*, *E*, and *Ep* are independent on each other"

What we meant by this sentence is that in general, studies on the Budyko framework have been carried out assuming that *P* and E_p are independent on each other. For example, the analytical derivation of the Budyko equation by Yang et al., (2008) was carried out under the assumption that $\partial P/\partial Ep = 0$. On the contrary, complementary relationship studies (such as the one by Lintner et al. 2015 and others) show that that *P* and E_p are indeed connected via actual evapotranspiration.

This paragraph has been reframed as: "Nevertheless, all of these studies have focused on the two dimensional approach of the Budyko hypothesis, assuming that *E*, *P* and E_p (but mostly *P* and *Ep*) are independent on each other. For example, the analytical derivation of the Budyko equation by Yang et al. (2008) (Eq. 4) was carried out under the assumption that $\partial P/\partial Ep = 0$. Such an assumption is questionable given the well-known complementary relationship of evapotranspiration (Bouchet, 1963; Morton, 1983; Hobbins et al., 2001; Xu & Singh, 2005; Szilagyi & Jozsa, 2009; Han et al., 2014 and Lintner et al. 2015), but also having in mind the important role of evapotranspiration in the recycling of precipitation (Shuttleworth, 1988; Elthair & Brass 1994; Dominguez et al., 2006; Zemp et al 2014)".

Line 15 p7: study THE water

"Motivated by Budyko's coupling between water and energy balances and considering the mutual inter-dependence between *E*, *Ep* and *P*, we propose to study water and energy balances on a 3-D space defined by three dimensionless parameters: $\Phi = Ep/P$, $\Psi = E/P$, and $\Omega = E/Ep$"

The paragraph has been modified as: "Motivated by Budyko's coupling between the water and energy balances and considering the mutual inter-dependence between *E*, *Ep* and *P*, we propose to organize the analysis within a 3-dimensional space defined by three dimensionless variables: $\Phi = Ep/P$, $\Psi = E/P$, and $\Omega = E/Ep$"

Line 23: again you should mention Lintner et al. 2015

"Briefly, this approach combines the water balance from Budyko's perspective with the energy balance from the perspective of the complementary relationship of evapotranspiration".

We agree. Both the studies by Lintner et al (2015) and Yang et al., (2006) bring together both perspectives (Budyko hypothesis + complementary relationship). Thus both studies will be mentioned as follows in line 23, P7:

"Briefly, this approach combines the analysis of annual water balance based on Budyko's perspective with the energy balance from the perspective of the complementary relationship of evapotranspiration, as has also been attempted previously by Yang et al. (2006) and Lintner et al. (2015)".

Before section 2.2: I have difficulties with the theoretical argument (the limit is correct though) because it is just a curve fitting at the end of the day and of course no places on earth as a 0 aridity index. You should reframe your argument.

"Section 2.1.2 A physical inconsistency of Budyko-type equations"

Following your observation, this section will be reframed for clarity. Previously we were attempting to show mathematically, that for very humid environments theoretical Budyko-type equations force the aridity index to be equal to zero using the limit of Φ→0, which for us implies a physical impossibility. However, it has been brought to our attention that it would be better to use the limit of the inverse function, that is, the limit of Ω→1. For this reason this section will be changed as follows:

2.1.2 A physical inconsistency of Budyko-type equations

The proposed 3-dimensional state space and its 2-dimensional projections (Ψ vs. Φ, Ψ vs. Ω and Φ vs. Ω) provide an interesting setting to test for the physical soundness of Budyko's original hypothesis. In terms of our dimensionless variables, Budyko's Eq. (3) and Yang et al.'s Eq. (4) can be written, respectively, as,

$$\Psi = [\Phi tanh(\Phi^{-1})(1-e^{-\Phi})]^{\frac{1}{2}} \quad \textbf{(5)}$$

and

$$\Psi = (1+(\Phi)^{-n})^{\frac{-1}{n}} \quad \textbf{(6)}$$

Using Equation (6), the relationship between $\Omega = E/E_p$ and $\Phi = E_p/P$ can be expressed as:

$$\Phi = \left(\frac{1}{\Omega^n} - 1\right)^{1/n} \quad \textbf{(7)}$$

Analytically, <u>for very humid environments, if $\Omega \rightarrow 1$</u> it can be demonstrated that:

$$\lim_{\Omega\rightarrow 1}\left(\frac{1}{\Omega^n} - 1\right)^{1/n} = 0$$

The same result is obtained if instead of Eq. (6) we use Eq. (5). Thus, the Budyko-type equations mathematically require that for humid environments, when $\Omega \rightarrow 1$, $\Phi=0$. This theoretical prediction of Budyko's framework entails a physical inconsistency in the relationship between $\Omega=E/Ep$ and $\Phi=Ep/P$, i.e., in the relationship between the partitioning of energy and the aridity index. Budyko-type equations (Eq. 7) suggest two possibilities for the case of $\Phi=Ep/P=0$: (*i*) that *Ep* can be zero (negligible atmospheric demand), or (*ii*) that *P* approaches infinity. However, even in the most humid regions of the world (i.e., Lloró, Colombia or Cherrapunji, India) there is always a potential for evapotranspiration, and even though rainfall is very high (up to 12,000-13,000 mm yr^{-1}) it is never infinite. We consider this to be a physical inconsistency of Budyko's theoretical framework for humid environments. Therefore, a different approach is in order: this provides the main motivation for this study.

Line 20 p9: the data

"Data used for this study consisted of 3123 agro-climatic stations from the CLIMWAT 20 2.0 database, a joint product of the Water Development and Management Unit and…"

Both "Data" and "The data" are grammatically correct, thus this line will remain unchanged.

Line 10 p10: you didn't mention storage

"*E* was also calculated using Budyko's Eq. (3) with data and estimates of mean annual P and Ep…"

For the data set provided by FAO no information about soil moisture was available, and no water balance equation was used since these are not catchments but in-situ "point" data. For this reason, for the estimation of annual *E*, only data pertaining to *P* and *Ep* were used.

Line 10 p11: why is this method the most appropriate (data limitation)

"*Ep* was calculated using the Hargreaves equation (Hargreaves et al., 1985) following Trabucco and Zomer (2009) and Vallejo-Bernal et al. (2015), who showed that for South America, particularly for the Amazon River basin, this model based on temperature and extra-terrestrial radiation is one of the most appropriate methods to estimate Ep"

Indeed, data limitation is one of the reasons why Hargreaves' equation is the most appropriate to estimate E_p in the Amazon River basin; nonetheless, it is not the only one. It has been shown that estimates of E_p from databases such as the Climatic Research Unit (CRU) underestimate E_p, as evidenced in the annual regime curves of E vs. E_p (Vallejo-Bernal et al., in preparation).

Equation 9: what is the advantage of the 3-D perspective for the curve fitting?

Equation 9: $\Psi = k\Phi^e$

On the one hand, the power law relationship provides better fits to the Budyko curves for catchments in the Amazon River basin than the traditional (Yang et al., 2008) and non-traditional (Cheng et al., 2011) Budyko-type equations. In addition, the advantage of this power law is revealed later on in the manuscript, when interannual variability is analyzed. Given the dependent nature of the considered variables, we demonstrate that the coefficient in the power law (*k*) is closely related to the partitioning of energy via evapotranspiration, that is, in terms of Ω in each sub-catchment ($\Omega = 0.994k$, $R^2 = 0.95$). It should also be pointed out that Ω is a variable from the 3-D space that does not appear in the power law. For this reason, we believe that our scaling approach (Eq. 9) implicitly incorporates the complementary relationship of evapotranspiration into the formulations of the Budyko curve. Thus, the parameter *k* could be deemed a sign of energy limitations in a catchment.

For clarity, the explanation presented above will be included in the manuscript.

Line 23 p14: remove

"Nevertheless, the linear relationship does not fully comply with the energy limit in the Budyko curve as can be seen in Fig. 5a".

We kindly ask the reviewer to explain this comment with more details so we can understand better why this line should be removed.

Fully remove line 24-25: it can be seen..., obvious it is fair to mention that you have two parameters

"In fact, it can be seen that in order to fulfill the energy limit, its intercept would have to be restricted to b = 0".

Again, we would like the reviewer to please expand his observation so we can understand his point of view regarding this line and thus, attend his observation. We have trouble following his line of thinking.

Line 6 p17: but Budyko only applies on long time scales, please justify

"In contrast, the scatter present in the year to year variations does affect the performance of Eq. (6), as reflected in a decrease of R^2"

Not necessarily. Even though Budyko was first designed for the long-term time scale (long mean annual water balance) it has been successfully used to assess the interannual variability of coupled water and energy balances, such as in the studies by Koster and

Suarez (1999), Sankarasubramanian and Vogel (2002), Yang et al. (2007), Wang et al. (2009), Potter and Zhang (2009), Cheng et al. (2011), Chen et al. (2013), Carmona et al (2014), among others.

Line 10 p19: again mention Lintner et al. 2015 who discusses this point over the Amazon further

"Also, it can be seen in Fig. 8a how Budyko-type equations suggest that for very humid environments ($Ep/P \rightarrow 0$) changes in E are equal to changes in Ep ($\partial E/\partial Ep = 1$), which is not necessarily true (Granger, 1989; Kahler and Brutsaert, 2006; Szilagyi, 2007)".

In Lintner et al.'s (2015) paper entitled "The Budyko and complementary relationships in an idealized model of large-scale land–atmosphere coupling", they do mention environments with increased precipitation and soil moisture, even though the Amazon basin was not specifically mentioned. Nevertheless, they do reinforce the fact that the complementary relationship is naturally asymmetrical, which they attribute to the dependence of this relationship to the Clausius-Clapeyron equation. Also, and very interestingly, they state that under a warming climate, the complementary relationship is expected to become more asymmetric as higher values of the slope imply a larger change in potential evaporation for a given change in evapotranspiration. For this reason, this paper (Lintner et al 2015) will be cited in Line 10, p. 19.

P25: again the justification on Ep/P goes to zero is a bit sketchy please make it cleaner

"We demonstrate analytically that the Budyko framework is unable to capture the physical limits of the relation Ω vs. Φ in humid environments, owing to the unfeasibility of $Ep/P \rightarrow 0$ at $E/Ep = 1$. This means that if Budyko-type equations are used to study the relationship between Ω and Φ a physical inconsistency is found, since Budyko-type equations suggest two things: (*i*) that *Ep* can be zero (non-existent atmospheric demand) or (*ii*) that *P* tends to infinity. However, even for the most humid regions of the world there is always non-negligible atmospheric demand and even though rainfall can be high it is never infinite".

According to one of the previous comments and the changes that were made in Section 2.1.2, this paragraph will be re-written as follows:

"By studying the mathematical limits of traditional Budyko-type equations (Eq. 3 and Eq. 4) we demonstrate that these relationships are unable to capture the physical nature of water balance in humid environments. This is because they theoretically require that when $\Omega \rightarrow 1$ (very humid environments), $\Phi = 0$. We believe this is not possible, given that (*i*) that *Ep* cannot be zero (non-existent atmospheric demand) and (*ii*) *P* is not infinite".