Referee Report on "Technical Note: Three-dimensional transient groundwater flow due to localized recharge with an arbitrary transient rate in unconfined aquifers" by C.-H. Chang, C.-S. Huang, and H.D. Yeh, offered for publication to HESS

4th January 2016

1 General comments

The authors have treated a difficult and complicated hydrological problem. The solution methods are of a standard mathematical nature, but by no means trivial. Their final soluton becomes a triple sum where zeros of transcedental equations have to be calculated. Moreover, the factors for the horizontal contributions $F_x(\alpha_m, \bar{x})$ and $F_y(\beta_n, \bar{y})$ are independent, but the term $\Phi(\alpha_m, \beta_n, \bar{z}, \bar{t})$ depends on α_m and β_n by means of the variable $f = \alpha_m^2 + \kappa_z \beta_n^2$. This analytical solution belongs to Class 2 according the classification in Veling and Maas (2009).

The style of the paper is straightforward and the derivation in the Appendix is intelligible.

In their sensitivity analysis the authors give useful dimensionless expressions with criteria when to use which approximation for given circumstances and when an approximation is not appropriate. Their sensitivity analysis could be extended even further by treating the boundary conditions in a different way.

The authors do not give much information about the numerical evaluation of the found analytical expression other than some details how the zeros of the transcedental equations have been found. A validation of solution has not been supplied other than comparisons with other published solutions of simpler problems. It is possible to make choices for the parameters such that this solution should be equal to earlier published ones (*e.g.* the recharge area is the whole aquifer). In that way an independent, partial check of this solution could be possible.

Can the authors give information about the performance of their code (calculation times, convergence properties of the triple sums) and about the availability?

The general impression is a good piece of technical work based on well-established equations and boundary conditions for such cases. This solution based on the inclusion of equation (8) (time dependent first order free surface equation) for the chosen finite aquifer with a finite recharge domain seems to be new.

2 Some specific remarks

Page 12249, l. 9: No mention is made of the work of Bruggeman (1999, 360 BIII-6, from p. 321) for comparable solutions in a finite strip .

Page 12252, l. 24: The introduction of the distance d is unclear in the case that the location of the observation well has coordinates (x_w, y_w) with $x_w > x_1 + a$, $y_w > y_1 + b$ or $x_w > x_1 + a$, $y_w < y_1$ or $x_w < x_1$, $y_w > y_1 + b$ or $x_w < x_1$, $y_w < y_1$. What should be the distance in such cases:

$$d = \min(|x_w - x - a|, |y_w - y_1 - b|, |x_w - x_1|, |y_w - y_1|)$$

or

$$d = \min \left(\sqrt{(x_w - (x_1 + a))^2 + (y_w - (y_1 + b))^2}, \sqrt{(x_w - (x_1 + a))^2 + (y_w - y_1)^2}, \right.$$
$$\left. \sqrt{(x_w - x_1)^2 + (y_w - (y_1 + b))^2}, \sqrt{(x_w - x_1)^2 + (y_w - y_1)^2} \right) \quad ?$$

Page 12254, l. 4: The symbol l for the recharge rate has been introduced earlier for the width in the x-direction of the rectangular aquifer.

Page 12254: l. 12: Remark the way of scaling: with d in the horizontal plane and with B in the vertial plane.

Page 12257, l. 1: It should be better to label f as $f_{m,n}$ to make clear the dependency on α_m and β_n. In fact, also λ_j should be better $\lambda_{j,m,n}$. In the current presentation the solution looks simpler that it is really!

Page 12258, l. 20: More explanation is needed for formula (23); specify a reference here for the use of Duhamel's Principle. Very likely, in the denominator ξ should be $\xi_t(0)$.

Page 12258, after Section 3.2: Some information could be given about the way the authors have treated the triple sum numerically. Did they use convergence accelerators?

Page 12261, l. 5: The mention of "Fig. 2" does not seem to be correct.

Page 12264, l. 18: The sensitivity analysis w.r.t. a: have the authors taken in consideration that by changing a also the scaling variable d changes too by the chosen location of the observation points/wells A and B?

3 Some minor remarks

Page 12248, l. 24: Change "the" into "a".

Page 12257, l. 1, formula (18o): It is more natural to introduce variables before and not after the introduction of the formulas where they are used explicitly. The same applies to formulas (18k) and (18m). As exhibited here in this paper the distance between use and definition is rather great.

Page 12257, l. 11: Change "first and second" into " second and third".

Page 12257, l. 12: Change "third" into "first".

Page 12260, l. 7: Very likely, the authors mean $10^{-3}P_c$ in stead of $10^{-3}\Delta P_c$.

Page 12264, l. 10: Change "squire" into "square".

Page 12271:, l. 3: Change "cauchy" into "Cauchy".

4 References

References

G. A. Bruggeman. *Analytical Solutions of Geohydrological Problems.* Developments in Water Science, nr. 46. Elsevier, Amsterdam, 1999.

E. J. M. Veling and C. Maas. Strategy for solving semi-analytically three-dimensional transient flow in a coupled N-layer aquifer system. *Journal of Engineering Mathematics*, 64(2):145–161, doi:10.1007/s10665–008–9256–9, 2009.